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PRACTICAL ELECTRICITY

AND

MAGNETISM

PHYSICAL AND ELECTRICAL ENGINEERING LABORATORY MANUALS

VOL. II.

PRACTICAL ELECTRICITY

AND

MAGNETISM

BY

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PREFACE TO THE SERIES

In bringing before the public these laboratory manuals, it has been the object of the authors to provide a course of instruction for carrying out a progressive series of experiments in Physics and Electrical Engineering, arranged so that the usual apparatus at the disposal of a laboratory, though not especially designed for any particular experiment, may nevertheless be used with advantage in a variety of ways.

Able courses of instruction in experimental work have already appeared, and have done a vast amount of good; but as these usually require expensive apparatus made and arranged for each experiment, they have not become so generally useful as otherwise might have been the case, especially in such instances where the scope of the work undertaken, precludes all possibility of separate and special apparatus being provided for each independent experiment. In technical work this is more particularly the case, seeing that in actual practice a set of instruments must be put to very divergent uses, in order that results may be obtained quickly and with sufficient accuracy for commercial work. This use of apparatus for ends not specially intended, is in itself a training of considerable importance, to any student who will afterwards, in his daily life, have to so adapt for different purposes, such instruments as may be available at the time.

It has not, however, been forgotten that most apparatus thus used is too often placed under circumstances inconsistent with accurate work, and to this end very careful instructions will be found in the more advanced volumes, for guarding against such disturbing influences as time, situation, temperature, and magnetic forces; this being too often neglected in general laboratory and commercial work, it being frequently forgotten that a set of apparatus arranged for a particular test, is sometimes not only a centre of disturbance itself, but is liable to disturbance from other apparatus in use in its neighbourhood. The precautions thus indicated are of especial importance in technical work, where the disturbing influences are of such a powerful nature as may be found in engine-rooms and dynamo-houses, where high and varying temperatures and leakage magnetic lines are very prevalent.

Another way in which an alteration has been effected is to, as far as possible arrange experiments where a student *working alone*, may be able to obtain satisfactory results. In a large proportion of the existing laboratory manuals, groups of students are expected to work together; but a number of years of practical experience with students of all kinds has convinced the authors, that habits of individual accuracy and self-reliance can only be acquired by separate and unaided work. Of course, in advanced work it is often necessary, that two or more students should work together, in order that simultaneous observations should be taken; but it is most desirable that students so combined, should have had considerable individual training and experience.

It is particularly desirable that every experiment should be repeated until a set of consistent results have been obtained. In this way only, can experience and accuracy be acquired.

This series is divided into two courses, to which Vol. I. is a general introduction. The future volumes will contain the

advanced work in both Physics and Electrical Engineering, and it is intended, that students should be able to take either the one or the other, thus specializing in Physics or Electrical Engineering; but they may combine the two courses with advantage, where time will admit.

J. HENDERSON.
S. JOYCE.

MANCHESTER, 1895.

PREFACE

THE present volume, which forms the second of the series of Physical and Electrical Engineering Laboratory Manuals, has been entirely devoted to practical work in electricity and magnetism, as it was found that the importance of this department of physical work demanded a volume to itself.

The arrangement and treatment of the subject-matter in the present volume is essentially different from that adopted in Vol. I., since it was found that, although it was extremely desirable to present instructions to elementary students in as brief and concise a manner as possible, it was impossible to follow the same method with more advanced work. A considerable amount of explanatory matter has therefore been added to the descriptions of the various measurements. In very few cases have proofs of formulæ assumed been given as it was found that this would lead to the book assuming unwieldly dimensions; besides, mathematical investigations do not come within the scope of a practical manual, but of a theoretical treatise. In order, however, to assist the student as much as possible in this direction, references are given to sources where the required proofs may be obtained.

One of the principal features of the book, and one which the author hopes will commend itself to others engaged in writing scientific works, is the list, given at the end of each chapter, of references to the more important original papers published in

the various scientific periodicals, which bear on the subject-matter of the chapter. So much valuable matter is buried in the "Proceedings" and "Transactions" of various learned societies, and in monthly and weekly scientific publications, which in many cases are only indexed for each volume, there being no general index, that it becomes a very formidable task, and one involving a great expenditure of time, to ransack perhaps many volumes of back numbers in order to get a single paper.

Feeling the want of such references himself, and knowing that others were necessarily in the same position, the author, after considerable labour, has compiled lists of references to the more important papers on subjects treated of in the present volume which have appeared in—

(1) The Philosophical Transactions of the Royal Society of London.

(2) The Proceedings of the Royal Society of London.

(3) The Transactions of the Royal Society of Edinburgh.

(4) The Proceedings of the Royal Society of Edinburgh.

(5) The Philosophical Magazine.

(6) The Electrician.

Several references have also been made to "Nature," the "B.A. Reports," the "Journal of the Institution of Electrical Engineers," and to a few foreign scientific periodicals. In many cases, where the same paper appears in several of the above publications, more than one reference is given, as the student may not have access to all the above-mentioned books.

These lists are not to be looked upon as giving all the references to the subjects with which they deal, but they will supply the student with a ground-work to start upon, and if he consults the papers mentioned he will in them find the further references which he requires. It is also hoped that

these lists will be found useful by students and others engaged in original research.

As regards the arrangement of the material in the present volume, the measurement of resistance is dealt with first, on account of its great importance. At the end of this chapter there is a very brief account of some of the methods of measuring resistance in absolute units; this is not given with the hope of the student making highly accurate determinations of the length of the mercury column which represents the practical unit of resistance, but simply to familiarize him with the methods of making such determinations, and also as an exercise in absolute measurement, albeit his results may not be correct to one or even ten parts in ten thousand.

In choosing methods for the various measurements it has been the author's aim to take only the best suited for the purpose, and the book does not pretend to be a dictionary of *possible* methods, or, as is sometimes the case with practical handbooks on physics, to show how many different experiments *could be performed* with any given piece of apparatus.

Figures and diagrams of representative pieces of apparatus have been included, and in a few cases they are taken from photographs of apparatus in the author's own laboratory.

The last chapter has been devoted to a description of experiments with electro-magnetic waves, this the author believes is also new to practical books on electricity, but the rapid growth of this section of the subject, and the many possible practical applications of electro-magnetic phenomena in the near future, is sufficient justification for its inclusion.

Tables of logarithms, etc., and of physical constants have been added for the convenience of the student.

The author takes this opportunity of acknowledging his indebtedness to Messrs. Elliott Bros., The Davies Motor Co., The Royal Society, and the Editor of the *Philosophical*

Magazine for permission to reproduce illustrations; also to Mr. Jas. White, Glasgow, the constructor of Lord Kelvin's current balances, for illustrations and for special permission to use a large part of his descriptive pamphlet on that instrument.

The author will be obliged to any of his readers who will point out errors in the text which have escaped correction in the proof-reading.

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PHYSICAL LABORATORY,
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November, 1897.

PREFACE TO THE SECOND EDITION

ADVANTAGE has been taken of the demand for a second edition of this book, not only to make several necessary corrections, but also to rewrite certain sections, and to introduce a good deal of new matter in order to bring the book thoroughly up to date. In particular, the chapter on Standard Cells has been entirely rewritten, and modern commercial methods of testing the magnetic qualities of iron and steel have been described in the section dealing with magnetism.

The author desires to acknowledge his indebtedness to several friends for assistance in preparing this new edition. Especially are his thanks due to his assistants, Messrs. H. S. Saunders and H. Nutton, for reading and correcting proofs, preparing the index, and drawing the new diagrams that have been added. Also to his friend and former student, Mr. H. Halliday, A.R.C.S., for pointing out several errata in the first edition.

Thanks are also due to Prof. Ewing for permission to use the blocks illustrative of his Magnetic Testing Apparatus, supplied by Messrs. Elliott Bros., and to Messrs. Nalder Bros. & Co. and Messrs. Crompton & Co. for the use of blocks for illustration.

JOHN HENDERSON.

PHYSICAL LABORATORY,
BOROUGH POLYTECHNIC INSTITUTE,
LONDON, S.E., *July*, 1904.

PREFACE TO THE THIRD EDITION

In bringing out a third edition of this book, advantage has been taken of the opportunity to make one or two small corrections in the text, and of adding new references to scientific papers so as to bring the book up to date. No new matter has been added, as a very thorough revision was made recently when the last edition was published.

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Physical Laboratory,
Borough Polytechnic Institute,
London, S.E., *July*, 1908.

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I.—Measurement of Resistance.

II.—Measurement of Current.

III.—Measurement of Electro Motive Force.

IV.—Measurement of Quantity of Electricity.

V.—Measurement of Capacity.

VI.—Magnetism.

VII.—Electro-Magnetic Waves.

APPENDIX.

REFERENCES TO SCIENTIFIC PAPERS

The following abbreviations are used in connection with the above references

The Philosophical Transactions of the Royal Society of London	(Phil. Trans.).
The Proceedings of the Royal Society of London	(Pro. Roy. Soc.).
The Transactions of the Royal Society of Edinburgh	(Trans. R.S.E.).
The Proceedings of the Royal Society of Edinburgh	(Pro. R.S.E.).
The Philosophical Magazine	(Phil. Mag.).
The Electrician	(Elect.).
The Reports of the British Association	(B.A. Report).
The Journal of the Institution of Electrical Engineers	(J.E.E.).
Poggendorff's Annalen der Physik und der Chemie	(Pogg. Ann.).
Wiedemann's Annalen der Physik und der Chemie	(Wied. Ann.).
Comptes rendus de l'Académies des Sciences	(Comp. Rend.).
Journal de Physique	(Jour. de Phys)
Wiener Berichte	(Wien. Ber.).

PRACTICAL ELECTRICITY AND MAGNETISM

I.

INTRODUCTION.

1. In all physical measurements, with any pretence to accuracy, and which are intended to be of any permanent value, the student must grudge no amount of time and trouble in making them. He must never be in a hurry. A week spent in discovering and overcoming some source of error will be well-spent time, and may be of more educational value than the performance of the original experiment itself. Above all things, however, the experimenter must be methodical; all results and measurements must be recorded *exactly as observed* No corrections, however simple, should be made mentally at the time of observation, and the results must be recorded *immediately* after they have been obtained—the memory ought never to be trusted.

In order to impress the student with the necessity of being methodical in his work, a few hints are given respecting the method of recording results.

The observation of general conditions which may affect the experiment—

(*a*) The date, time, and place where the experiment is performed should invariably be recorded.

(*b*) The temperature of the room during the experiment, and the barometric pressure, should be noted in cases where a variation of either of these quantities might affect the results.

(*c*) There should also be a complete description of all the apparatus employed, with the reference numbers of the various

instruments, and a diagram showing the disposition of the apparatus, and giving some idea of the relative distances of various pieces of apparatus from one another more especially when there is reason to suspect that one instrument may exert an effect on others near it. The great importance of these precautions is very evident. Suppose, for instance, that after some experiment it was found that a resistance coil in a box was faulty, then unless the exact part played by that particular coil in the experiment was known, a correction for the error so introduced would be impossible. Too great elaboration cannot be given to this somewhat tedious and uninteresting part of experimental work, and the author would advise every student who intends to devote his time to experimental physics, to first of all read Faraday's "Experimental Researches in Electricity," not only for the sake of the valuable information contained in them, but also to learn how to record results of experiments, nothing being too unimportant to record to that prince of experimenters, even to the lengths of the connecting wires employed.

2. Before commencing any experiment, it is assumed that the student is perfectly familiar with its object. He should also read as much of the literature bearing on the subject as he can lay his hands on. Should any irregularities occur, the causes of them must be hunted down and *proved*—they should never be shelved as being merely "experimental errors." *Nothing must ever be taken for granted*, everything must be *conclusively proved.* No deductions should be made from results at the time of recording, as it is better that the mind should be entirely given up to accurate observation; then afterwards, on reading over and studying his results, the student will be better able to make deductions, and to devise further experiments to test uncertain points, or to be crucial tests of any theory he may have proposed.

Adjustment and Care of Apparatus.

3. *Galvanometers.*—The number of forms which galvanometers may take is legion, but there are a number of remarks on their adjustment and care which apply equally to all forms.

The table or stand on which the galvanometer is placed should be separate from that on which the rest of the apparatus is arranged, otherwise it is liable to be shaken when adjusting the other instruments. The table or stand must be arranged to be as free from vibration as possible. In some laboratories the room in which the sensitive galvanometers are kept is in the basement, the stands being built up specially from the foundation, in order to secure freedom from vibration. This cannot always be managed, and very often small brackets are attached to the walls, on which the galvanometers may be placed; provided the walls are thick, and there is no heavy running machinery near them, this makes a very satisfactory stand. All large masses of iron which would influence the galvanometers must be removed from the neighbourhood of the testing-room. Stationary masses of iron, such as iron pillars, are not so important, as the effect due to them will be constant, unless the magnetic field around them alters in value. Also movable magnets, or anything of a magnetic nature, such as knives, keys, watches, etc., must be placed where they will exert no influence on the instruments.

Great care must be taken to prevent the magnetic effect of the current in the other pieces of apparatus affecting the galvanometer. In order to ascertain whether or not there is any such action, it is advisable to send a current through the apparatus, having previously disconnected the galvanometer. Any deflection obtained under these circumstances points to the magnetic effect of some part of the apparatus on the galvanometer, and the apparatus must be rearranged until no such effect is observed. For this reason it is advisable in some cases to twist the wires leading to the galvanometer together for some distance from it.

4. In some experiments, as for instance the measurement of insulation resistance, the insulation of the galvanometer becomes very important, and the instrument must be tested for leakage. This can best be done by connecting up one terminal of the galvanometer to one pole of a battery of several cells, the other pole of the battery being earthed, whilst the other galvanometer terminal is insulated. A deflection of

the needle under these circumstances points to a leakage to earth from some part of the galvanometer coil. This test should be made to each terminal of the galvanometer separately, since the leak might be close to one, when of course no deflection would be obtained on the battery being connected to that terminal. Should a small leakage be found which would interfere with the accuracy of the measurements, the galvanometer should be placed on blocks of freshly scraped paraffin wax, one under each levelling screw; this will completely insulate the instrument from the earth.

5. The galvanometer must be set up so that the needle oscillates freely when disturbed from its zero position, the instrument being levelled so as to bring the needle into the centre of the field. In the case of a reflecting galvanometer, any friction between the needle and the coil can be detected by observing the motion of the spot of light on the scale when the needle is set oscillating, an irregular, jerky motion denoting friction between the two.

It is also advisable, whenever possible, to so arrange the galvanometer that the needle will point to the zero on the scale when under the influence of the earth's field alone. This is not essential, as the needle can always be brought to zero by means of a bar magnet, but it is often desirable.

6. Most galvanometers are supplied with a movable directing magnet, attached to the frame of the instrument, and by means of which the needle may be adjusted to zero by turning the magnet on its axis, or the controlling force may be altered by altering the distance between the needle and controlling magnet. Such an arrangement is shown in Fig. 1. Here the directing magnet is attached to the top of the case of the galvanometer, and the rotation is given to the magnet by means of a worm and worm-wheel, whilst the controlling force is altered by sliding the magnet up or down the vertical brass rod. The great objection to this arrangement is that it is almost impossible to alter the position of the magnet without seriously shaking the whole instrument, thus making it very difficult to rapidly vary the controlling field so as to give a long period of swing to the needle. This difficulty may be overcome by mounting

the directing magnet on a separate stand. A simple form employed by the author is shown in Fig. 1. A represents a sliding board, moving in the frame F, between the guide bars G, G; to it is fixed the vertical rod P, on which the magnet slides, so as to adjust it to the height of the gal-

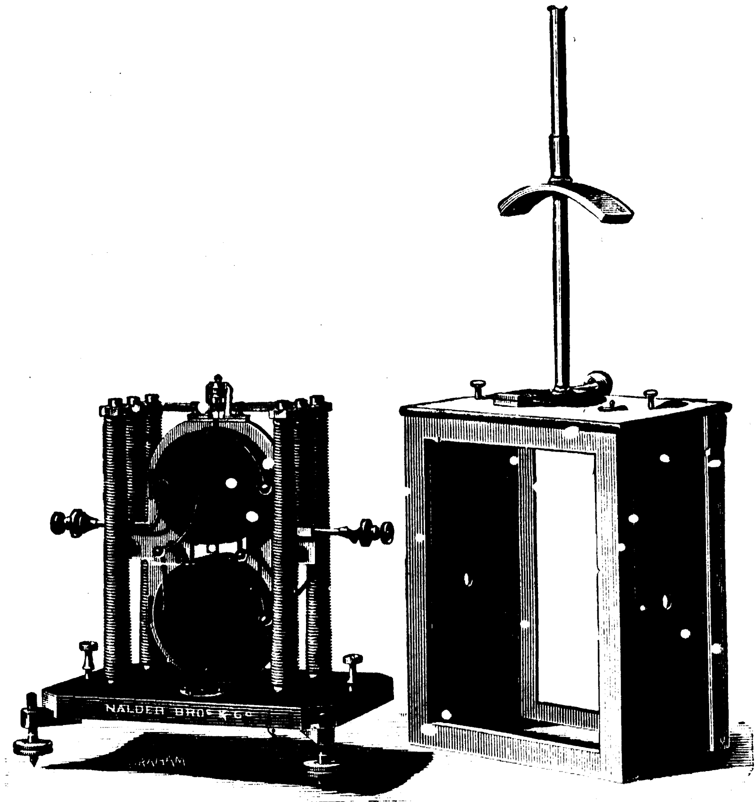


FIG. 1.

vanometer needle. The rod P is free to rotate in its socket, and is turned by means of a small band passing round the pulley S and the pulley T, to which a milled head is attached. The to-and-fro motion is given by means of a band, B, attached to A, and passing over two rollers, R, R, to one of which a milled head, M, is attached. The whole

apparatus may be made in the laboratory workshop. In use it is placed behind the galvanometer, and the magnet adjusted on the rod P until it is on a level with the needle. The milled head M is then turned so as to pull the magnet nearer the galvanometer until the required sensitiveness is obtained.

7. *Suspension of the Magnetic System.*—The suspension of the needle of a galvanometer is a very important matter. The function of the suspending fibre may be twofold—either simply to suspend the needle, or, in addition, to supply the controlling force—and of necessity the nature of the suspension varies with the above conditions. If a suspension only is required, the fibre must be as torsionless as possible; if

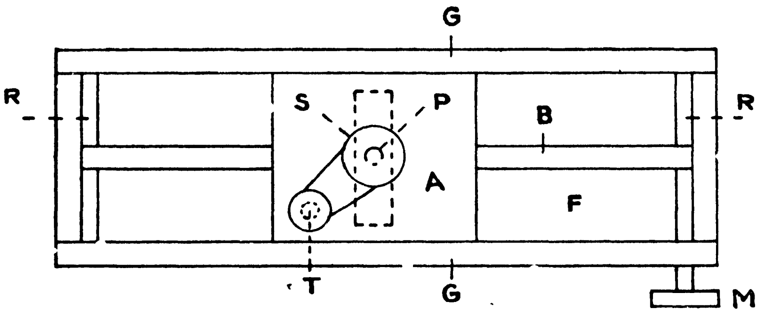


FIG. 2.

a controlling force is required, that must be supplied by the torsion of the fibre. In the first case it is necessary to get some substance which, when of sufficiently small diameter, is practically free from torsion, and is at the same time strong enough to support the weight of the suspended system. One such substance is unspun cocoon silk. This is produced by the silkworm as a double thread, each part having a diameter of about 0·0005 inch. The threads are separated from each other by washing them with warm water, painted on from a camel's-hair brush, in order to dissolve the gum by which they are fastened together. One such thread has been found to support a weight of about 5 grammes before breaking.[1] A number of such fibres should be kept ready for use, suspended inside a glass tube, with small

[1] See Gray's "Absolute Measurements in Electricity and Magnetism," vol. i. p. 241.

weights attached to them to keep them stretched. The torsion in silk fibres is very small, but the fibre is affected by both temperature and moisture, so that the zero of instruments in which such suspensions are employed is liable to alteration.

8. A much better material for a suspension is a thread from a garden-spider's web; this, although possessing the properties of an almost ideal suspension, seems to be very little used. It is very strong, considering its size. Joule found that it could support a weight of 2 grammes without breaking.[1] The greatest point in its favour is that experiments go to show that it is almost absolutely torsionless.[2] Experiments by Bottomley and Tanakadate go to show that a spider line capable of carrying a mirror and magnet weighing 0·2 gramme has a torsional rigidity $\frac{1}{700}$ that of a single cocoon silk fibre. Its extension with temperature is also very small, a fibre 23 inches long, loaded with a weight of 1 gramme, not altering more than about 0·7 % in length for a rise of over 50° C., a lengthening of about 2 % taking place when dry air was changed for moist. This, however, need be no drawback, since, on account of its great freedom from torsion, very short fibres may be employed.

9. Another material for galvanometer needle suspension is quartz fibre, originally prepared by Professor C. V. Boys, F.R.S.[3] Such fibres can be obtained of any desired degree of fineness. As regards strength, Professor Boys has shown that, for a fibre nearly $\frac{7}{10000}$ of an inch in diameter, the breaking stress was 51·7 tons per square inch. In addition to its other properties, quartz fibre is an almost perfect insulator even in a damp atmosphere, which makes it invaluable as a means of suspending charged bodies, such as the needle of an electrometer. The following hints regarding the method of suspending substances by quartz fibres have been taken from Professor Boys' Cantor Lectures on "Instruments for the Measuring of Radiant Heat":—

"Having chosen a fibre of the right diameter, and longer

[1] Joule's "Scientific Papers," vol. i. p. 479.

[2] Cantor Lecture on "Instruments for the Measuring of Radiant Heat," Boys, p. 11; also Bennet, *Phil. Trans.*, 1792.

[3] See *Pro. Roy. Soc.*, vol. 46, p. 253; also *Phil. Mag.*, Aug., 1889.

than is ultimately required, the first thing to do is to fasten a small fragment of gummed stamp-paper to one end. This acts as a weight and makes the following processes more easy. The upper or fixed support must next be fastened to the free end of the fibre. I prefer a common blanket-pin passing through a cork to any of the more elaborate contrivances in common use; however, whatever is going to be used for the fixed support should be pointed at the lower end. If the needle or other thing to be supported is very light, *i.e.* nowhere approaching the breaking weight of the fibre, shellac varnish is the best thing to use as the cement. Just moisten the last five millimetres above the pin with this varnish—holding the fibre near its free end in one hand, and the pin in the other, with the little finger of one hand resting against the little finger of the other for the sake of steadiness—immediately apply the fibre to the varnished surface, to which it will stick. Then pull it endways through a distance of half a millimetre about, to make sure that when all is dry there will be no sudden bend in the fibre. A hot piece of wire, or knife, or pair of pliers must then be applied to the pin above the varnish, so that the heat may be conducted down slowly to drive off the remaining alcohol and melt the shellac. After this the fibre is securely held at that end. If the thing to be supported is very heavy, varnish is not so good as melted shellac, but this is much more difficult to apply. The pin must be warmed and smeared near the joint, and while hot the fibre must be applied and slightly pulled. Assuming one end of the fibre is properly fastened, the next thing is to determine the exact length required. For this purpose I always make a drawing on a perfectly clean and smooth board, showing the point of the pin at one end, the extreme end of the thing to be supported, and the position of the mirror or whatever else determines the length of the fibre. The holding pin is then raised up until the paper weight is hanging in the air. This is then allowed to rest on the board, and slowly dragged along until the point of the pin is exactly over the corresponding mark on the board, and the paper wafer is in the line of, but beyond the other mark.

"The fibre is then straight, and must pass over the mark which indicates the upper end of the apparatus that is to be suspended, though of course nothing can be seen. A knife is then drawn across the board, say 5 millimetres beyond this mark. By this means the fibre is cut, and the 5 millimetres are left for the purpose of attachment. The needle or suspended thing is then fastened the same way as the pin."

10. Metallic suspensions are employed in galvanometers built on the D'Arsonval principle, in which case they have not only to supply the controlling force, but also to make electrical contact between the suspended coil and the terminals. Flat strips of phosphor bronze have been found very satisfactory for suspensions of this kind, but it has been shown that a better material would be platinum silver for both coil and suspension, since the strip is liable to have its torsional rigidity altered by change of temperature, and in the case of platinum silver the percentage diminution of torsional rigidity is almost equal to the percentage increase of electrical resistance; so that the same potential difference at the terminals of a galvanometer possessing such a suspension would always produce the same deflection throughout a considerable range of temperature, the one variation being made to compensate for the other.

11. In making connection to the galvanometer, fine double silk-covered wire (about No. 26) should be used. If thick wires are employed, mechanical vibrations may be transmitted along them from the rest of the apparatus, which will keep the galvanometer needle continually shaking; also the wires leading to the galvanometer should not be coiled into spirals, as these form electro-magnets which produce a disturbing effect on the galvanometer needle, but should rather be twisted together, or, at least, run parallel to one another close together, so that their magnetic effects will tend to neutralize.

12. In some cases, especially where a busy street runs near the laboratory, it may be found impossible to get a support for the galvanometer which is free from vibration. In the event of this, it is sometimes the practice to place the galvanometer on a board suspended from the roof by long indiarubber ropes,

which absorb the greater part of the vibration. A new trouble is, however, sometimes introduced in the shape of a pendulum swing of the whole apparatus.

12*a*. The deflections of a sensitive galvanometer are generally observed by throwing a beam of light, from an oil, gas, or electric lamp, on a small concave mirror attached to the galvanometer needle or moving coil. The light is reflected back on a graduated scale placed in front of the galvanometer. The most satisfactory arrangement is to use a small 5 or 8 candle-power straight-filament electric lamp as the source of light, and a semi-transparent glass graduated scale as the screen, on which the image of the filament is thrown by the mirror. This has two very important advantages over the ordinary scale, inasmuch as it admits of readings being taken in daylight, without darkening the room, and the experimenter makes his observations from behind the scale, being therefore as far as possible from the galvanometer. In Fig. 2*a* a representation of such a scale

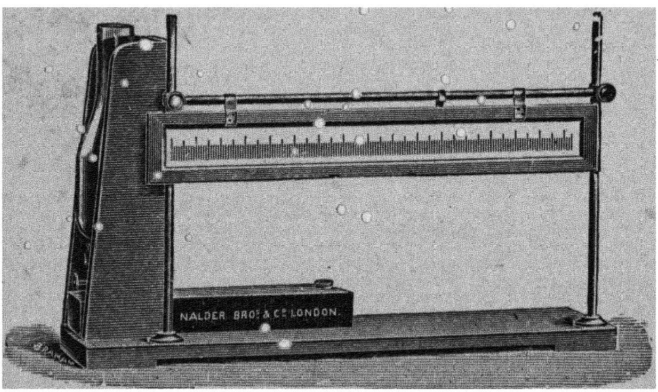


FIG. 2*a*.

is given. It must always be borne in mind, however, that the angular movement of the beam of light is twice that of the mirror.

12*b*. *Shunts.*—In order to decrease the sensibility of a galvanometer, and therefore increase its range of measurement without interfering with its controlling force, it is usual to shunt a known fraction of the current through a resistance placed in

parallel with the galvanometer. If the resistances of the galvanometer and shunt are known, the current in each can easily be calculated. Thus in Fig. 2*b*, if the galvanometer resistance is g and the shunt resistance S, the fraction of the total current C that will traverse the galvanometer coils will be $\frac{S}{S+g}C$, or if G is the galvanometer current, then—

$$C = \frac{S+g}{S}G$$

The quantity $\frac{S+g}{S}$ is known as the "multiplying power" of the shunt.

It is usual to construct small shunt boxes containing three or

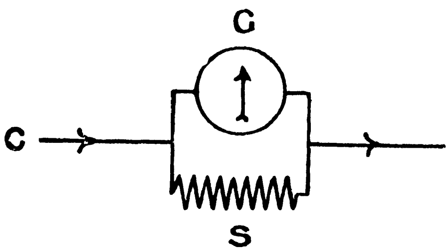


FIG. 2*b*.

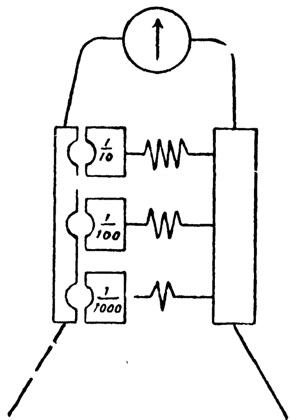


FIG. 2*c*.

four coils for each galvanometer, these coils being marked either $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, or $\frac{1}{9}$, $\frac{1}{99}$, $\frac{1}{999}$, according as markings are intended to specify the fraction of the total current that flows in the galvanometer coils, or fraction of the galvanometer resistance to which each shunt coil is adjusted respectively.

Thus the resistance of the shunt coil that will make $G = \frac{1}{10}C$ must be $\frac{1}{9}g$.

Fig. 2*c* shows the arrangement of three shunt coils in a box, so that by means of a single plug any one may be placed across the galvanometer terminals.

12*c*. It will be obvious from the above that each galvanometer must have its own shunt box, and that the shunt for one galvanometer will not do for any other, unless it has exactly the same resistance and is wound with wire of the same temperature coefficient. This difficulty has now been over-

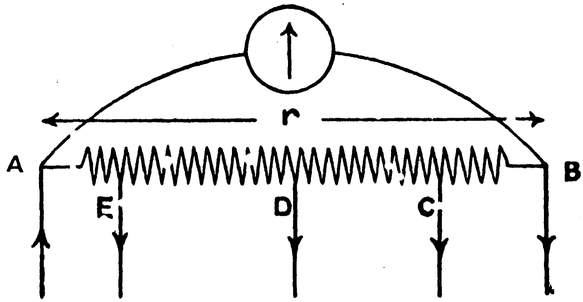


FIG. 2*d*.

come by Professor Ayrton and Mr. Mather,[1] who have designed a "universal" shunt box. In Fig. 2*d* a diagram of the connections in such a box is given. The galvanometer is connected *permanently* across the ends of a resistance r, usually about 10,000 ohms, whilst the outside circuit may be connected to AB, AC, AD, or AE. If the resistance AC is $\frac{1}{10}r$, the galvanometer current, when the outside circuit is connected to A and C, will be $\frac{1}{10}$ of its value when the external circuit is connected to A and B. Similarly, if AD and AE are $\frac{1}{100}$, and $\frac{1}{1000}$ of r respectively, the galvanometer currents, when the outside circuit is connected to these points, will be $\frac{1}{100}$ and $\frac{1}{1000}$ of the galvanometer current when across AB. This relationship will be seen to be quite independent of the

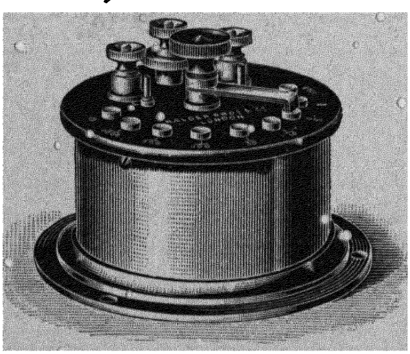


FIG. 2*e*.

[1] *Jour. Elect. Eng.*, vol. 23, p. 314.

galvanometer resistance, so that one shunt box will do for galvanometers of various resistances. Fig. 2*e* illustrates the dial pattern of universal shunt box.

Whenever a shunt box such as either of the two just described is used, it must be borne in mind that the total resistance of the curcuit will be altered with each alteration of shunt resistance.

12*d*. *Batteries.*—For all ordinary testing, such as bridge work, etc., where the constancy of the current is not of great importance, any form of primary battery may be employed, the most convenient and cleanest being some form of so-called dry cell, of which many types will be found on the market. These cells will be found to supply a more or less constant current for some considerable time, but eventually they will develop a high internal resistance.

For insulation testing or voltmeter calibration where high E.M.F.'s of constant value are required, a set of 50 or 100 small accumulator cells made up in boxes supplied with terminals which admit of 10, 20, 30, 40, etc., cells in series being obtained will be found very suitable. These cells should not be used for some little time after they have been charged, as the E.M.F. is found to drop rapidly at first, until the cells are partially discharged, or have stood for some time, after which the E.M.F. will be more or less constant.

For the supply of heavier currents, such as may be required in calibrating ammeters, etc., a battery of large secondary cells should be used. As it is not advisable to carry these about the laboratory, they should be permanently fixed up in some well-ventilated room, from which leads are run to various parts of the laboratory. In the illustration below (Fig. 2*f*) is shown a switchboard designed by the author for the electro-chemical laboratory in the Borough Polytechnic Institute By means of this board, any one, or any number in series of the twelve secondary cells can be connected simultaneously to any one of the three laboratory circuits, the connections being entirely made by metal plugs, no mercury contacts being employed at all. Also, by means of the "parallel battery plug" any number of cells may be connected in parallel, should very heavy currents

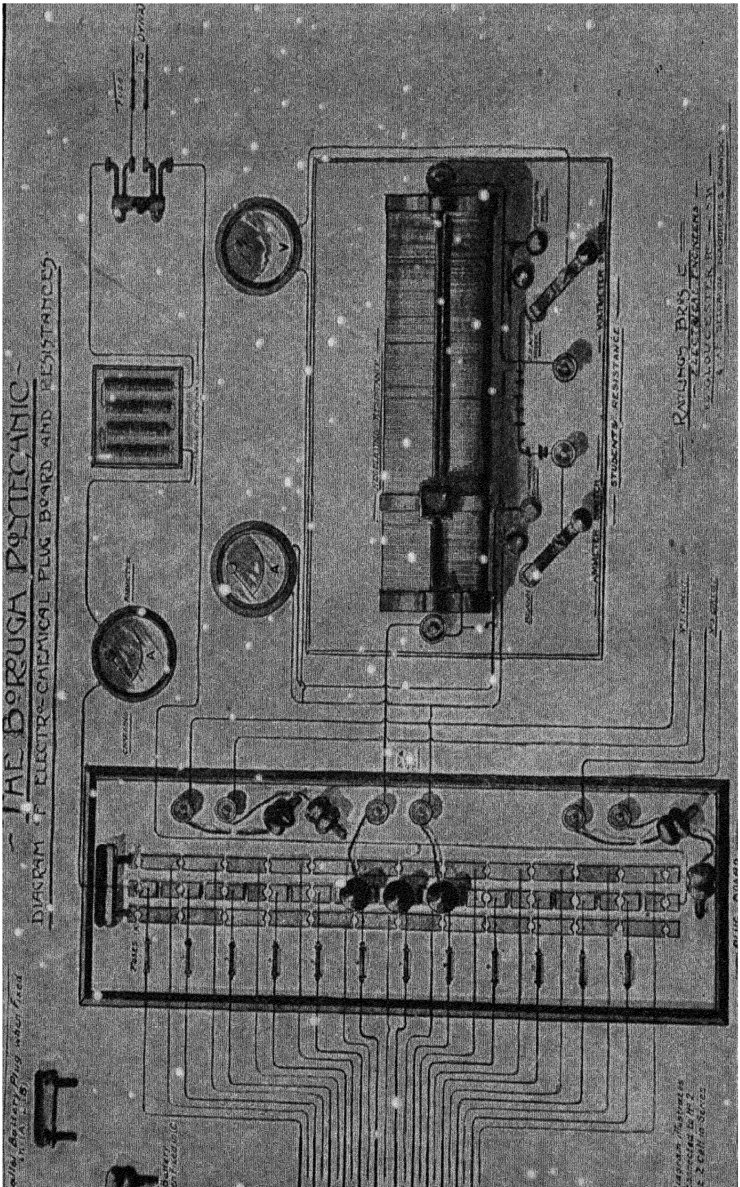

THE BOROUGH POLYTECHNIC
DIAGRAM OF ELECTRO-CHEMICAL PLUG BOARD AND RESISTANCES

Fig. 2*f*.

be required. The diagram also illustrates the charging arrangements, and the student's resistance with ammeter and voltmeter.

Contacts, Connecting Wires, etc.

13. All connections should be made with well-insulated copper wire; and it is not a bad plan to record the lengths and diameters of the various connecting wires employed in any experiment, as some point might turn up afterwards in which a small correction had to be applied for the resistance of the leads, and if this was not measured at the time, it may be estimated afterwards from the record of the nature and distribution of the connecting wires.

In cases where the resistances of leads, connecting wires, etc., become important, however, it must be borne in mind that to have the resistances of the connecting wires small is not enough, since the resistance introduced at the contacts between the wires at terminals and binding screws, often far exceeds the resistance of the leads, and especial care has to be exercised in order to make all such contacts as perfect as possible. Two methods of connecting wires together are available: (1) Mercury contacts, (2) metallic contacts.

14. *Mercury Contacts.*—In mercury contacts a cup of some insulating material is required to hold the mercury. Such a cup may be cut out of a solid, homogeneous block of paraffin wax, this block being afterwards fixed to a wooden stand; at the bottom of the cup is placed a plate of well-cleaned copper, which has been amalgamated by placing in a vessel containing dilute nitric acid and mercury, and rubbing the mercury over the surface of the plate On the top of this plate is poured pure mercury, into which the ends of the wires to be connected together, and which have also been amalgamated, dip, pressing on the copper plate. A cover should be placed over the cup to keep dust and dirt from getting at the mercury. This form of mercury cup will probably give as good a contact as it is possible to obtain, so long as the mercury and the wires remain clean. It is, however, so difficult to manage this, and mercury

is so difficult to purify, that this form of connection is not recommended for ordinary laboratory work.

15. *Metallic Contacts.*—In order to get a good metallic contact, the surfaces in contact should be large and perfectly clean. Such a contact may be obtained between two copper plates, which have had their surfaces filed and scraped true, and which are then bolted together with a strong bolt having a large nut, the under surface of which has also been filed and scraped true, so that it makes contact all over. Such surfaces may be easily cleaned from time to time, but after cleaning should never be touched by the hand, as a thin film of oil is liable to get on the contact surface, thus increasing its resistance. Metallic contacts such as this will in general be found much more satisfactory than mercury ones.

16. *Keys.*—In all circuits which have to be made and broken, suitable keys must be employed.

I. *Simple Make-and-Break Key.*—This may take the form of either a plug key (see Fig. 3), which may be arranged as a one-way or multiple-way key, or a tapping key (see Fig. 4).

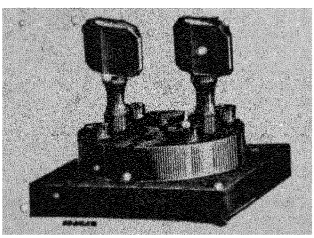


FIG. 3.

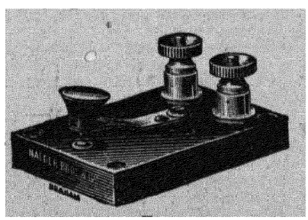


FIG. 4.

In the first form, the metal part consists of brass blocks fixed to an ebonite base, and insulated from one another. They may be connected together by means of a plug, in the same way as the brass blocks in a resistance box are. The tapping key consists of a brass spring attached to an ebonite base; to the lower side of the spring a plantinum contact piece is attached, which, when the key is depressed, makes contact with a similar platinum contact piece on the base, which is in connection with the other terminal of the key. The contact points should be as large as possible, and arranged so that

they may be easily accessible, in order to clean and adjust them. The brass spring has an ebonite button attached to it for the finger to press on.

17. II. *Reversing Keys.*—One form of reversing key is shown in Fig. 5, and is similar to the tapping key, only there are two tappers instead of one. Normally, both springs press on the upper contacts, which are attached to the brass bridge over the key. When the keys are depressed they make contact with the lower contacts, which are connected to the terminal on the far side of the key. In using this key, the battery must not be connected to the two tappers, otherwise, when the key is not depressed, it will be short circuited, but to the terminals connected to the upper and lower contacts respectively, the circuit

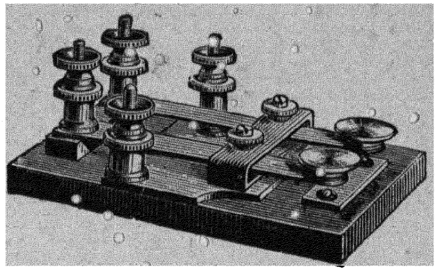


FIG. 5.

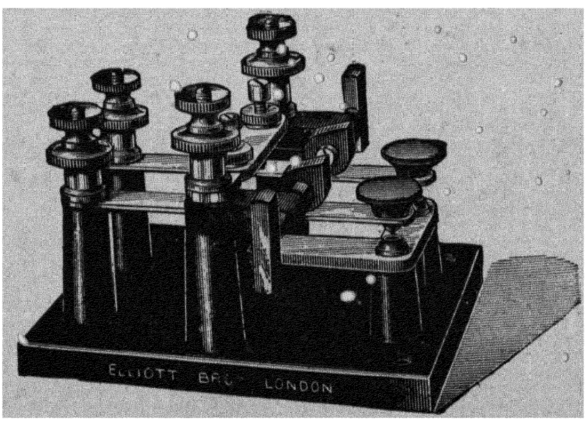


FIG. 6.

in which the current is to be reversed being connected to the terminals attached to the tappers. A very highly insulated form of reversing key is also shown in Fig. 6, which is supplied with cams to hold the tapping springs in any position.

Another very useful form of reversing key is Pohl's mercury commutator (see Fig. 7). This key may not only be used as a reversing key, but also for connecting either of two circuits to a third, which is very useful, for instance, in comparing the fall of potential in an unknown resistance with that in a standard resistance. The following diagrams show the connections in the two cases:—

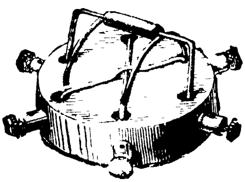


FIG. 7.

(1) As an ordinary reversing key (Fig. 8). In one position of the rocker, 1 is connected to 3 and 2 to 4; in the other position, 1 is connected to 5 and 2 to 6; contacts 3 and 6, also 4 and 5, being permanently connected by copper straps.

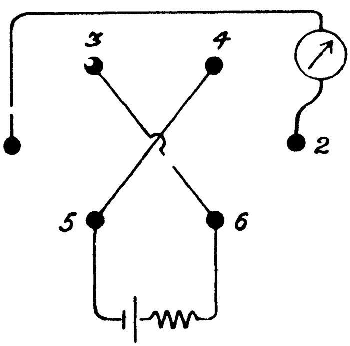


FIG. 8.

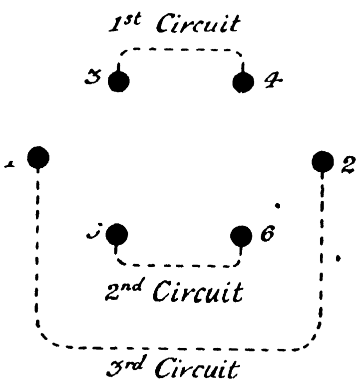


FIG. 9.

(2) For connecting either of two circuits to a third, the two copper straps are removed (Fig. 9).

When 1 is connected to 3 and 2 to 4, the first and third circuits are connected together; also where 1 is connected to 5 and 2 to 6, the second and third circuits are connected.

18. *High-Insulation Keys.*—In cable and condenser work, keys of special construction and of high insulation are required. One form largely used is shown in Fig. 10. This consists of a movable brass tongue attached to a long ebonite pillar. The tongue moves between an upper and lower contact, both of which are highly insulated from the base; there

are also two detents attached to levers marked "discharge" and "insulate." When the tongue is depressed, it is held on the lower contact by the detent attached to the "insulate" lever. If this lever is depressed, the tongue springs up until it is caught by the other detent, which holds it midway between the two contacts. When the lever marked "discharge" is depressed, the tongue springs up to the upper contact. The ebonite insulating pillars should be corrugated, in order to offer a greater resistance to surface leakage; and the screws by which they are attached to the base should only enter the ebonite a short distance, otherwise the advantage of all the insulation is lost. Such keys, and in fact all keys,

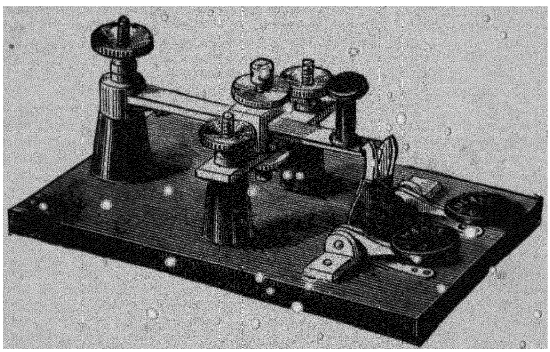


FIG. 10.

should be carefully cleaned before being used. The ebonite should occasionally be well washed with soap and distilled water and dried before a good fire; drying by rubbing is apt to electrify the ebonite, and this in some cases would introduce errors into the experiments. Such instruments also should *always* be lifted by the metal part, as the hand is apt to leave a film of moisture on the ebonite which might spoil its insulating properties.

19. *Batteries.*—In many experiments one of the necessary requirements is a battery of constant E.M.F. No ordinary primary battery will fulfil this condition, unless the current taken from it is very small and is only required for a short interval of time. When large currents are required, or a constant E.M.F., for any length of time, a secondary battery

must be employed. Small-size secondary batteries of about 14 ampère hours' capacity will be found very useful for general laboratory work, and they may be relied upon to maintain a constant potential difference at their terminals, provided they are not used immediately after being charged, but after having been partially discharged. Great care must be taken not to accidentally short circuit such cells, since the internal resistance is very small, and a large current might not only damage the cell but also the external resistance. It will sometimes be found a good plan to make up sets of three such cells in boxes, inside which is a resistance in series with them, sufficient to protect the cells from a dangerous short circuit.

Dry batteries and Leclanchés will be found very useful for ordinary testing and bridge work, where the constancy of the current is not of such vital importance. The latter cells will be much improved if potassium permanganate crystals are mixed up with the manganese doxide in the porous pot; and if this cannot be done, the cells may be "revived" from time to time by pouring a solution of potassium permanganate into the porous pot through one of the air tubes. Other forms of battery, such as Daniell's and Latimer Clark's cells, will be dealt with later on in connection with standard cells.

20. In many cases an E.M.F. very much smaller than that given by any of the above batteries, but of known value, is

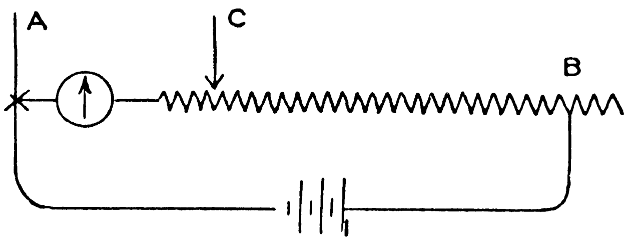


FIG 11.

required; this may be obtained as follows. A current from an ordinary cell is sent through a very high resistance, say 10,000 ohms, in series with a low-resistance sensitive galvanometer (see AB in Fig. 11). The terminals A, C supply a potential difference which may be any desired fraction of AB, by simply

moving C nearer to or farther from B. The object of the galvanometer is to enable the potential between A and C to be kept constant when the resistance across AC varies; as the external circuit attached to AC diminishes in resistance the potential difference between A and C will fall, and this will be indicated by the galvanometer deflection falling off. The point B must now be adjusted so as to diminish the total resistance between A and B until the galvanometer deflection is restored to its original value, the P.D. between A and C then being the same as before, and the same fraction of the original value of AB, the fall of potential down which may be taken as equal to the E.M.F. of the cell.

21. *Resistances.*—In addition to the ordinary resistance boxes, post-office bridges, etc., which are accurately adjusted, but meant only to carry small currents (in no case more than 0·001 ampère should be allowed to pass through them), resistance coils are often required, which, while still being accurate, are capable of

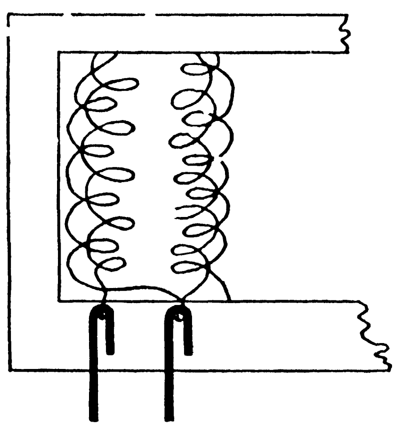


FIG. 12.

carrying much stronger currents without heating to such an extent as to alter their resistance appreciably. Such coils may be made of manganin, wound in non-inductive spirals and stretched on a large wooden frame, so as to expose a large cooling surface, the ends of the coils being soldered to thick copper wires; a very convenient form used by the author being that

shown in Fig. 12, one of the copper end pieces being shown in Fig. 13, and consisting of a piece of thick copper wire first doubled in two, the end of the resistance coil being soldered into the bend, and then one limb being bent at right angles at the middle point, B, the object of this being to enable the current to be sent in at A, and B to be used as a potential terminal when it is required to measure the fall of potential down the coil.

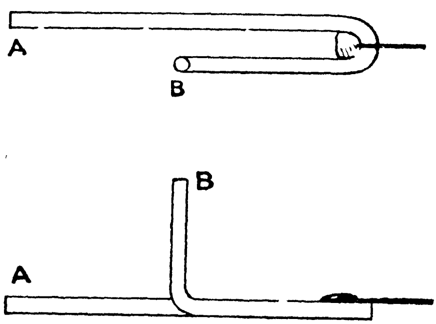


FIG. 13.

The following table gives the sizes of wires that may be employed for the various resistance coils :—

Resistance.	Size in B.W.G
1-5 ohms	20
10-50 ,,	26
100-500 ,,	30
500-1000 ,,	32
1000-5000 ,,	36
10.000 ,,	40

The wires in all cases are double silk covered. Also, in cutting the lengths of wire for the coils, it is advisable to test each gauge for specific resistance, as various specimens of manganin are found to vary among themselves. The coil, before being finally adjusted, should be artificially aged by heating to a temperature of 140° C. in an air-bath for 5 hours; some authorities also recommend coating the wire with shellac varnish, to protect it from the oxidizing action of the air. The advantage of using manganin is its high specific resistance and negligible temperature coefficient.

22. In addition to such resistances as the above, variable unknown resistances are often required to regulate the current in a circuit. For purposes of this kind, Varley's carbon rheostat (see

Fig. 14) will be found very useful. This consists of a number of discs of carbonized cloth, through which the current passes, and by compressing the discs the resistance can be diminished. Ranges from 500 ohms to $\frac{1}{10}$ ohm may be obtained with such instruments.

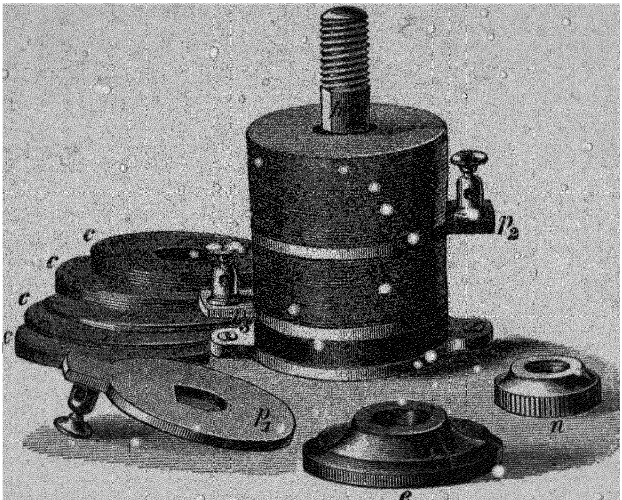


FIG. 14.

Liquid resistances will also be found very useful. A simple form which can be easily constructed in any laboratory is shown in Fig. 15. A is a long glass specimen jar, fitted with an india-rubber cork, B, through which a hole has been bored and fitted with a brass connector, C, the lower screw of which has been removed; through this passes a copper rod, R, to the end of which is attached a copper plate, P, forming one electrode; the other electrode, P′, is connected to a wire, W, which is well insulated down to the point where it is attached to P′. A solution of copper sulphate is employed. To vary the resistance, the rod R is pushed in or pulled out, so causing the plate P to approach or recede from P′, the upper screw of the connector being employed to clamp the rod R in any position. Such a resistance is capable of considerable variation, not only by altering the distance apart of the plates, but also by employing solutions of various densities.

THE CALIBRATION OF A SENSITIVE GALVANOMETER.

23. In the majority of experiments in which the indications of a galvanometer, reflecting or otherwise, are employed to denote the relative values of the currents passing through it, the assumption is frequently made (except in the case of a standard galvanometer as a tangent or sine) that the currents are directly proportional to the deflections. This may be true for the majority of reflecting galvanometers for small deflections on either side of the zero, but, as the deflections increase, the needle of the galvanometer may move out of the uniform magnetic field, and then this assumption is no longer correct. In all experiments of an accurate nature it is desirable to calibrate the galvanometer employed, and to deduce the relationship between the currents and the deflections from the calibration curve. The shape of this curve depends on the winding of the galvanometer, the relative positions of the fixed and movable magnetic systems, and the nature and strength of the controlling force. Consequently, if any of these are altered, the calibration curve is also liable to alter. It is therefore necessary to carefully specify the nature of these conditions at the time of calibration, so that it will be possible to readjust them to their original values should they at any time be altered.

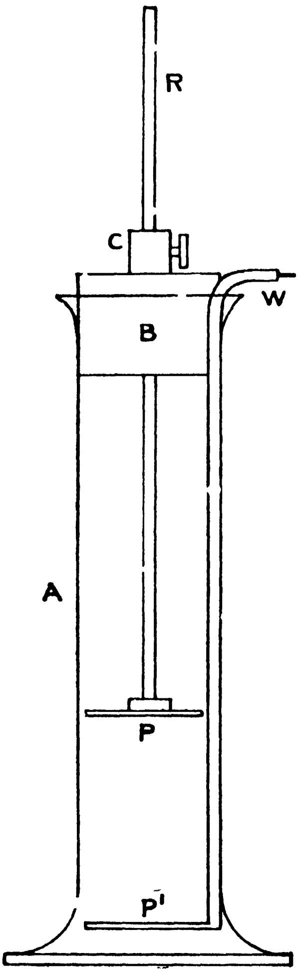


FIG. 15.

In the following method of calibration, advantage is taken

of the relation that the current in a simple circuit varies inversely as the total resistance of the circuit, when the E.M.F. acting in it remains constant. Therefore, by comparing various deflections with the values of $\frac{1}{R}$ corresponding to them, where R represents the total resistance of the circuit, the calibration of the instrument may be effected.

Since the galvanometer itself is part of the circuit traversed by the current, it will be necessary to determine its resistance. The most satisfactory method of doing this being, to connect it up to a Wheatstone bridge and measure its resistance in the ordinary way, taking the precaution in the case of a suspended-

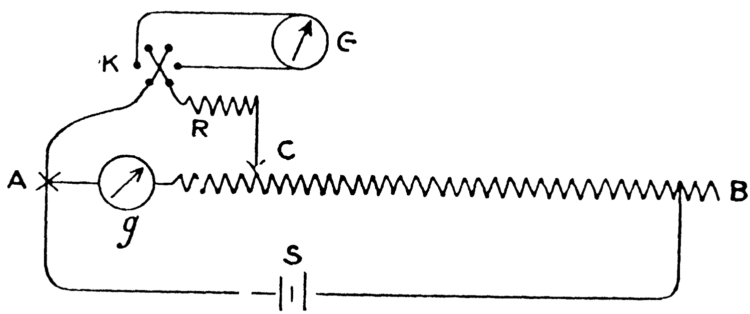


FIG. 16.

needle galvanometer of removing the needle before making the measurement, and in the case of a D'Arsonval galvanometer of fixing the movable coil, so as to prevent the suspension from being strained by the relatively large testing current employed. Having obtained the galvanometer resistance and the temperature at which the measurement was made, the calibration is next proceeded with. Since the galvanometer is a sensitive one, it will be impossible to connect it up in series with a battery, unless there are available some very high and accurately adjusted resistances. It is therefore more convenient to use the method described in par. 20, of employing a small fraction of the total E.M.F. of the battery. Fig. 16 shows a diagram of the connections. S represents a secondary battery connected to the resistance AB, which should not be less than

10,000 ohms. In series with this resistance, and forming part of it, is the low-resistance galvanometer g, to show variations in the current strength; and once the experiment is started, the deflection of g must be kept constant by altering, if necessary, the value of AB, *i.e.* by bringing the point B nearer A. G is the galvanometer to be calibrated, and is connected, through the reversing key K and resistance R, to A and C; AC representing a small fraction of the total resistance in the battery circuit AB. The galvanometer g may be dispensed with if the resistance of the galvanometer G is not less than a hundred times AC, as in this case the percentage variation of P.D. across AC, with a variation of resistance in the galvanometer circuit, is exceedingly small. The galvanometer G having been adjusted properly, R is increased until a small deflection is obtained on G; this is recorded. The reversing key K is then rocked over, and the deflection on the opposite side of the zero obtained; this is also recorded, along with the value of R. R is then varied so as to increase the galvanometer deflection by a few degrees, and deflections to right and left of zero are again obtained, and so on, until the deflection is as large as can be measured. The temperature of the coils R and of the galvanometer must be determined, also the time of swing of the galvanometer needle must be taken. The needle is set swinging, the galvanometer having been first disconnected from the rest of the apparatus, and a stop-watch is started just as the spot of light or galvanometer needle is crossing the zero on the scale; the time that elapses until the next transit is made in the same direction is the periodic time of swing. By taking ten or twenty such swings, and dividing the total time by the number of complete swings, a more accurate determination will be got. The periodic time specifies the relative value of the controlling force acting on the needle or movable magnetic system. The distance of the mirror from the scale must be measured, in the case of a reflecting galvanometer, and the length of a scale-division, in centimetres. The readings may then be tabulated thus—

Resistance R corrected for temperature.	Deflection to right.	Deflection to left.	Galvanometer resistance corrected for temp. g.	Total resistance $R + g$.	Values of $\frac{1}{R+g}$.

Periodic time of swing =
Length of a scale division =
Distance of mirror from scale =
Temperature during test =

The calibration curves are obtained by plotting the deflections to right and left respectively against the corresponding values of $\frac{1}{R+g}$, values of $\frac{1}{R+g}$ being taken as ordinates and the deflections as abscissæ. Two curves will thus be obtained; but if the deflections on either side of the zero are identical, they will coincide.

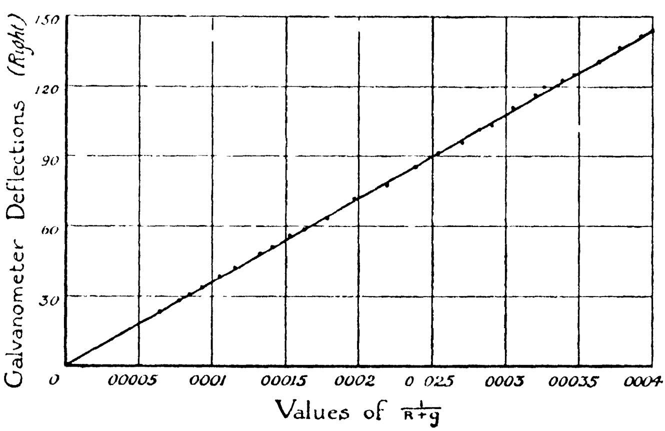


FIG. 17

24. The following data were obtained in calibrating a reflecting galvanometer:—

The galvanometer resistance = 1609 ohms at 14° C. The

temperature during the calibration was 9° C. Using as temperature coefficient of copper 0·388 % per 1° C., the galvanometer resistance corrected for temperature was 1578 ohms. Since the box of known resistances was adjusted for a temperature near that of the room, no temperature correction was made on them, as the galvanometer was not sensitive to that amount.

Resistance R.	Deflection to right.	Deflection to left.	Galv. resist. g.	Total resist. $R + g$.	$\frac{1}{R+g}$
13330	23·5	24·0	1578	14908	0·000067
11000	28·0	29·0	1578	12578	0·000080
10000	30·5	31·0	1578	11578	0·000087
9000	33·5	33·5	1578	10578	0·000095
8000	37·5	37·0	1578	9578	0·000104
7000	41·5	41·5	1578	8578	0·000116
6000	47·5	46·5	1578	7578	0·000132
5500	50·5	50·0	1578	7078	0·000141
5000	55·0	54·0	1578	6578	0·000152
4500	59·0	58·5	1578	6078	0·000165
4000	64·5	63·0	1578	5578	0·000180
3500	71·0	69·5	1578	5078	0·000197
3000	79·0	77·0	1578	4578	0·000219
2600	86·0	84·5	1578	4178	0·000240
2400	91·0	88·0	1578	3978	0·000252
2200	95·5	92·5	1578	3778	0·000266
2000	101·5	97·5	1578	3578	0·000280
1900	104·0	100·5	1578	3478	0·000289
1700	111·0	107·0	1578	3278	0·000306
1600	114·0	110·0	1578	3178	0·000316
1500	120·0	112·5	1578	3078	0·000326
1400	123·5	116·5	1578	2978	0·000337
1300	127·5	119·5	1578	2878	0·000349
1200	133·0	124·0	1578	2778	0·000362
1100	138·5	128·5	1578	2678	0·000375
1000	143·0	133·5	1578	2578	0·000390

Periodic time of swing = 6·5 seconds.
Distance of mirror from scale = 1 metre.
Length of one scale-division = 1 mm.
Temperature during test = 9° C.

Measurement of Resistance.

25. One of the most important of all electrical measurements, and perhaps that most frequently made, is the measurement of the resistance of a conductor. In 1827 Ohm first

stated the relationship, which is now known as Ohm's law, that for any given material, provided its physical conditions remain unaltered, there is a constant ratio between the potential difference at its ends, and the steady current flowing through it due to that potential difference. This ratio is defined as the resistance of the conductor.

The methods employed in determining this ratio in absolute measure will be given later; at present we are only concerned with the methods of comparing different resistances with one another, and of expressing the resistance of a conductor in terms of a suitably chosen standard, this latter process being meant when we speak of measuring a resistance.

26. The following measurements will be dealt with in this chapter:—

(*a*) The measurement of resistances of ordinary value.

(*b*) The measurement of very low resistances.

(*c*) The measurement of very high resistances.

(*d*) The measurement of liquid resistance.

(*e*) The measurement of battery resistance.

(*f*) The measurement of specific resistance.

(*g*) The variation of specific resistance with temperature.

(*h*) The variation of specific resistance with molecular change.

(*j*) Fault testing.

(*k*) The absolute determination of resistance, and construction of standards.

27. Two of the methods employed in the measurement of resistances of ordinary value, and in the comparison of resistances which are nearly equal to one another, have already been mentioned in Vol. I. (p. 47). The first of these, the substitution method, is not susceptible of the same accuracy as the Wheatstone bridge method, and we will therefore pass it over in favour of the latter, the proof of which we will now proceed to give. Four coils, A, B, C, D, are connected together, as shown in Fig. 18. A galvanometer, G, is connected across one diagonal of the diamond-shaped figure from *c* to *b*, and a battery across the other from *a* to *d*. If now the resistances are so arranged that when the battery circuit is closed and then the galvano-

meter circuit, there is no deflection in the galvanometer, the points c and b must be at the same potential. Hence the current in A will be the same as that in C, and that in B the same as that in D. Let c_1 be the current flowing in A and

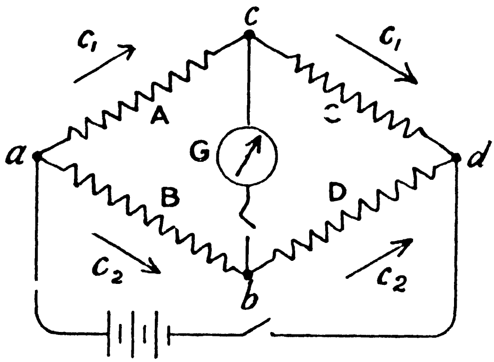


FIG. 18.

C, and c_2 that flowing in C and D. Also let a, b, c, d represent the potentials at the corners of the figure, and A, B, C, D the resistances from a to c, a to b, c to d, and b to d respectively. Then by Ohm's law—

$$A = \frac{a-c}{c_1}, C = \frac{c-d}{c_1}, B = \frac{a-b}{c_2}, D = \frac{b-d}{c_2},$$

$$\text{and } c_1 = \frac{a-c}{A} = \frac{c-d}{C}; \text{ also } c_2 = \frac{a-b}{B} = \frac{b-d}{D}$$

But since there is no galvanometer deflection, the potential at c is equal to that at b, and $a - c = a - b = P_1$, while $c - d = b - d = P_2$.

$$\text{Therefore } \frac{P_1}{A} = \frac{P_2}{C} \text{ and } \frac{P_1}{B} = \frac{P_2}{D}$$

$$\text{Hence } P_1 = \frac{AP_2}{C} = \frac{BP_2}{D}$$

$$\text{or } \frac{A}{C} = \frac{B}{D}$$

It will be observed here that in order to calculate the resistance of any one coil, we do not need to know the resistances

of the other three, but only that of one of them, provided we know the ratio of the resistances of the other two.

28. An apparatus for measuring resistances according to this method, and known as the post-office resistance box, is shown in Fig. 19, and consists of three rows of resistance coils attached

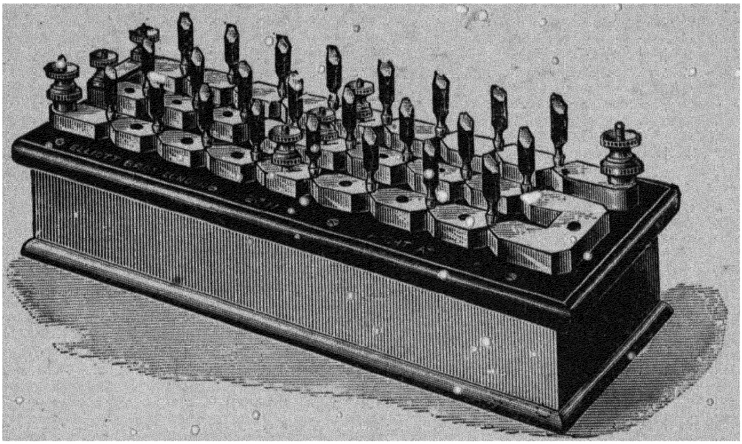


FIG. 19.

to brass blocks fixed on an ebonite plate, and so arranged that by placing a brass plug in between two blocks the coil between them is short circuited. The values of the coils and the method of connection are shown in Fig. 20, the lettering being

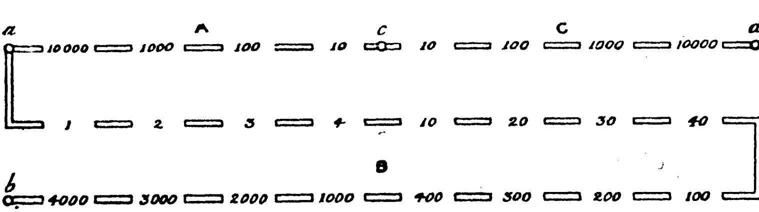


FIG. 20.

the same as that employed in Fig. 18. The coil to be measured is connected from *b* to *d*, the galvanometer from *b* to *c*, and the battery from *a* to *d*. The coils from *a* to *c* and *c* to *d* represent coils A and *c* in the other figure, and are known as the ratio coils or the proportional arms. The maximum possible

limits to which the bridge could measure would be, for high resistances, when the arms were in ratio, $\frac{A}{C} = \frac{10}{10{,}000}$, and B had all the plugs withdrawn, giving 11,110 ohms; then—

$$\frac{10}{10{,}000} = \frac{11{,}110}{D}$$

$$\text{or } D = 11{,}110{,}000$$

$$= 11 \text{ megohms practically;}$$

and for the inferior limit, $\frac{A}{C} = \frac{10{,}000}{10}$; then—

$$\frac{10{,}000}{10} = \frac{1}{D}$$

$$\text{or } D = 0{\cdot}001 \text{ ohm.}$$

The above measurements could not, however, be made with any pretence to accuracy, the practical limit of working of the above bridge being from about 0·1 ohm to a megohm.

It will be observed, in Fig. 19, that a hole is drilled in the centre of each block to admit of a plug being inserted. This is exceedingly useful, as it allows of sets of coils being compared against one another, say the 1, 2, 3, 4 against the 10, etc., and also it admits of a fall of potential down a known resistance being taken from the box.

The wires of which the coils are wound consist, in the more expensive boxes, of platinum-silver, and in the cheaper forms, of platinoid, German silver, or manganin. They are adjusted so as to be correct at the temperature stated on the box. A hole is provided in the ebonite top for the insertion of a thermometer to measure the temperature of the coils.

29. When making a measurement of resistance, if an exact balance cannot be obtained, the galvanometer deflections for resistances above and below the true value are taken, and the exact value of the resistance obtained by interpolation, as explained in Vol. I.

In using a post-office box of coils, great care must be exercised in seeing that the plugs are fitting tightly into the holes, and make good contact. For this reason the metal part of the plugs should never be held in the hand, since a film of oil is

apt to get over the surface and affect the contact between the plug and the brass blocks.

30. In order to minimize the effect of the plugs and reduce their number as much as possible, the form of bridge shown in Fig. 21 is sometimes employed: this is known as the dial bridge. The coils in this form are arranged in four sets—units, tens, hundreds, thousands. The ten coils attached to each dial are all of the same value, and are in series with one another; so that by means of one plug, one coil, or any number up to ten in series, can be inserted into the circuit. This arrangement

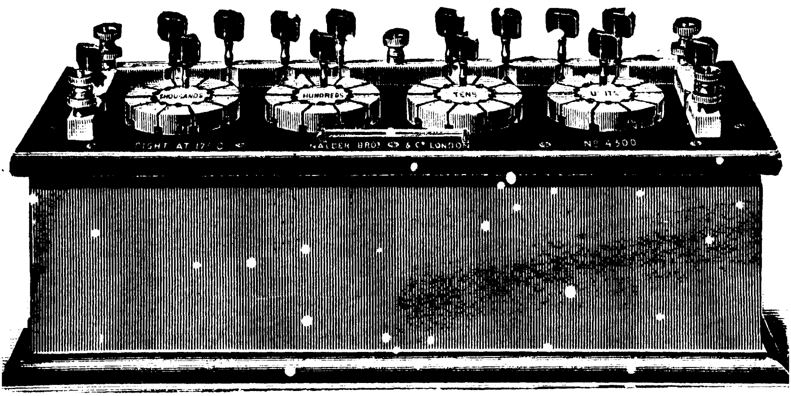


FIG. 21.

is convenient for the inter-comparison of the coils with one another, since the ten coils in series in one dial should equal the first in the next above it.

THE METRE BRIDGE.

31. If it is desired to compare the resistances of two coils, or to determine the value of an unknown resistance in terms of that of a standard coil, the most satisfactory and accurate method to adopt is to compare the resistances on the wire or metre bridge.

In this apparatus, so called because the wire employed is usually 1 metre in length, the ratio of the resistances required is obtained in terms of the ratio of two parts of the bridge wire.

The theory of the instrument is the same as that of the Wheatstone bridge just described, the wire taking the place of the proportional arms in that arrangement. A rough outline of the method of using the wire bridge has already been given in Vol. I.,[1] which contains a description of an apparatus

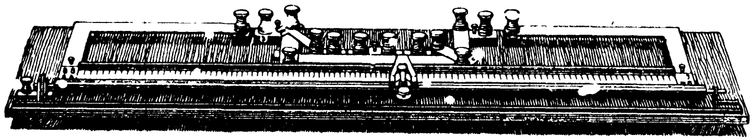


FIG 22

capable of giving results sufficiently accurate for most commercial purposes; for more accurate work, however, such as the standardization of coils, etc., a more carefully made and elaborate apparatus is required. The following is the description of such a bridge, which has been designed by the author

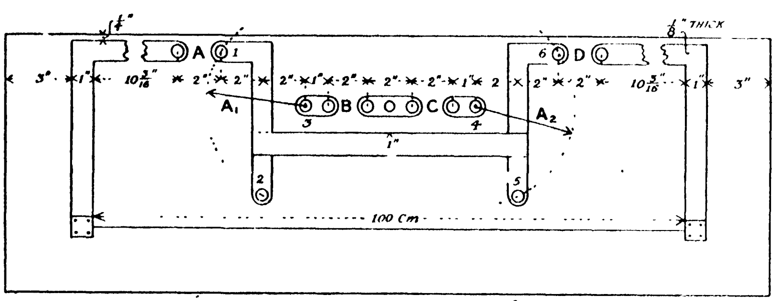


FIG 23.

for his own laboratory, and in which there is a special commutating device for interchanging the coils without having to employ mercury contacts, designed by the author's colleague, Mr. S. Joyce.

Fig. 22 gives a general view of the bridge, while Figs. 23, 24, and 25 give dimensioned sketches of various parts.

The base of the instrument is of teak, and the copper conductors are fastened to it by screws which pass through ebonite washers, thus completely insulating them from it, while

[1] See vol. i. p. 47.

allowing the whole of the bridge to be taken to pieces in a very short time for cleaning. The scale, which is of boxwood, is also insulated from the base by pieces of ebonite.

There are in all four gaps in the bridge for the insertion of coils, these are shown in A, B, C, D (Fig. 23), the coils, the ratio of whose resistances is required, being placed in the gaps

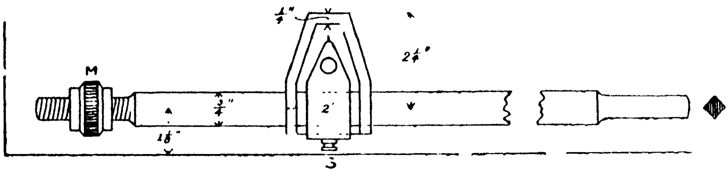


FIG. 24

B and C; then, when the copper connecting arm A_1 is turned so as to connect terminals (1) and (3), and the arm A_2 connects (4) and (6), the resistance in the gap B is next to that in gap A, and the resistance in gap C is next to that in gap D. But by

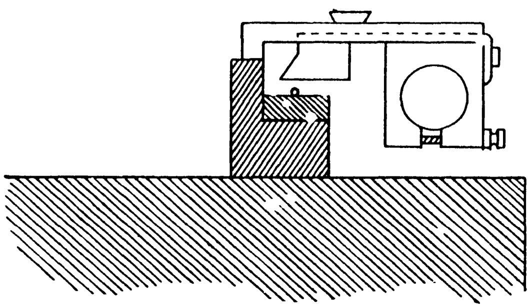


FIG. 25.

simply swinging the arms round so as to connect (2) and (3), and (4) and (5), the coil in C is next A, and that in B is next D, or the coils B and C have been interchanged with respect to the other conductors in the bridge. The rotating arms A_1 and A_2 make contact with large terminal screws, so that the contact resistance is small, also the time occupied in making the change over is very small. The galvanometer is connected to the blocks to which terminals (3) and (4) are attached; one of the battery terminals is connected to the terminal on the central

small block between B and C and marked b, the other being attached to the sliding contact.

The bridge wire is not soldered at its ends to the copper bars, but clamped by means of small copper plates screwed down on the copper bars.

The details of the sliding contact are shown in Figs. 24 and 25. The tapping contact slides along a brass rod, $\frac{3}{4}$ inch external diameter, and can be clamped tightly to it by means of the small set screw S, the final adjustment being made by means of the micrometer screw M, which moves the whole rod.

A wooden cover encloses the whole bridge when not in use, thus keeping it clean and protecting it from damage. Also on the inside of the cover is pasted the calibration curve of the stretched wire.

32. *Calibration of a Bridge Wire.*—Before describing the methods of comparison of resistances on a wire bridge, we must first consider the various sources of error likely to affect such a measurement.

The method of comparison is essentially the determination of the ratio of the resistances of the two coils in terms of the resistances of two portions of a stretched uniform wire. If the wire is uniform, and has the same physical properties throughout its length, then the resistances of the portions will be directly proportional to their lengths. This is the first assumption, and a set of experiments made with a view to discover whether or not it is warranted, is usually termed the *calibration of the bridge wire.*

33. The first method of calibration to be described is a modification of that due to Mattheissen and Hockin.[1] Two coils, A and B (see Fig. 26), of equal resistance, which should be about twenty times that of the wire, are placed in the outer gaps of the bridge. Two other coils of resistances, R_1 and R_2, differing from each other by about $\frac{1}{10}$ per cent., are placed across the inner gaps, the galvanometer and battery being connected up in the usual way, a balance is obtained at some point, B_1, on the scale. A small known resistance, r, is then inserted

[1] See *Report of Electrical Standards Committee of British Ass.*, 1864, Appendix C.

in series with A, and the balance is altered to some point, B_2. R_1 and R_2 are then interchanged, and a third balancing point, B_3, is obtained. Then, if the resistance of the bridge wire and end contacts is represented by w, and the length of

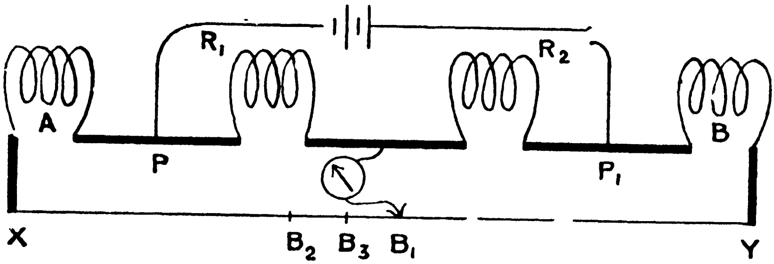


FIG. 26.

the bridge wire between B_2 and B_3 in millimetres by l, it can be shown that the resistance of the part B_2B_3 of the wire is equal to—

$$\frac{R_1 - R_2}{R_1 + R_2}(A + B + w + r)$$

and the resistance per millimetre of this part of the wire equals—

$$\frac{R_1 - R_2}{l(R_1 + R_2)}(A + B + w + r).$$

In order to proceed with the calibration of the wire, the value of r is gradually increased, and so the balance is obtained at various parts of the wire. To get the value of $(A+B+w+r)$, the coils R_1 and R_2 should be disconnected, also the battery and galvanometer, and the resistance between P and P_1 measured on a post-office bridge in the usual way. A table giving resistance per millimetre, at various parts of the wire, may be made, and a calibration curve plotted from it with resistance in ohms per millimetre for ordinates, and scale-divisions for abscissæ.

34. *Calibration of a Wire by fall of Potential.*—The second method of calibrating a bridge, or any other wire, is a fall of potential method. A steady current from a secondary battery, with resistance in series with it, is sent through the wire to be calibrated, the current being kept small in order to prevent any

serious heating in it, which might alter its resistance. The fall of potential along the wire is then proportional to its resistance; so that by comparing the P.D.'s per centimetre run of the wire, we get a series of numbers proportional to the resistance per centimetre of the wire.

In order to tap the wire at points 1 cm. or other convenient distance apart, the following arrangement is adopted. Two small thin brass or copper plates are attached to two separate small rectangular blocks of ebonite or wood well boiled in paraffin (see Fig. 27). The brass springs are attached so as to project over the ends of the ebonite blocks. To the under surface of the brass plates are soldered short straight pieces of stout platinoid wire, which have been filed to form knife-edges. These small blocks are laid on the bridge in such a way that

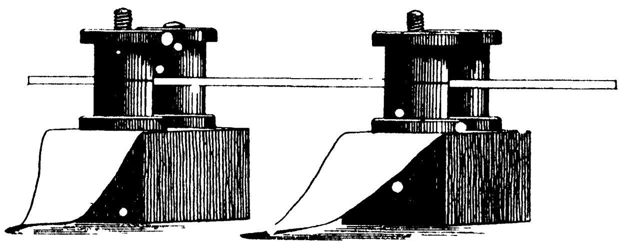


FIG. 27.

the platinoid knife-edges press on the wire, the pressure being maintained by the springs to which they are soldered, and the ends of the knife-edges projecting on to the scale indicate their position on the wire. The two blocks are then arranged at a suitable distance apart on the wire, and are clamped in position by means of a small wooden clamp. The galvanometer is connected to the terminals attached to the brass springs, and should have a high resistance compared with that of the length of wire experimented upon, so as not to sensibly affect the potential difference by the current which it takes; the galvanometer must have been previously calibrated, since the values of the P.D.'s are to be deduced from its deflections.

The apparatus is connected up as shown in Fig. 28. The secondary battery, B, is connected to the ends of the bridge

wire through a regulating resistance, R, and a key, K. Starting at one end with the tapping contacts, deflections are obtained for every centimetre length of the wire. The first reading should be repeated frequently, to test for constancy in the current flowing through the wire. The readings obtained in this way give us a means of calibrating the wire relatively to one of the centimetre lengths. If the calibration is required to be absolute,

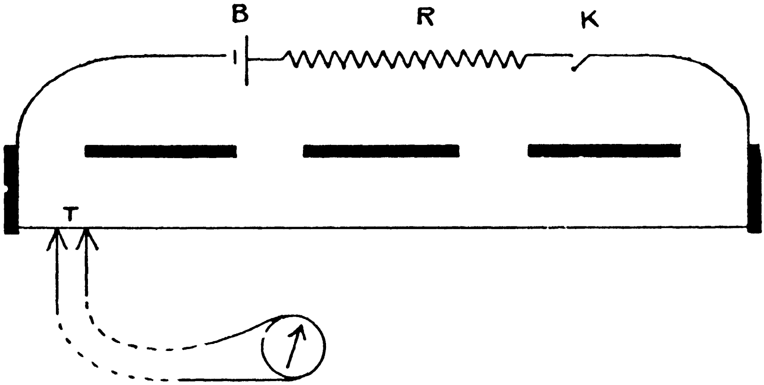


FIG. 28.

that is, to determine the ohmic resistance per centimetre of the wire, a standard resistance of $\frac{1}{10}$, $\frac{1}{100}$, or $\frac{1}{1000}$ ohm must be included in series with the wire during the test, and the fall of potential between its ends measured on the galvanometer, thus enabling the ohmic resistances of the sections of the wire to be calculated from the deflections produced by them. The results should be tabulated thus—

Part of wire under test.	Galvanometer deflection.

The sum of all the deflections represents the deflection that would have been obtained if the galvanometer had been connected to the ends of the bridge wire.

35. The calibration curve is obtained by plotting the lengths of

the bridge wire as ordinates against the sum of the deflections obtained from the commencement to the end of these given lengths as abscissæ. Thus the ratio of the length of the abscissa at any part of the curve to the total length of the horizontal scale minus that length, will be the ratio of the resistances of the two portions into which the wire has been divided at that point. Thus let Fig. 29 represent such a curve, and let the sum of all the deflections come to 865. Then, if the balance on the wire came at 500 in a metre bridge, the ratio of the other resistances is not $\frac{500}{500}$, but $\frac{400}{465}$, on account of the inequality of the wire, 400 being the length of the abscissa

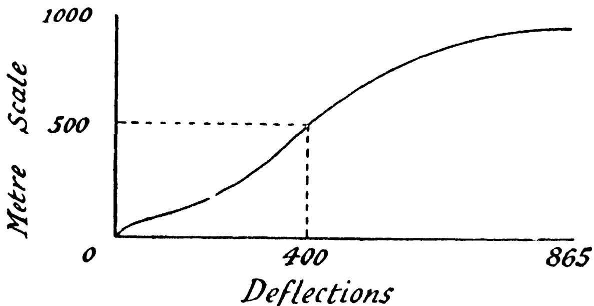


FIG. 29.

at the 500 point, and 465 is the total length of the horizontal scale minus the abscissa, or $465 = (865 - 400)$.

Such a curve having been drawn for a bridge wire, it may be pasted on a wooden board and kept beside the bridge, and when a measurement is made on the bridge, the *effective* lengths of the two portions of the wire on either side of the balance point are obtained from the curve, instead of taking the actual lengths as determined from the scale over which the wire is stretched. In all experiments like the above the hand must not be brought near the contacts, as thermo-electric currents will be set up which will interfere with the results. To make quite sure that no such effect is taking place, the battery circuit should be broken at K (see Fig. 28), when, if there is no disturbing effect in the tapping circuit, the galvanometer needle should remain at zero.

36. The following are the data obtained in the calibration of the wire of a metre bridge, the sensitiveness of the galvanometer being such that a variation of the distance between the tapping-points of 1 mm. represented a variation of about six scale-divisions on the galvanometer scale :—

Tapping-points.	Galvanometer deflection.	Sum of deflections.
(centimetres).		
0–6	418	418
6–11	336	754
11–16	337	1091
16–21	336	1427
21–26	335	1762
26–31	330	2092
31–36	330	2422
36–41	330	2752
41–46	338	3090
46–51	335	3425
51–56	336	3761
56–61	336	4097
61–66	337	4434
66–71	337	4771
71–76	337	5108
76–81	325	5433
81–86	334	5767
86–91	330	6097
91–96	335	6432
96–100	251	6683

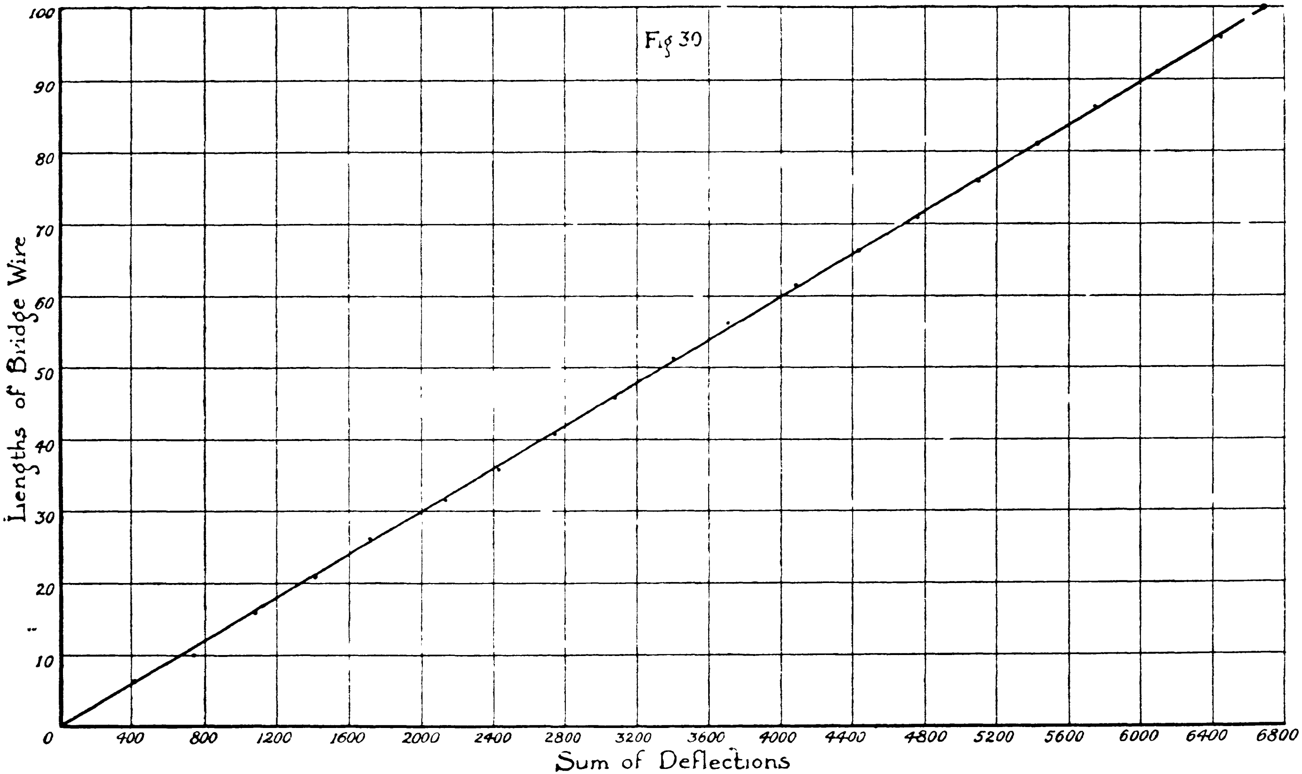
Fig. 30 represents the curve drawn from the above data.

In this case it will be seen that if balance was obtained at the middle point on the wire, instead of the resistances being equal they would be in the ratio of—

$$\frac{3360}{6683 - 3360} = \frac{3360}{3323} = \frac{1\cdot01}{1}$$

In the following modification of the last method of calibrating the wire, which, if not so accurate, has the advantage of being simpler and of not requiring any special tapper, the sliding contact of the bridge is itself used as a movable contact-piece, the other contact being fixed permanently to one end of the bridge wire. Fig. 31 shows the connections of the apparatus. The battery B, with a large resistance, R, in series with it, is

Fig 30


Lengths of Bridge Wire
100
90
80
70
60
50
40
30
20
10
0
400
800
1200
1600
2000
2400
2800
3200
3600
4000
4400
4800
5200
5600
6000
6400
6800
Sum of Deflections

connected to the ends of the wire XY through the key K. One terminal of the galvanometer is connected to X, while the other goes to the movable contact-piece P of the bridge. Readings are taken with the contact-piece P at various positions between X and Y, say every centimetre, until the middle point of the wire is reached, when the wire from the galvanometer is disconnected from X and connected to Y, the readings being then continued until P reaches Y. The object of changing the galvanometer terminal from X to Y when half the wire has been calibrated is

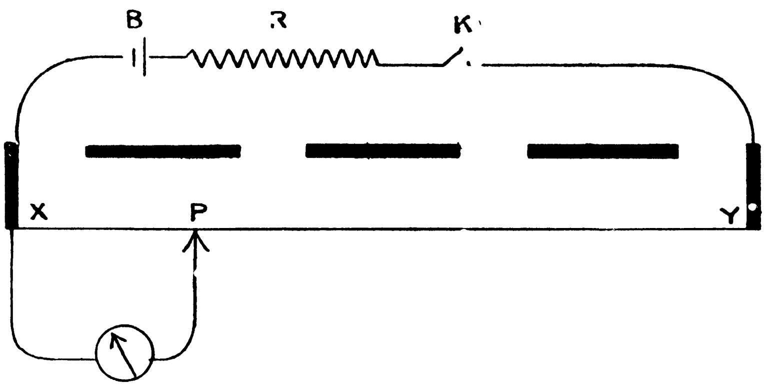


FIG 31.

in order to allow larger deflections to be obtained on the galvanometer, since the maximum P.D. at the terminals of the tapping circuit will then be half the total fall down the wire, whereas, if the change had not been made, the total P.D. at the tapper and down the wire would have been the same. The change of course alters the direction of the current in the galvanometer, unless a reversing key is placed in its circuit. The results are tabulated thus—

Distance, XP.	Galvanometer deflection.

From this table the deflections corresponding to equal lengths of the wire may be calculated, and the result plotted graphically, as in the last case. The remarks on thermo-electric effects at the tapping-point apply equally to this method, and the test for such an effect should be applied between each pair of readings. If such an effect persists it must be allowed for by adding or subtracting the deflection produced by it from the deflection when the current is flowing in the wire, according as it acts against, or with, the main effect.

The galvanometer deflections are supposed to be proportional to the currents flowing through it; the instrument must be calibrated, however, and, if necessary, corrections applied from its calibration curve, should it not be a straight line.

37. The following numbers were obtained in calibrating a wire according to the above method. On account of a deflection due to thermal currents in the conductors a correction had to be applied to each reading.

Distance along XY	Deflection due to thermal effect.	Deflection on tapping.	Corrected deflection.	Deflection per 10 cm.
0–10	−1·0	+34·5	35·5	35·5
0–20	−1.5	+68·5	70·0	35·5
0–30	−2·0	+103·5	105·5	34·5
0–40	−2·0	+139·5	141·5	36·0
0–50	−3·0	+175·0	178·0	36·5
100–90	−5·5	+29·5	35·0	35·0
100–80	−4·5	+64·5	69·0	34·0
100–70	−4·5	+99·0	103·5	34·5
100–60	−4·5	+134·5	139·0	36·5
100–50	−3·5	+171·5	175·0	36·0

From these numbers the following table can be calculated from which the curve is plotted (see Fig. 32), a previous experiment having shown that the galvanometer deflections were proportional to current:—

Part of wire.	Deflection.
0–10	35·5
0–20	70·0
0–30	105·5
0–40	141·5
0–50	178·0
0–60	214·0
0–70	250·5
0–80	285·0
0–90	319·0
0–100	354·0

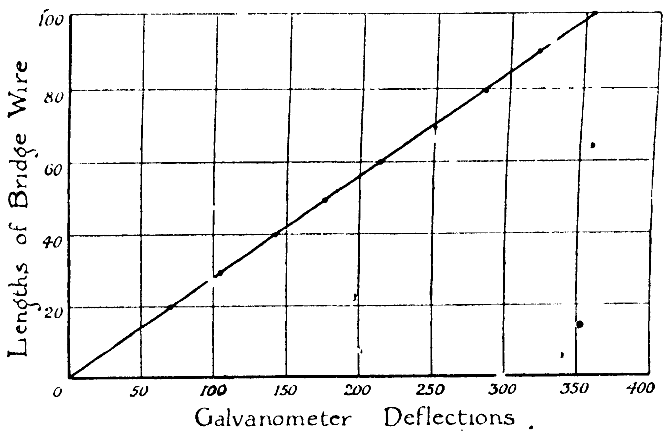


FIG. 32.

38. *Correction for the Contacts at the Ends of the Wire and the End-Pieces of the Bridge.*—In bridges where the wire is soldered at X and Y, or as in Fig. 23, where it is clamped by copper clamps, if the contact is not good a small resistance is introduced at each end of the wire in addition to the resistance of the copper end-pieces, which practically lengthens it. The value of the resistance thus introduced is best determined in terms of a length of the wire, and the correction is applied as such. One method of determining this is to insert known resistances R_1 and R_2 in the gaps; the ratio of $\frac{R_1}{R_2}$ should be

large. Let the balancing-point be B_1, and let the distance $XB_1 = x_1$, also let the length $XY = l$, σ_1 = resistance at the end, X and σ_2 = resistance at the end Y, expressed in terms of a length of bridge wire. Having got a balance, R_1 and

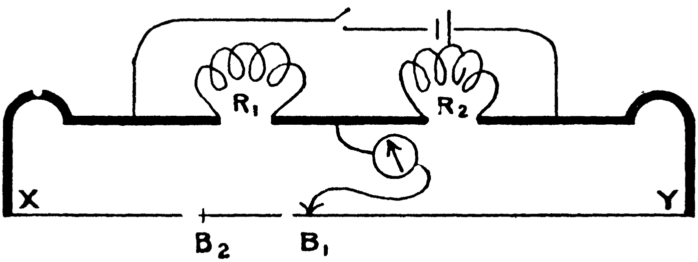


FIG. 33.

R_2 are interchanged, and a second balancing-point, B_2, is obtained, let $XB_2 = x_2$. Then—

$$(1)\quad \frac{R_1}{R_2} = \frac{x_1 + \sigma_1}{(l - x_1) + \sigma_2}$$

and—

$$(2)\quad \frac{R_1}{R_2} = \frac{(l - x_2) + \sigma_2}{x_2 + \sigma_1}$$

Calling the ratio $\frac{R_1}{R_2} = K$, we get—

$$K(x_2 + \sigma_1) = l - x_2 + \sigma_2$$

$$\frac{1}{K}(x_1 + \sigma_1) = l - x_1 + \sigma_2$$

Whence subtracting we get—

$$\sigma_1 = \frac{x_1 + K(x_1 - x_2) - K^2 x_2}{K^2 - 1}$$

In a similar manner we get—

$$\sigma_2 = \frac{K(x_2 - x_1) - l(1 - K^2) + x_2 - K^2 x_1}{1 - K^2}$$

We thus have the effects of the end resistances expressed in terms of the effective lengthening which they produce on the bridge wire. In making the above measurement it is assumed

that when taking the bridge wire readings corresponding to the ratio of the resistances R_1 and R_2, allowance is made for any irregularity in the bridge wire by the aid of a previously determined calibration curve.

39. The following data were obtained in order to determine the resistances of the side-pieces and end-contacts in a metre bridge :—

R_1 was a standard coil of 100 ohms resistance,
R_2 was a standard coil of 10 ohms resistance.

The temperature at which the measurement was made was 15·5° C., and was that at which the coils were correct.

The first balance on the wire was obtained at 91·15 cm. The second balance, after interchanging R_1 and R_2, was at 8·90 cm.

$$\text{Hence } \sigma_1 = \frac{91{\cdot}15 + 10\,(91{\cdot}15 - 8{\cdot}90) - 100 \times 8{\cdot}90}{10^2 - 1}$$

$$= 0{\cdot}23 \text{ cm.}$$

$$\text{and } \sigma_2 = \frac{10(8{\cdot}90 - 91{\cdot}15) - 100(1 - 10^2) + 8{\cdot}90 - 10^2 \times 91{\cdot}15}{1 - 10^2}$$

$$= 0{\cdot}28 \text{ cm.}$$

40. *Fall of Potential Method of finding the End Resistances on a Metre Bridge.*—The following method may be employed instead of the last, in order to determine the resistances of the

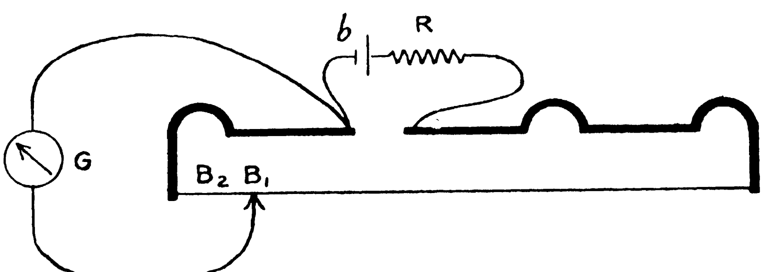


FIG. 34.

ends of the bridge. The bridge wire is connected in series with a battery b of constant E.M.F., and a large resistance R (see Fig. 34). The galvanometer G has one of its terminals

connected to one of the bridge side-pieces and the other to the movable tapping-contact, and its sensitiveness is so adjusted that when the tapping-point B_1 is about 1 cm. from the end of the wire a large deflection is obtained. Galvanometer readings are taken with the tapping-point at B_1, B_2, etc., points about 0·1 cm. apart. The first reading should from time to time be repeated in order to test the constancy of the current in the wire. A similar set of readings are taken with the galvanometer, at the other end of the wire.

If curves are now plotted with the galvanometer deflections for ordinates, and the tapping-points as abscissæ, they will be found to run as shown in Fig. 35. The curve obtained does

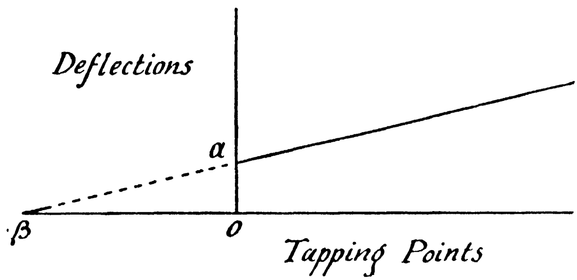


FIG. 35.

not run to zero, but to some point, α, on the vertical scale; this is on account of the resistance of the end-contact. If we project the curve backwards till it cuts the horizontal axis at β, then the length $o\beta$, measured on the same scale as the horizontal axis to the right of the zero, represents the length of bridge wire which would have the same resistance as that of the end-contact.

This method will be found very convenient, and its accuracy depends on the number of readings taken at each end of the wire.

41. To determine the resistances at the ends of a bridge wire, the following data were obtained from the calibration of a centimetre of wire at each end of the bridge by means of the fall of potential method :—

I. Left-hand end of wire—

Division of wire.	Galvanometer deflection.
centimetre.	
0–0·1	86
0–0·9	78
0–0·8	72
0–0·7	65
0–0·6	59
0–0·5	50
0–0·4	42
0–0·3	36
0–0·2	29
0–0·1	22

II. Right-hand end of wire—

Division of wire.	Galvanometer deflection.
centimetres	
99·0–100	92
99·1–100	82
99·2–100	74
99·3–100	65
99·4–100	55
99·5–100	47
99·6–100	38
99·7–100	30
99·8–100	22
99·9–100	13
99·95–100	8

The curves drawn from these data are shown in Figs. 36 and 37.

42. *The Tapping Error.*—A source of error is sometimes introduced into bridge measurements on account of the pointer which indicates the position of the tapping knife-edge on the wire, not being situated immediately above the knife-edge, but a little to one side or other of it; this is known as the tapping error, and may be eliminated by obtaining two balances with the coils interchanged. Thus let R and x in Fig. 38 represent the two coils whose resistances are to be compared. Also

let B_1 represent the balancing-point, as indicated by the pointer attached to the tapping-key, and let the true point of contact of

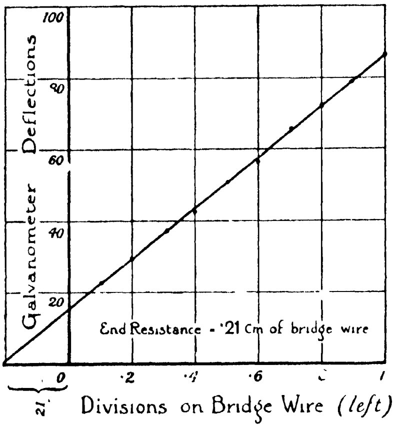


FIG. 36.

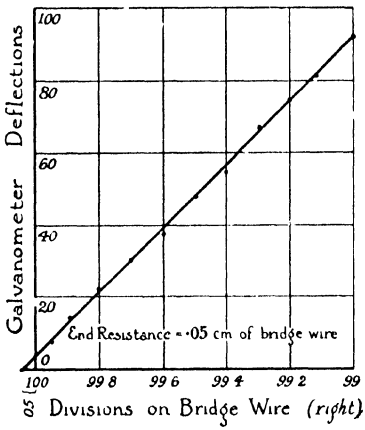


FIG 37.

the knife-edge be at a distance τ on the Y side of B_1. Then—

$$\frac{R}{x} = \frac{XB_1 + \tau}{l - (XB_1 + \tau)}$$

is the length of the bridge wire XY.

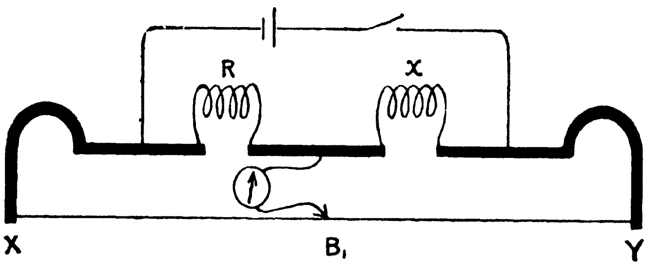


FIG. 38.

On interchanging the coils R and x, a second balancing point, B_2, will be obtained, and—

$$\frac{R}{x} = \frac{l - (XB_2 + \tau)}{XB_2 + \tau}$$

By adding together the numerators and denominators of these two equations, we get—

$$\frac{R}{x} = \frac{XB_1 + l - XB_2}{l - XB_1 + XB_2}$$

An expression which, it will be noticed, is free from τ. So, by taking the mean value of the resistance of the coil x in the two positions, the tapping error, if any exists, will be eliminated.

43. *The Thermo-Electric Effect.*—When a metre bridge is connected up to measure the resistance of a coil, the circuit may include several junctions of dissimilar metals where thermo-electric effects might occur. There are always the contacts between the stretched wire and the copper end-pieces; and if the wire to be measured is not of copper, it will also introduce two such junctions; besides these, there is the junction between the wire and the tapping knife-edge. Should these junctions be at different temperatures, thermo-electric currents will be set up in the network which will disturb the balancing of the resistances. The tapping-point is one of the most likely places for such an effect to occur, since it is liable to get heated by radiation from the hand; it is therefore advisable to work the tapper by means of a long ebonite rod, so as to keep the hand as far as possible from the point of contact. The whole of the bridge should be kept as far as possible at the same temperature, and thus minimize as far as possible the thermo-electric effects at the other junctions. It must, however, be borne in mind that the passage of the testing current itself will tend to produce differences of temperature at the various junctions on account of the reversible Peltier effect; it should therefore be kept small, and only allowed to flow for as short a time as possible. The presence of thermo-electric effects may easily be observed by opening and closing the galvanometer circuit, the battery being disconnected; if any deflection is obtained it must be due to thermo-electric currents. If these effects are present they may be allowed for by taking the deflection due to the thermo-electric currents as the true zero on the scale, and balancing so as to obtain that deflection. A more satisfactory method is to reverse the battery current, and take two readings

for each position of the coils, since in one case the thermo-electric effects will act with, and in the other against, the battery current, so that the mean of the two results will be free from this error. In order to diminish as far as possible any thermo-electric effect at the tapping-point, the galvanometer may be replaced by the battery in the tapping-circuit; any such effect then will only increase or decrease the battery current, and the balancing-point is independent of the current. An objection sometimes urged against this method is, that local heating might occur at the point of contact between the tapping knife-edge and the wire, and so injure the latter; this, however, is very unlikely, on account of the very small value of the bridge currents employed. There is, however, one point that must be borne in mind if this method is adopted, namely, that if there is self-induction in the circuit whose resistance is being measured, the galvanometer connection must not be made until after the battery circuit has been closed, otherwise a swing will be obtained on the galvanometer, due to the back E.M.F. of self-induction, which might be mistaken for a want of balance. This effect is only instantaneous, and occurs at the moment of making or breaking the battery circuit.

44. *Sensitiveness of the Wheatstone Bridge.*—In Vol. I. the student, when making measurements of resistance with the post-office bridge, would find out the best arrangement for the arms of the bridge, experimentally. It can, however, be shown mathematically[1] that the bridge is most sensitive when all four arms are of equal resistance, and the battery and galvanometer resistance each equal to that of one of the arms. This arrangement is, of course, not always practicable, nor can we always manage to have the battery and galvanometer resistance variable. Generally speaking, the galvanometer will be wound with two coils, one of high resistance and the other of low resistance; the unknown resistance may, however, have a value halfway between these. When this is the case, it can be shown[2] that the best arrangement for the bridge is to connect the battery or galvanometer—whichever has the higher

[1] See Gray's "Absolute Measurements," vol. i p. 331.
[2] See Kemp's "Handbook of Electrical Testing," p. 195.

resistance—across from the junction of the two higher resistances in the bridge arms to the junction of the two lower resistances.

In the wire bridge, for maximum sensitiveness, the wire should have twice the resistance of the coil to be measured. In general, the wire has a low resistance, but this can be increased by placing equal resistances in R_3 and R_4, thus virtually lengthening the wire, the resistances R_3 and R_4 being expressed either in ohms or in terms of so many centimetres of bridge wire. By employing an arrangement of this kind, the sensitiveness of the bridge is greatly increased, but, at the same time, the range of resistances capable of being compared is very much smaller, since the arrangement is equivalent to a bridge wire, perhaps ten or a hundred times the length of the bridge, but of which only a small portion in the middle is available for tapping on.

The resistances R_3 and R_4 are generally wound on the same bobbin, so that they are both subject to the same temperature variations, and therefore the ratio of their resistances will remain constant

It may be found convenient to make a set of coils of this kind for use with the bridge. one pair of 10 ohms each, one of 100 ohms, and one of 1000 ohms, the resistances being carefully marked on them in terms of centimetres of bridge wire.

45. In making a measurement of resistance, after obtaining a balance, the tapping-key should be displaced until a deflection on the galvanometer is just noted; this gives a practical test of the sensitiveness of the arrangements, and may be expressed as so many scale-divisions deflection per millimetre of bridge wire.

46. *Measurement of Temperature.*—One of the most important precautions necessary in an accurate comparison of resistances is that required in the measurement of temperature. The temperature of each of the coils employed must be accurately known; this is liable to alter, both on account of the coil being above or below the temperature of the surrounding space, and on account of the heat generated in it by the testing current. This latter cause of heating may be reduced to a minimum by keeping the testing current small, and by only allowing it to flow for very short intervals of time.

In cases where the testing current might cause an alteration of temperature of the coil, an estimate of the amount may be made if we know the value of the current employed. Calling C the current in ampères flowing in the coil, and R its resistance in ohms, the energy expended in the coil per second which causes heating equals C^2R; this produces a rise of temperature of the wire, which continues until the rate of loss of heat by cooling equals the rate of production. The heat lost per second by cooling being represented by $(t_2° - t_1°)SK$, $t_1°$ and $t_2°$ are the temperatures of the space outside the wire and the wire itself respectively, S the surface from which the radiation takes place, and K a constant depending on the radiating surface, the construction, and surroundings of the coil, and represents the number of units of heat dissipated per square centimetre per second per degree excess temperature.

47. It is almost impossible to give data for K, since it depends so largely on the construction of the coil, and should be determined experimentally from a cooling curve. The following numbers are given by Dr. St. Lindeck[1] for a manganin standard coil immersed in paraffin oil, similar to that described in par. 125:—

Excess temperature of wire.	K.
1·1° C.	0·0022
6·7°	0·0035
25·0°	0·0048
44·0°	0·0055

In order to insure constancy of temperature of the space round the coil, it should be immersed in a vessel of paraffin oil, which latter may be jacketed by a much larger vessel of water, the oil being kept constantly stirred, and a thermometer placed in it to register the temperature.

In cases where the coil has to be kept at 0° C., it is usually arranged so that it may be placed in a vessel, and packed round

[1] See *Electrician*, vol. xxxvi. p. 509.

with melting ice. Even this, however, is not always sufficient to insure a temperature of 0° C., since heat is liable to be conducted to the coil from the outside along the heavy copper connecting wires.

This uncertainty in measuring the temperature is a strong argument in favour of adjusting standard coils to a temperature near or a little above that of the average temperature of a room, in which case the standard temperature can almost always be reproduced.

48. In cases where the temperature of a coil requires to be adjusted and kept constant for some time at different values, the most convenient method is to employ some form of automatic gas regulator or "thermostat." This may take various forms, but that devised by Ostwald, which the author uses in his laboratory, is perhaps the most satisfactory. In this instrument the expansion of some liquid is employed to regulate the gas-supply to the Bunsen burner.

The liquid is contained in a long thin-walled glass vessel (a test-tube answers very well), which is immersed in the water in the outer jacket of the heating vessel; this is connected to the side tube of a U-tube by means of a long small-bore glass tube (see Fig. 39). In the bottom of the U-tube there is placed a little clean mercury. The gas-supply enters by a glass tube, which slides through a cork in the other limb of the U-tube, the end of the glass tube being cut off at right angles to the axis of the tube, the gas to the burner escaping through the side tube.

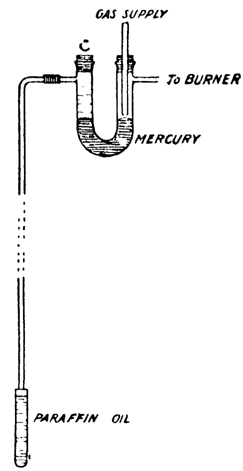


FIG. 39.

To set up the apparatus, the test-tube and connecting tube are filled with petroleum, and connected to the U-tube by means of a small piece of rubber tubing; the cork *c* is then removed, the space above the mercury filled with petroleum, and the cork replaced, care being taken not to admit any air into the tube. As the temperature rises the petroleum expands,

and forces the mercury up to the open end of the gas inlet tube, thus diminishing the gas-supply, and lowering the Bunsen flame. The temperature at which this regulation takes place may be altered by sliding the gas-supply tube further in or out of the U-tube. When working properly, the temperature may be regulated to a fraction of a degree, and maintained at that temperature for many hours.

The sensitiveness depends, to a certain extent, on the diameter of the U-tube, and Ostwald gives 3 mm. as the best size. The author, using a tube of half-inch bore, has succeeded in maintaining a temperature constant to half a degree Centigrade. The temperatures with which this regulator may be employed cannot be very high, on account of the low boiling-point of petroleum; if a high temperature is required, the petroleum may be replaced by a ten per cent. solution of calcium chloride.

In addition to the regulator the Bunsen burner should be provided with a by-pass, so as to relight the gas in the event of its being shut off at any time by the regulator, the flame of the Bunsen should also be protected from draughts and air-currents by a small metal screen.

49. All thermometers employed in the measurement of the temperatures should be graduated to $\frac{1}{10}^\circ$ C., should be compared with a standard thermometer, and, if necessary, corrections applied to their readings.

50. When the final tests are being made, no two successive tests should be made with a smaller interval of time than a quarter of an hour between them, in order to allow the coils to settle down to a steady temperature.

51. *Resumé.*—In making an accurate measurement of the resistance of a coil by means of a wire bridge, a rough measurement is first made to determine approximately what its value is. Then, if necessary, the resistances R_3 and R_4 of the proper magnitude are placed in position, and a standard resistance chosen, which is, as nearly as possible, equal in resistance to the unknown coil. The galvanometer coil most suitable is chosen, and connected to the bridge in the manner described previously. A balance is then obtained, the battery current is

reversed, and a second balance got, thus correcting for any thermo-electric effect. The standard coil and the unknown are then interchanged, and a third balance obtained, this reversal eliminating the tapping error if it exists, a reversal of the battery current, and a fourth balance again eliminates thermo-electric effects. The resistance in each case should be calculated, the effective lengths of bridge wire, as taken from the calibration curve, being used, and the correction made for the resistances of the contacts at the ends of the wire. The mean of these determinations is taken as the true resistance of the coil. The resistances of the standard and R_3 and R_4 are, of course, corrected for temperature before making the calculation, and the temperature at which the measurement is made is carefully taken.

52. The following data were obtained in the accurate measurement of a resistance.

A preliminary determination of resistance proved that the resistance to be measured was approximately 100 ohms, and the bridge wire having a resistance of 1·738 ohms, coils of resistance, approximately 100 ohms each, were introduced at either end of the bridge wire. These were as follows: R_3 = 101·76 ohms = 5855·00 cm. of bridge wire, and R_4 = 101·56 ohms = 5843·50 cm. of bridge wire; these being of manganin, as well as the bridge wire, they had no appreciable temperature correction. A previous measurement had shown the end resistances of the bridge to be respectively α = 0·23 cm. of bridge wire, and β = 0·28 cm. of bridge wire. The bridge wire was uniform. The standard resistance of 100 ohms balanced against the coil was correct for a temperature of 15·5° C., and was of platinoid.

Temperature of coil during experiment = 22·0° C.
Therefore the true resistance of standard = 100·169 ohms.

First balancing-point	19·60 cm.
Balancing-point with current reversed	19·60 „
Interchanging coil and standard, balance	70·30 „
Reversing current, balancing-point	70·30 „

$$\text{Hence (1)} \quad x = \frac{100{\cdot}169\,(5843{\cdot}50 + 0{\cdot}28 + 80{\cdot}40)}{(5855{\cdot}00 + 0{\cdot}23 + 19{\cdot}60)}$$
$$= 101{\cdot}01 \text{ ohms @ } 22^\circ \text{ C.}$$
$$\text{(2)} \quad x = \frac{100{\cdot}169\,(5855{\cdot}00 + 0{\cdot}23 + 29{\cdot}70)}{(5843{\cdot}50 + 0{\cdot}28 + 70{\cdot}30)}$$
$$= 101{\cdot}04 \text{ ohms @ } 22^\circ \text{ C.}$$

Therefore the mean value is $x = 101{\cdot}02$ ohms @ 22° C.

The Carey Foster Bridge.

53. The method of using the wire bridge due to Professor Carey Foster is specially applicable to the measurement of low resistance, since the resistance to be measured is expressed in terms of a certain length of the bridge wire. One advantage of the method is that it is independent of the contact resistances at the ends of the wire; it, however, assumes that the bridge wire has been carefully calibrated.

The connections for this method of using the bridge are shown in Fig. 40. In the two middle gaps of the bridge are

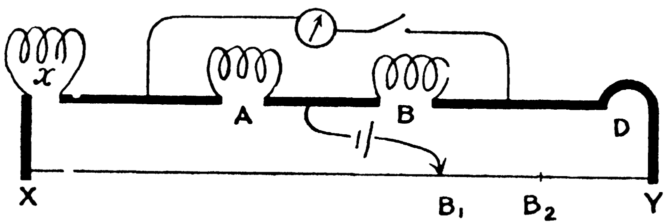


Fig. 40.

placed two coils, A and B, of known resistance, nearly equal to one another, and such that the ratio of $\frac{A}{B}$ does not differ from unity more than does the ratio of x to the resistance of the bridge wire. A and B may with advantage be wound on the same bobbin, so that they are both subject to the same temperature changes. The coil x represents the resistance to be measured, while D is a thick copper strap of negligible resistance. The connections are made as shown, and a balance is obtained at some point, B_1, on the wire. The coil x and the

strap D are then interchanged, and a balance is again got at some other point, B_2. Then we have—

$$(1)\quad \frac{A}{B} = \frac{x + XB_1}{l - XB_1}$$

where l is the length of XY;

$$(2)\quad \frac{A}{B} = \frac{XB_2}{x + (l - XB_2)}$$

From (1) and (2) we get—

$$x = XB_1 - XB_2$$

or is equal in resistance to that part of the wire between the two balancing-points, the value of which may be got from a measurement of the resistance of the whole wire, and from the calibration curve; the resistance of $XB_1 - XB_2$ bearing the same ratio to resistance of XY as the deflection produced by the fall of potential between B_1 and B_2 does to the sum of all

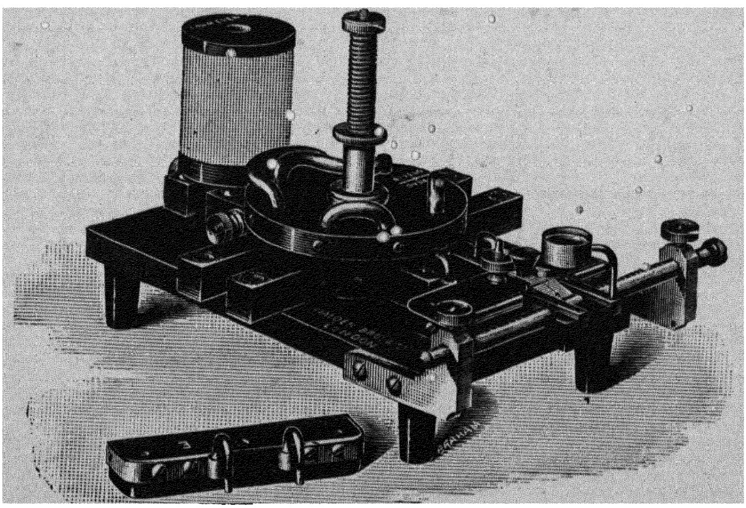


FIG 41.

the deflections obtained between X and Y in the calibration (see par. 34).

54. The above method of using the wire bridge is specially useful in comparing standard coils, or in standardizing resistance

coils, since the difference in the resistances is then very small. A special form of bridge for comparing coils is shown in Fig. 41, and was described by Mr. F. H. Nalder in the Physical Society, 1893.[1] It consists of an ebonite base, on which are fixed thick copper bars with mercury cups at their ends. The connections being as shown in Fig. 42, AA′ and BB′ are

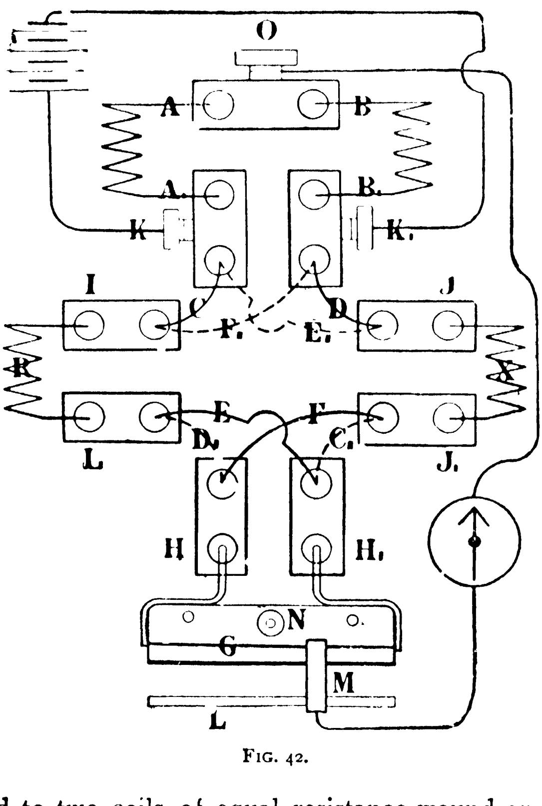


FIG. 42.

connected to two coils of equal resistance wound on the same bobbin. The standard coil is connected to IL, and the coil to be compared with it to JJ′. The bridge wire, which is very short, is mounted on an ebonite plate, and is detachable from the rest of the apparatus, so that various wires of different resistances may be used as occasion demands; it is connected

[1] See *Electrician*, vol. xxxi. p. 241.

to HH_1. The commutator for interchanging R and X consists of a circular ebonite plate, rotating about a vertical axis, to which the copper connectors, which dip into the mercury cups, are attached, a spring keeping them pressed down into the cups. When it is desired to interchange the coils, the plate is raised, then rotated through 180° and lowered. The whole instrument is very compact, and the short bridge wires greatly reduce the labour of calibration.

55. *Standardization of a Coil by the Carey Foster Bridge.*—In order to standardize a coil of resistance, say 1 ohm, by means of this method, two coils of approximately equal resistance, which should be about 1 ohm, are attached to AA′ and BB′, a standard 1-ohm coil is connected to IL, and the coil to be standardized to connected to JJ′; a bridge wire of low resistance, the value of which is known, and which has been carefully calibrated, is used. The battery and galvanometer connections are made, and a balance is obtained at some point on the wire. Should it be found impossible to get a balance on the wire, then the resistance of the unknown coil must differ from that of the standard by an amount greater than the resistance of the bridge wire, and the coil must be removed and roughly readjusted until a balance can be obtained on the wire. The position of the standard and unknown coil is then interchanged, and a fresh balancing-point obtained on the wire. The resistance of the bridge wire between the two balancing-points represents the difference in resistance between the standard and unknown coil, and can, as previously stated, be determined from the resistance of the whole length of the wire and the calibration curve.

In making such a comparison, all the corrections employed in making a measurement with the metre bridge equally apply. It will be noticed that it is not necessary to know the resistances of the two middle coils, or even their ratio, provided the latter keeps constant during the measurement, and for this reason it is usual to wind them together on a single bobbin, so that temperature changes will affect both equally.

56. The following experiment was made to determine the

resistance of a piece of copper wire 64 cm. long and 0·114 cm. diameter.

The coils A and B were standard 1-ohm coils of platinoid, both at the same temperature, which was also the temperature of the rest of the apparatus and of the copper wire, viz. 21·5° C. D consisted of a very thick strap of copper, the resistance of which was negligible.

First balance	50·18 cm.
Balance with current reversed	50·14 ,,
Resistances interchanged balance	49·60 ,,
Balances with current reversed	49·59 ,,

Mean balance in first case	50·16 cm.
Mean balance with coils interchanged	49·59 ,,
Difference	0·57 ,,

The resistance of the wire was therefore equivalent to the resistance of 0·57 cm. of the bridge wire. This resistance had previously been determined to be 1·738 ohms, and the wire being of manganin, the temperature variation of resistance is negligible. The ratio of the resistance of the part of the bridge wire between 49·59 and 50·16 to the resistance of the whole wire, was found from the calibration curve to be the ratio of 36 to 6683. Hence the resistance of the copper wire is—

$$= \frac{1·738 \times 36}{6683}$$

$$= 0·00936 \text{ ohm @ } 21·5° \text{ C.}$$

Measurement of Very Low Resistance.

57. In measuring accurately resistances below $\frac{1}{10}$ ohm, it is advisable to employ either the wire bridge, after the method of Carey Foster, or one of the fall of potential methods to be described. The first of these involves the reading of the deflections of a galvanometer, the other is a zero method.

In the first method, the resistance to be measured is connected in series with a standard resistance, which should have a value as nearly as possible equal to it (the method of constructing such low-standard resistances being described in

pars. 61–63), a current from a battery, which has a regulating resistance, r, in series with it, being sent through the two resistances (see Fig. 43). Wires from the ends of the two resistances X and R are taken to a Pohl's commutator, K, to which the galvanometer G is also connected, so that it may either be placed across the ends of X or R, according as the rocking lever of the key is turned to one side or the other. The galvanometer G must be made very sensitive, so that when connected across the ends of the smaller of the two resistances a considerable deflection may be obtained, whilst the current flowing through the coils must not be allowed to be so large as to alter their resistance by heating. The galvanometer is connected across the ends of X, and the deflection recorded. The key is then quickly turned over so as to connect the galvanometer across the ends of R, and the deflection obtained is again recorded. To insure that in the mean time the current from the battery has remained constant, the reading across the ends of X is repeated; if it is still the same, the current flowing may be assumed to have remained the same. If the galvanometer deflections are directly proportional to the currents passing through it, then the two resistances are directly proportional to the deflections; if not, the relative values of the two potential differences may be obtained from the calibration curve of the galvanometer.

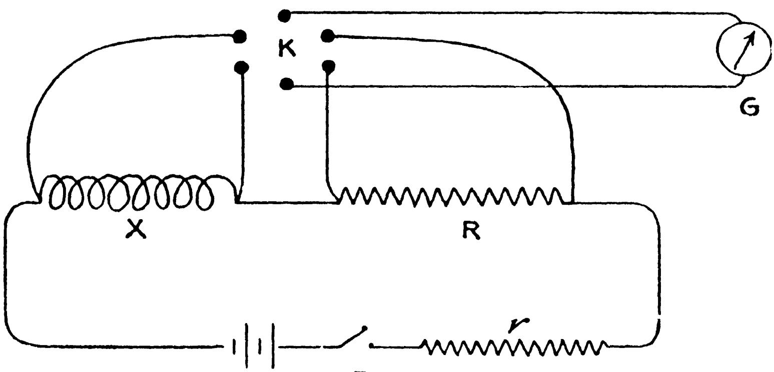


Fig. 43.

58. The following experiment was performed in order to measure the resistance of a manganin resistance coil.

The coil to be measured was placed in series with a standard coil of manganin, whose resistance at 15° C. was 0·00999 ohm, an additional resistance, and a secondary battery and key. The galvanometer employed was a sensitive D'Arsonval instrument. On completing the battery circuit, and connecting the galvanometer across the terminals of the standard coil, a deflection of 185 scale-divisions was obtained; while when placed across the terminals of the coil to be measured, it gave 190 scale-divisions. On repeating the first experiment, the deflection was found still to be 185, thus showing that there had been no alteration of resistance due to heating. On consulting the calibration curve of the galvanometer, it was found that deflections in the ratio $\frac{190}{185}$ corresponded to currents in the ratio of $\frac{0·000201}{0·000196}$

$$\text{Hence } \frac{x}{0·00999} = \frac{0·000201}{0·000196}$$

$$\text{and } x = 0·01019 \text{ ohm}$$

The temperature during the experiment was 15·6° C., so that no correction was required for the resistance of the manganin standard.

59. The other fall of potential method consists in comparing the fall of potential down the unknown resistance and standard with that down a calibrated wire, the ratio of the resistances being the ratio of the lengths of calibrated wire down which there is the same fall of potential. A calibrated Wheatstone bridge wire may be employed as follows. A battery is connected up so as to send a constant current through a bridge wire (Fig. 44), the gaps in the bridge being connected over with thick copper straps, except one where the battery is inserted, and one where a regulating resistance, r_1, is included. The resistance to be measured and the standard resistance are connected in series with a battery and regulating resistance, r_2. Wires from the ends of these are brought to a Pohl's commutator, K, whilst the other terminals of the commutator are connected, one with the end of the bridge wire at A, and the other

through a sensitive galvanometer. G, with a tapping contact, B. The currents in the two circuits must be arranged so that the fall of potential down the bridge wire is greater than the fall down the larger of the two resistances, X and R; also so that the value of the current in either circuit is not likely to produce heat sufficient to alter the value of the resistance, and that the currents in the two are in such a direction that a balance is

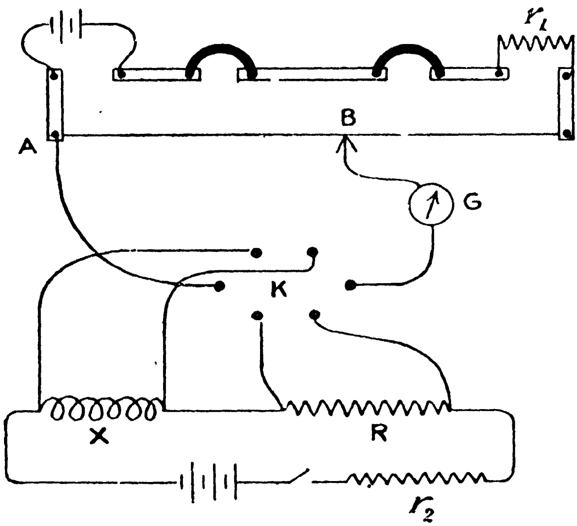


FIG. 4.

obtainable. The commutator K is then arranged so that the ends of the resistance X are connected to A and B, and a balance is obtained by adjusting the position of the contact B until there is no deflection in the galvanometer; the tapping-point is noted and recorded. The key is then altered so that R is connected to A and B, and a second balancing-point obtained. To insure that the currents have remained constant during the change, the first reading is repeated. If this is the same, then the resistances of X and R are directly as the lengths of wire AB on which a balance was obtained for each respectively, assuming the wire uniform; if not uniform, the relative lengths are obtained from the calibration curve of the wire. It must be remembered in this measurement that

the contact resistance at the end of the bridge wire is included in addition to the resistance of the portion of wire AB.

60. The following comparison of resistances was made by this method.

A platinoid standard coil of 0·0100 ohm resistance at 15·5° C. was placed in series with the coil to be measured, the rest of the connections being as shown in the above diagram. A balance was obtained for the standard coil when the tapper of the bridge was placed at 85·35 cm., and for the unknown coil at 86·01 cm. The temperature of both coils was 15·6° C. No temperature correction was therefore judged necessary for the platinoid standard. The left-hand end contact of the bridge had a resistance equivalent to 0·22 cm. of bridge wire. The calibration curve of the wire showed that the lengths of wire, 85·35 and 86·01, were proportioned to resistances in the ratio of 4262 and 4298 respectively. So that

$$\frac{x}{0 \cdot 0100} = \frac{4262 + 0 \cdot 22}{4298 + 0 \cdot 22}$$

$$x = 0 \cdot 00991 \text{ ohm @ } 15 \cdot 6^\circ \text{ C.}$$

61. *Differential Galvanometer Method of comparing Resistances.*—In the preceding sections the various modifications of the Wheatstone Bridge method of comparing resistances have been fully discussed; this method, especially the Carey Foster modification, being by far the most satisfactory one to employ when the resistances to be compared are nearly equal in value. When, however, we have to compare resistances of widely different values—such, for instance, as when a $\frac{1}{10}$ ohm coil is to be compared with a 1 ohm standard, the bridge method is not very satisfactory, and the following method, employing a differential galvanometer, should be adopted. Before describing the method, it will be as well to indicate that a differential galvanometer is one wound with two independent coils of exactly equal resistance, and each producing exactly the same magnetic effect on the needle when carrying the same current strength. The method of comparison is as follows. The resistances to be compared are connected in series, with a battery, key, and adjustable resistance (see Fig. 45). The

differential galvanometer coils are connected one across the ends of each of the resistances X and R, that across the ends of R having an adjustable resistance, R_1, added. The coils are so connected that the currents circulating in them will produce

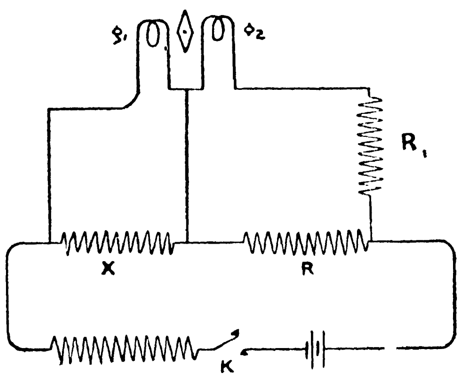


FIG. 45.

a differential magnetic effect on the needle. The resistance R_1 is adjusted until no deflection is obtained when the key K is closed, indicating that currents of equal strength are circulating in the galvanometer coils. If g_1 and g_2 are the resistances of the galvanometer coils, then

$$\frac{X}{R} = \frac{g_1}{R_1 + g_2}$$

and

$$X = R \frac{g_1}{R_1 + g_2}$$

The resistances of the galvanometer coils must be known accurately at the temperature at which the instrument is during the test, and as these coils will in all probability be of copper wire and have a large temperature coefficient, it will be necessary to place a thermometer inside the case of the instrument, and to calculate the true resistance at the indicated temperature. In some cases where the ratio between X and R is very large, it may be found necessary to shunt the coil g_2 as well as add a resistance R_1 in series with it. This, of course,

will make the calculation of the resistance R a little more complicated.

62. Before making the test indicated above, it will first be necessary to test the galvanometer for adjustment; in other words, to find if it is truly differential. This is very easily done by the two following tests made in the order named. The two galvanometer coils are first connected in series, but so that the current in them will produce a differential magnetic effect on the needle; a small current is then sent through them. If a deflection is obtained, it indicates that the coils do not produce equal magnetic effects on the needle when carrying the same current. The adjustment to perfect equality is usually made by adjusting the position of a small auxiliary coil, attached to one of the galvanometer coils, with respect to the needle till the spot of light on the galvanometer scale returns to zero. The test for equality of resistance is then made by connecting the galvanometer coils in parallel (see Fig. 46), and sending a current from a battery through them so as to produce a differential magnetic effect at the needle. If a deflection is obtained, it indicates a difference of resistance of the coils, and resistance must then be added to one of the coils until this deflection is reduced to zero.

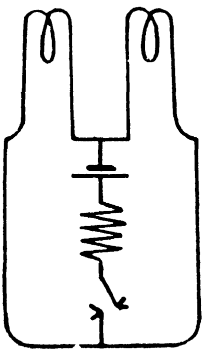


FIG. 46.

62*a*. The adjustment of a differential galvanometer as above is a somewhat tedious operation, and in cases where the highest accuracy is not required, a most useful modification of the method has been proposed by Mr. C. W. S. Crawley,[1] which greatly facilitates the rapidity of testing. In Mr. Crawley's method no attempt is made to get an accurate magnetic balance at all between the two coils. The galvanometer coils are wound with twin wire to a resistance of 100 ohms; they are then joined up in parallel as in Fig. 47, so that the magnetic effects of the two coils on the needle are differential.

In series with one of the coils is a $\frac{1}{10}$-ohm resistance with a

[1] *Jour. Elect. Eng.*, vol. 30, p. 908.

key, which admits of it being short-circuited, also a slide wire S of 1·5 ohms resistance. In circuit with the other coil is a resistance-box to enable a balance to be obtained. When connected, as in Fig. 47, the contact on the slide wire is adjusted till a balance is obtained. The $\frac{1}{10}$-ohm coil is now short-circuited; this disturbs the balance by $\frac{1}{10}$%, and causes a deflection of some hundreds of scale divisions. It is now known that two equal and opposite E.M.F.'s at the terminals

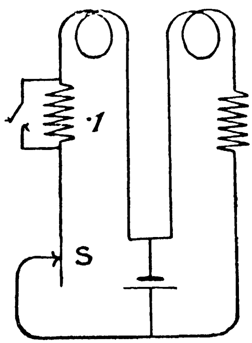


FIG. 47.

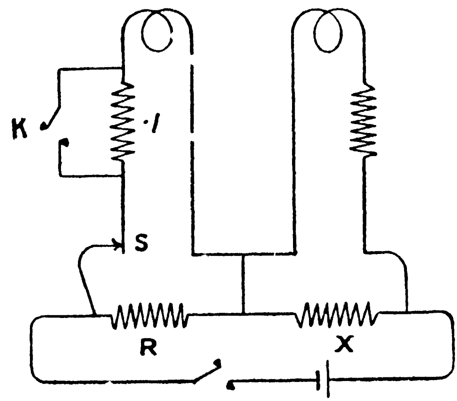


FIG. 47*a*.

of the two coils will balance. To compare the resistances of two coils the connections are made as in Fig. 47*a*.

If the two coils R and X are not equal in resistance, there will be a deflection which is noted. Closing the key K gives the deflection for $\frac{1}{10}$ % difference between them, so that the ratio of the two resistances can be very rapidly arrived at. Coils of large ratio as 2 to 1 or even 10 to 1 may be compared by the same method. As to sensibility, the following numbers have been given by Mr. Crawley. With 0·1 volt at the terminals of the coils R and X on a galvanometer with a 10-second swing, a deflection of 450 scale divisions was obtained for a difference of $\frac{1}{10}$ %, so that two coils of $\frac{1}{1000}$ ohm each could be compared, using a current of 10 ampères, and a deflection of 4·5 scale divisions would indicate a difference of 0·01%

63. A form of low resistance which is very easy to make and adjust, and which will carry strong currents, can be constructed out of a sheet of manganin as follows. A rectangular sheet of manganin of proper thickness has a series of saw-cuts made in it, as shown in Fig. 48, thus forming a zigzag strip of manganin, as shown by the shaded portions. Large terminals, A and B, are attached to the ends of the strip to convey the current to it, while small potential terminals, T_1, T_2, are bolted to the plate at some distance from them. The resistance of the strip is taken between T_1 and T_2. The end strips of the resistance are at least twice the breadth of the others, and the holes into which T_1 and T_2 are bolted are oblong, so as to admit of a small adjustment. A preliminary adjustment is made by altering the positions of T_1, T_2 in the rectangular holes, the final adjustment being obtained by filing the saw-cuts deeper. The end strips are screwed to ebonite blocks raised above the surface of the base-board, so as to allow the air to circulate about the resistance, which may be coated over with lamp-black, in order to make it a better radiator of heat.

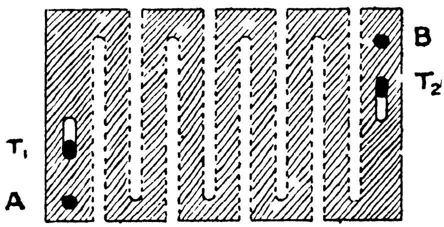


FIG. 48.

64. *Making and adjusting an Ordinary Resistance Coil.*—In making a resistance coil of an ohm or a few ohms in value, a piece of well-insulated, double silk-covered manganin wire is selected of a length considerably more than will be required. This must first be artificially aged by heating in an air bath to 140° C. for several hours; its resistance is then measured roughly, and a piece cut off having a resistance slightly above that required. The wire is then doubled on itself and wound on a bobbin or frame of ebonite; the ends, and the part of the wire at the bend, where it is doubled on itself, are left free. The ends may now be soldered to two thick copper wires or terminals, and the whole wire varnished over with shellac varnish and again heated in the air bath at

$140°$ C. for several hours. If it is required to be very accurate, the coil should be reheated several times, extending over a few weeks, in order to allow it to recover from strains that may have been set up in winding, and to come to a permanent state. This latter condition may be found by measuring the resistance of the coil from time to time against a standard coil, it always being at the same temperature during the measurement, until it shows no permanent alteration in resistance after heating and cooling. In order to finally adjust it to the correct value we may proceed in two ways.

(1) The coil should be connected up to a Metre or Carey Foster bridge and placed against a standard resistance coil, both coils being placed in thermostats adjusted to temperatures, which in the case of the standard is that at which it is correct, and in the case of the coil to be measured should be about the temperature of the room (*i.e.* $15°–16°$ C.). The tapping-contact of the bridge is adjusted to the reading at which it should be when the coil is exact, allowance being made for inequalities in the bridge wire and for end contact resistances, etc. The current is then momentarily sent through the bridge; if the galvanometer deflects, the coil is not exact. The free end at the bend is now bared of its insulation, and the wires twisted together by means of a pair of pliers till a balance is obtained. A drop of solder is then run into the twisted wire to keep it in position, and the bared part carefully insulated and varnished over with shellac. Should the drop of solder make the resistance slightly too low, it may be readjusted by scraping the wire a little, either with a knife or with sandpaper, until exact balance is obtained.

65. (2) The other method of adjusting, although simpler to carry out, is apt to make the coil bulky and, consequently, slow to alter in temperature.

The coil is placed in a thermostat as before, regulated to the temperature at which it is desired that it should be exact. Its resistance is then accurately measured on a bridge, and its excess over the desired value calculated. A simple calculation is now made to find the resistance of a wire which, if placed as a shunt across the coil, will make it the exact value. If S is

the resistance of the shunt coil required, r the resistance of the coil as measured above, and R the correct value required, then—

$$S = \frac{rR}{r - R}$$

The wire for the shunt coil should be of much smaller gauge than the wire of the coil, and should be of manganin which has been artificially aged by heating. The required length is soldered on to the main terminals and wound non-inductively round the bobbin. This method of shunting cannot well be applied to coils above 10 ohms resistance.

MEASUREMENT OF VERY HIGH RESISTANCE.

66. In the case of resistances too high to be measured accurately on the Wheatstone bridge, such, for instance, as those over a megohm, special methods have to be employed and special precautions taken. Also in the measurement of the dielectric resistance of cables, the E.M.F. applied in the test must always be in excess of the E.M.F. likely to be employed with the cable; thus electric light leads intended to carry current at a pressure of 100 volts, say, must be tested for insulation with an E.M.F. of 100 or 200 volts at least, and not of 1 or 2 volts, since the insulation might be sufficient for a low voltage, whereas it would be broken down by the greater strain of the high.

The first method to be described, which is also the simplest, and applicable for measuring all except exceedingly high resistances, is that known as the direct deflection method, the connections being as follows (see Fig. 49). A battery, B, of suitable E.M.F., is connected to the ends of a very high resistance, R_1, which should be about 10,000 or 100,000 ohms, and may consist of two or more resistance boxes in series. To the ends of R_1 is connected the resistance X to be measured, in series with a high-resistance galvanometer, G, and high-insulation key, K. The deflection obtained on the galvanometer is noted. Should this not be large, the sensitiveness of the galvanometer, or else the value of the resistance R_1, must be increased until a good deflection is obtained: call this deflection

δ_1. The resistance X is then removed, and a resistance box put in its place, the connections being altered as in Fig. 50. The galvanometer and resistance R_3 being placed across only a small part, R_2, of the large resistance R, by altering R_3 and R_2 a convenient deflection may be obtained on the galvanometer;

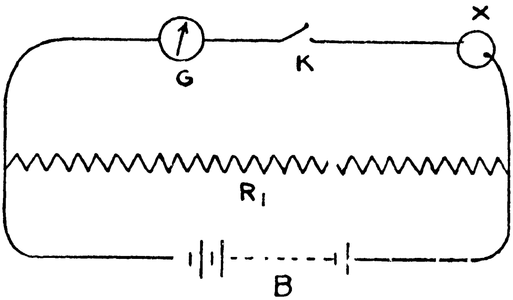


FIG. 49.

this may be arranged to be the same or nearly the same as δ_1: call it δ_2. Then, assuming the currents in the galvanometer to be proportional to the deflections (the galvanometer must be previously calibrated, and, if necessary, the relative currents obtained from the calibration curve), and calling E the E.M.F. of the

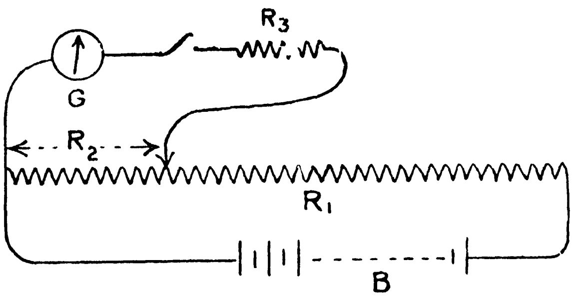


FIG. 50.

battery, in the second experiment the E.M.F. at the ends of the galvanometer circuit $= \frac{R_2}{R_1}E$, and hence we have, calling X the unknown resistance, and g the galvanometer resistance—

$$(1)\quad \delta_1 \propto \frac{E}{X + g}$$

$$(2)\quad \delta_2 \propto \frac{\frac{R_2}{R_1} E}{R_3 + g}$$

$$\text{Hence } \frac{\delta_1}{\delta_2} = \frac{(R_3 + g)\, R_1}{(X + g)\, R_2}$$

$$\text{and } X = \frac{R_1\, \delta_2}{R_2\, \delta_1} (R_3 + g) - g$$

It is here assumed that the E.M.F. of the battery B remains constant during the change, and also that $R_3 + g$ is great in comparison with R_2.

67. *Precautions.*—In all high-resistance measurement there are several practical points that must be attended to, or else the measurements will be of little value. First the experimenter must be sure that it is the dielectric resistance of the *specimen under test* that he is measuring, and not that of the apparatus employed; consequently all keys, wires, and connections from which leakage might occur, must be very carefully insulated. Should wires have to stretch long distances, they must be supported on glass or ebonite rods, or, better still, suspended by silk fibres, and in no case must the insulation of the wire alone be depended upon. Keys must be carefully cleaned and dried before use. The galvanometer should be well insulated from earth, as should also the battery and resistance R. The apparatus should be arranged so that the resistance X is between the key and galvanometer, and the high-potential terminal of the battery, as then the tendency to leakage at the key and galvanometer will not be so great.

68. If the resistance to be measured is the dielectric resistance of a long cable, it should be coiled up and placed into a large metal tank filled with water, completely covering it, except only the two ends, which are left outside. These ends require very carefully insulating, in order to prevent surface leakage.

One method of preparing the ends is to strip off 3 or 4 inches of insulation from each end, and to scrape and clean the insulation for at least 6 inches beyond that point. The whole 9 or

10 inches i then immersed repeatedly in a bath of melted paraffin wax until it is completely encased in a thick layer of insulation. The wax is then partly cut off, so as to bare the core at one end to make contact to the battery with; the other terminal of the resistance being the metal vessel which, through the medium of the water, is in contact with every part of the surface of the insulation.

69. A much simpler and better method of preventing surface leakage is that due to Mr. W. A. Price,[1] and is known as the guard-ring method. Leakage can only take place between two points at different potentials; if, therefore, the outer surface of the insulation near the ends is raised to a potential equal to that of the inner coating, there will be no tendency for leakage to take place along the end surfaces. This method of preventing leakage in the case of a cable consists in baring the ends of the cable for 2 or 3 inches, and then over the insulation, and close to the end, is placed the guard-ring, consisting of a few turns of fine bare copper wire (Fig. 51). The connections in the direct-deflection method of measuring the resistance are then as shown in Fig. 52. The guard-rings are connected directly to the positive pole of the battery, and the galvanometer, which in this case must be highly insulated from the ground, is placed between the positive pole of the battery and the cable. This, it will be seen, insures that the guard-rings and inner core are at the same potential, and consequently there will be no surface leakage over the insulation at the bared ends of the cable. This method has a great advantage over the other in being much simpler and more satisfactory.

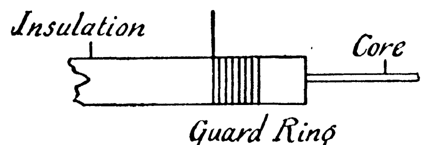


FIG. 51.

In making the test it will probably be found that the galvanometer deflection does not remain steady, but gradually diminishes; this diminution is not due to any alteration in the resistance, but to the effects of electrical absorption in the

[1] *Electrical Review*, vol. xxxvii. p. 702, 1895; also *Phil. Mag.*, vol. xlii., August, p. 150.

dielectric, and may continue for some hours. Thus it is possible, by taking readings on the galvanometer with the current left on for different lengths of time, to get a number of results which differ considerably from each other. Strictly speaking, the true resistance should be calculated only from the galvanometer deflection when it has become steady, but since this may not occur for a considerable time it is usual to

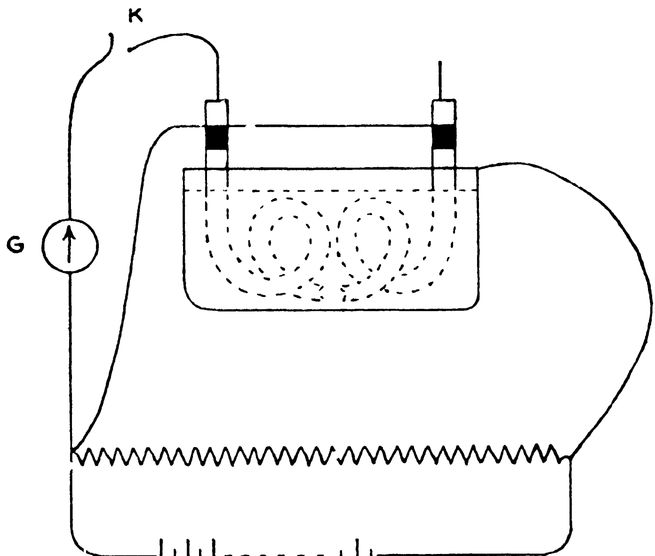


FIG. 52.

take the deflection after the current has been running for some definite time, usually one minute, this "time of electrification" of the cable, as it is called, being specified along with the resistance.

70. The temperature at which the test is made, is important, not so much on account of the variation of the specific resistance of the insulation, but on account of the large variation in the electrical absorption of the dielectric with temperature, which in some cases is as large as two per cent. per 1° C. between 0° C. and 24° C., and is greater at lower than at higher temperatures. The rate at which absorption goes on varies very

much with the material, being much larger in the case of india-rubber than with guttapercha, while it is very small in the case of mica.

Since the resistance of the galvanometer enters into the calculation, and since it is likely to be high, the temperature of the coils should be taken, in order to allow for the variation of its resistance with temperature, should the resistance have been measured at some other temperature. The insulation of the galvanometer must be tested before making the experiment, the method of doing this being given in par. 4.

71. Finally, in testing a long cable, or any resistance the electrostatic capacity of which is large, the galvanometer should be short-circuited during the time the circuit is being completed at K, otherwise the needle will receive a violent shock, due to the electrostatic charge which the cable receives.

72. The following test was made of the dielectric resistance between two coils wound on the same bobbin. The battery of two Hellesen cells in series was connected across the ends of a resistance of 10,000 ohms @ 15·5° C. The galvanometer, of resistance 6091 ohms @ 14° C., was placed in series with the unknown resistance across the ends of the 10,000 ohms resistance, and the mean of three deflections taken was 40·1 after one minute electrification. The unknown resistance was then removed, and the galvanometer terminals connected across 1 ohm of the 10,000 ohms, and the mean of three readings was 50·5. The temperature of the galvanometer, boxes of coils, and unknown resistance was 22° C.

The temperature coefficient of the 10,000-ohm coils was 0·026 per cent. per 1° C.; hence the correct value of this resistance = 10,017 ohms. The temperature coefficient of the galvanometer was 0·38 per cent. per 1° C.; hence its resistance corrected to 22° C. = 6276 ohms. Then—

$$x = \frac{R_1 \delta_2}{R_2 \delta_1}(R_3 + g) - g$$

$$= \frac{10017 \times 50{\cdot}5}{1 \times 40{\cdot}1} \times 6276 - 6276$$

$$= 78697000 \text{ ohms}$$

$$= 78{\cdot}7 \text{ megohms practically}$$

72*a*. The guard-ring method can also easily be applied to the testing of sheets of insulating material. The sheet of insulating substance is placed on a brass plate fixed to a wooden stand, and on the top of the sheet is placed a circular brass plate about $\frac{3}{8}''$ thick, and of such diameter that its surface area is 100 sq. cms. This is then surrounded by a brass ring of the same thickness—the guard-ring. The arrangement of the apparatus is shown in Fig. 52*a*.

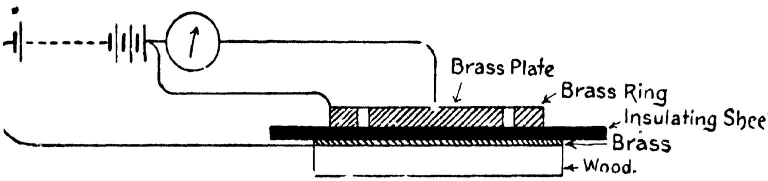


FIG. 52*a*.

73. *Loss-of-Charge Method of measuring High Resistance.*—In cases where the resistance is too high to be measured accurately by means of the direct-deflection method, such as in the case of a very short length of cable dielectric or the dielectric resistance of a condenser, some other more delicate methods have to be employed; one of these is known as the loss-of-charge method. The principle involved is briefly as follows. If one plate of a condenser is charged with a certain quantity of electricity, its potential rises, and the value of its potential is a measure of the quantity of electricity that has been given to it. If the insulation was perfect, the charge would remain on the plate, and the potential would keep constant; if, however, the charge leaks slowly away, the potential will fall, and from the measured fall of potential in a given time the quantity of electricity lost, and therefore the insulation resistance, may be calculated. Thus let R = the dielectric resistance in ohms, K = electrostatic capacity of the condenser in farads, V_1 and V_2 the values of the potential of the charged plate at the beginning and end of a time = t seconds, q = the instantaneous value of the quantity of electricity in the condenser, and v = the instantaneous value of the potential, then—

$$q = \mathrm{K}v$$

$$\text{also } dq = \mathrm{K}dv$$

But by definition of current we have—

$$c = \frac{dq}{dt} = \frac{v}{R}$$

$$\therefore \frac{K dv}{dt} = \frac{v}{R}$$

$$\text{and } \frac{dt}{KR} = \frac{dv}{v}$$

Integrating we get—

$$\frac{t}{KR} = \log_\epsilon \frac{V_1}{V_2}$$

$$\therefore R = \frac{t}{K \log_\epsilon \frac{V_1}{V_2}}$$

If K is given in microfarads, R will work out in megohms. From this it will also be seen that, since the potential is proportional to the quantity of electricity in the condenser, we might measure the quantity of electricity in the condenser at the beginning and end of the time t, instead of the potential. This may be done by means of a ballistic galvanometer, the throws of which may be taken as proportional to the quantity of electricity passing through it, as will be shown later. Calling δ_1 and δ_2 the throws obtained at the beginning and end of time t, we may write our formula—

$$R = \frac{t}{K \log_\epsilon \frac{\delta_1}{\delta_2}}$$

74. The measurement of the electrostatic capacity of the resistance, if not too small, may be determined by comparing it with a standard condenser, according to some of the methods given in Chapter V. If the capacity is very small, then it may not be possible to determine it in this way, and the following variation of the method may then be adopted. The resistance of a standard condenser is determined as above; then the resistance to be measured is placed as a shunt across its terminals, and the joint resistance of the two in parallel measured, the electrostatic capacity being taken as that of the

condenser alone. From these two measurements the unknown resistance can at once be calculated.

75. In making the measurement by observing the fall of potential, an electrometer is employed, the needle being charged heterostatically, the connections being as shown in Fig. 53. R is the resistance to be measured, the terminals of which are connected to the electrometer E, one being also earthed at *e*,

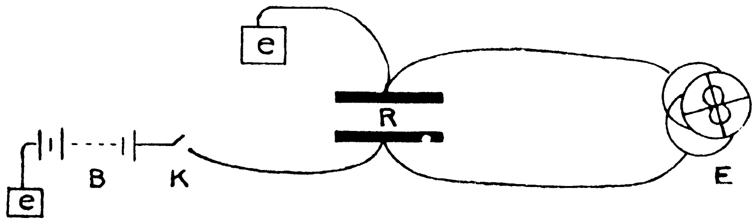


Fig. 53.

whilst the other is connected to the battery B through the high-resistance key K, the other battery terminal being earthed.

The key K is depressed, and the electrometer reading taken; K is then opened, and time readings of the electrometer deflection taken. To insure that the leakage does not ake place at the electrometer, one pair of quadrants should be charged, and then insulated, the other pair being earthed, and the deflection of the needle observed. If the insulation is good this will remain constant; if slight leakage occurs, time readings of the deflection should be taken. Two curves are then plotted on the same sheet, one giving the fall of potential,

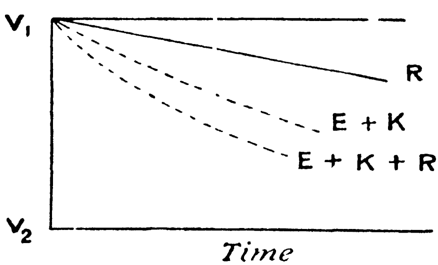


Fig. 54.

due to the combined leakage of R, K, and electrometer, and the other the leakage from K and electrometer only. By adding the difference between this latter curve and the initial value of the potential V_1 to the first curve, we get a curve representing the fall due to the resistance R alone (see Fig. 54). The

resistance may now be calculated by taking a number of pairs of readings for V_1 and V_2, the value of t, the time elapsing between them, being obtained from the curve. In each case, also, the total time that has elapsed since insulating R must be specified, in order to give some idea of the effect that electrical absorption may have on the results.

The particular advantage of the method is that the progress of the leakage can be watched over a considerable interval of time, and a large number of data obtained from which to calculate the resistance. All the disadvantages consequent on the use of an electrometer are, however, attached to the method; these will be dealt with more fully later on (see Chapter III.).

Should the guard-ring method of preventing surface leakage be employed, the guard-ring, after being raised to the same potential as the core of the cable, must be insulated from it and the charging battery, but the paraffining will be more satisfactory in this case.

76. A slight modification of this method which is sometimes employed when the rate of fall of potential is very small, is due to the late Professor Fleeming Jenkin, being known as the "Inferred zero method."[1]

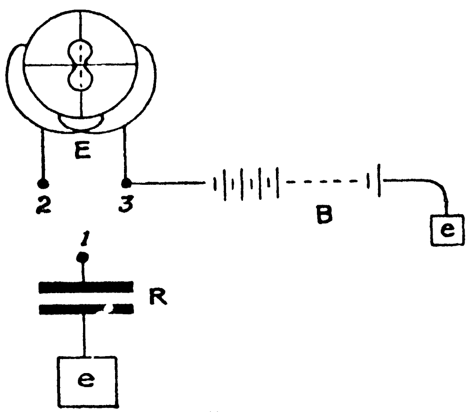


FIG. 55.

The arrangement of connections is as follows (see Fig. 55). The electrometer E, used heterostatically, is made very sensitive, and the defection produced by 1 cell of a battery, B, is noted: let it be $= \delta_1$. The positive pole of the whole battery B, of n cells, is then connected to terminals 2 and 3, which are at first connected together, the other battery pole being earthed. This brings both pairs of quadrants to the same potential, which we will call V_1, and which is n times that of one cell. If

[1] See Kemp's "Electrical Testing," p. 362.

the electrometer is properly adjusted, no deflection will be obtained. Terminal 1 is next connected to the two others, and then the connection between 2 and 3 is broken, leaving 2 connected to 1, and 3 to the battery. Leakage of the charge from the pair of quadrants connected to terminal 2 now takes place through R, and the potential of that pair of quadrants consequently falls; the other pair, being still connected to the battery, maintain their original potential; the electrometer needle therefore begins gradually to deflect. Let the deflection at the end of t seconds be δ_2, then $\frac{\delta_2}{n \times \delta_1}$ represents the fraction of its original value by which the potential has fallen in that time. We can therefore, knowing the value of K, the electrostatic capacity, determine the resistance by the aid of the relation—

$$R = \frac{t}{K \log_e \frac{V_1}{V_2}}$$

Since the rate of fall of potential is slow, the sensitiveness of the electrometer can be adjusted so that, say, a variation of one per cent. in the potential is represented by a deflection the whole length of the scale, the true zero being a long way past the end of the scale and is said to be inferred. If the cells composing the battery are all in good condition, they should have the same E.M.F. If great accuracy is desired, they must be tested individually on the electrometer, and the E.M.F. of all in series deduced from the separate deflections.

The same precautions apply to the insulation of the electrometer and keys as in the last case, and the readings may be taken from curves drawn as before.

77. In measuring the dielectric resistance of a 1-microfarad condenser by means of an electrometer, the following data were obtained.

The electrometer and key were connected to the battery, and a deflection of 100 scale-divisions was obtained; the battery was then disconnected, and the following time-readings of the electrometer deflection were taken:—

Time.	Electrometer deflection.
minutes.	
0	100·0
1	99·6
2	99·4
3	99·2
4	99·0
5	98·8
6	98·6

The condenser was then placed in circuit, as shown in Fig. 53, and the following readings weie got :—

Time.	Electrometer deflection.
minutes.	
0	100·0
1	98·4
2	97·2
3	96·2
4	95·2
5	94·2
6	93·2

The curves of fall of potential plotted from these readings

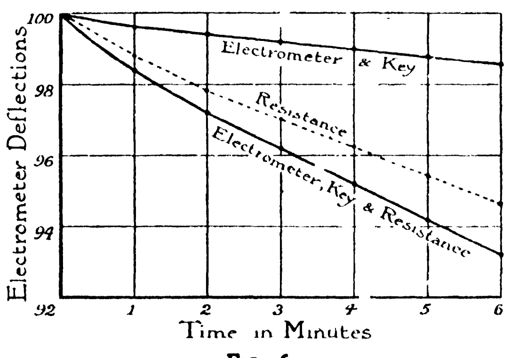


Fig. 56.

are shown in Fig. 56, the dotted curve representing the curve due of the resistance of the condenser alone.

The resistances of the condenser calculated from these readings by the aid of the relation—

$$R = \frac{t}{K \log_\epsilon \frac{V_1}{V_2}}$$

are as follows:—

Time of electrification.	Resistance in megohms.
1	5550
2	6066
3	7905
4	7905
5	7905
6	7905

78. *Loss-of-Charge Method, using a Ballistic Galvanometer.*—It has been already shown that the dielectric resistance of a cable or other high resistance may be measured from a knowledge of the loss of charge in a given time, as well as from the fall of potential, the measurement of quantity of electricity being made on a ballistic galvanometer.

The arrangement of the apparatus for this measurement is as shown in Fig. 57. G represents a highly insulated sensitive

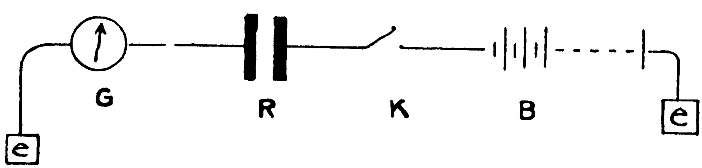


FIG. 57.

ballistic galvanometer, which is connected in series with R, the unknown resistance, K, a high-resistance key, and B, a battery; one terminal of the battery and one of the galvanometer being earthed.

The key K is closed, and the first throw of the ballistic needle δ_1, measured, this is proportional to the quantity of electricity given to R. K is now opened, and the time noted.

Opening K disconnects R from the battery, and consequently the charge on R slowly leaks away. After the lapse of t seconds, K is again closed, and the throw on the galvanometer taken: let it be δ_2; this is proportional to the quantity that has leaked away, consequently $\delta_1 - \delta_2$ will represent the quantity of charge remaining. Determining K as before, we have—

$$R = \frac{t}{K \log_e \frac{\delta_1}{\delta_1 - \delta_2}}$$

The object of charging and recharging the resistance, instead of charging and discharging, is in order to have the throws always to the same side of the scale, and to a certain extent to avoid some of the effects produced by electrical absorption.

All the apparatus, such as keys, galvanometer, and wires, must be highly insulated. It will be found better to place the galvanometer on the earthed side of R rather than between R and B, since then the chances of leakage are diminished.

79. In all measurements of this kind simplicity and cleanness in connections and apparatus is of the very greatest importance; elaborate and complicated keys, for instance, are to be avoided, as introducing great opportunities for leakage, unless kept very clean and dry. A key for a make and break, for instance, as in the last experiment, cannot be surpassed for high insulation by a block of ebonite into which two holes are drilled to contain mercury, and connections are brought to these, and the circuit may be made or broken by raising or lowering a bridging contact of copper wire, the ends of which dip into the mercury cups, and which is attached to a glass or ebonite rod by silk threads.

80. The following experiment was made in order to determine the dielectric resistance of a one-third microfarad condenser. Five sets of readings were taken, each set consisting of three readings. In every case the condenser was connected for one minute to the battery terminals before insulating; it was then insulated for a certain interval of time and recharged, the resistance being calculated from the formula—

$$R = \frac{t}{K \log_e \frac{\delta_1}{\delta_1 - \delta_2}}$$

The battery employed was a single Hellesen cell.

Throw on charge.	Time of insulation.	Throw on recharge.	Resistance.
	minutes.		megohms.
211	1	24	1588
210	1	23	1588
209	1	23	1588
216	2	45	1557
212	2	46	1506
210	2	43	1613
215	3	63	1569
215	3	63	1569
213	3	62	1569
215	4	78	1596
214	4	77	1619
214	4	78	1596
216	5	90	1677
214	5	89	1696
215	5	90	1696
Mean value ...	...		1601

Temperature = 23° C.

81. The dielectric resistance of the insulation of a short piece of copper wire covered with indiarubber and two layers of paraffined cotton was then determined according to the modification of the loss-of-charge method mentioned in par. 74. The wire was 78 cm. long, and was immersed in a copper vessel containing water, the ends being carefully insulated and brought out of the water. The following are the means of three separate readings when shunted across the condenser terminals:—

Throw on charge.	Time of insulation.	Throw on recharge.	Resistance.
	minutes.		
212·3	1	31·3	1148
211·0	2	54·0	1243
212·6	3	82·0	1126
211·3	4	98·0	1160
211·0	5	113·6	1166
Mean value ...	...		1168

Since $\frac{1}{A} + \frac{1}{x} = \frac{1}{B}$

$$\frac{1}{x} = \frac{1}{B} - \frac{1}{A}$$

$$x = \frac{AB}{A - B}$$

$$= \frac{1168 \times 1601}{433}$$

$$= 4318 \text{ megohms at } 23^\circ \text{ C.}$$

Measurement of Liquid Resistance.

82. With respect to their electrical conductivity, liquids may be divided into three classes: (*a*) Bad conductors, such as oils, etc.; (*b*) conductors such as mercury and fused metals; (*c*) electrolytes, or those which conduct and at the same time decompose, as in the case of salt solutions.

(*a*) In order to determine the resistance of liquids in the first class, those methods which have already been described for the measurement of dielectric resistance may be employed, the liquid being contained either in a tube into which dip electrodes, or else held as a thin film between two metal plates by its own surface tension. In either case great care must be taken in the measurement of the cross-sectional area of the liquid through which the current flows. This will be discussed later, in the chapter dealing with Specific Resistance Measurement.

(*b*) Good conducting liquids, like mercury, present no particular difficulty in the measurement of resistance, except that of the exact measurement of the length and area of the column tested.

(*c*) The last class of liquids—electrolytes—however, present certain difficulties which prevent ordinary methods of measurement from being applicable. When a current is sent through an electrolyte the liquid decomposes, and the polarization at the electrodes sets up a back electromotive force, which, if ordinary methods of measurement were employed, would be measured as a resistance; consequently it becomes necessary

to adopt some method in which polarization is either prevented from taking place or else is allowed for. An example of the latter method has been given in Vol. i. p. 56. It is not, however, very reliable, since the assumption that the polarization remains constant during the various measurements is found not always to be borne out in practice.

83. Regarding the methods in which the polarization is prevented from taking place, the best known is that due to Kohlrausch.[1] In this method the liquid is contained in a tube, into which suitable electrodes are attached. This is connected up to a metre wire bridge, just like an ordinary resistance; instead, however, of using a battery to send a continuous current through the bridge, an alternating current is used, this being usually taken from the secondary of a small induction coil. Currents in opposite directions following one another sufficiently rapidly are found to neutralize each other's polarizing effect. A balancing-point on the stretched wire is obtained, as in the other case, only this cannot be found by using a galvanometer, the current being alternating, not continuous; a telephone, therefore, is used instead. There should, theoretically, be silence in the telephone when the balance is obtained.

The objection to the method is that the ohmic resistance is not the only determining factor in the balance, the self-induction of the various arms of the bridge also having an effect; the resistances might all be the same, yet if the self-inductions of the different arms were not the same, no point of absolute silence in the telephone would be found. The electrostatic capacity of the arms also has an effect, but in the opposite direction to that of self-induction, the two effects tending to counteract one another. In practice, the balancing-point is one where the sound in the telephone is a minimum.

Again, the induction coil employed to send the alternating current through the bridge must have the condenser attached to it removed, otherwise the currents in reverse directions will be unequal, and consequently polarization will occur. Kohlrausch

[1] *Pogg. Ann.*, vol. cxxxviii.; also Kohlrausch, "Physical Measurements," p. 317.

employs a magnet rotating inside a coil of wire to produce the current; and he also finds that large electrodes are necessary to get consistent results.

84. A modification of Kohlrausch's method, which has been employed successfully at Cambridge,[1] employs a galvanometer instead of a telephone, but arranged with a commutator so that the galvanometer terminals are reversed at the same instant that the battery current is reversed; this insures the current in the galvanometer always flowing in the same direction. The difficulty with respect to the self-induction is got over by arranging that the balance is at the same point on the wire at all speeds of reversal of current.

85. *Fall-of-Potential Method.*—A very satisfactory method of measuring liquid resistance, which is free from many of the objections to which the previous methods are open, is the fall-of-potential method, due originally to Branly.

Fig. 58 represents the form of apparatus employed by the

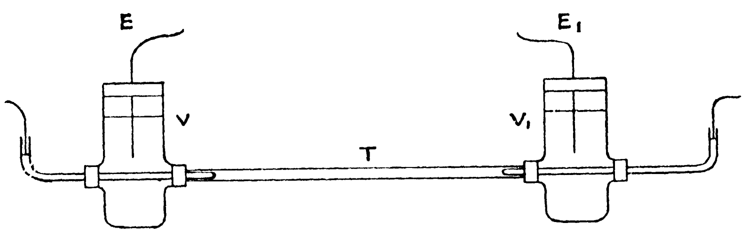


FIG. 58.

author. T represents the tube containing the liquid whose resistance is to be measured. This may consist of a graduated burette tube with the ends cut off. At each end are fitted on glass vessels, V, V_1, also filled with the liquid, and into which dip the electrodes E, E. There are two openings near the bottom of the glass vessels, which are opposite one another. Into one the end of T is fixed by means of a cork, while into the other passes a fine glass tube, *g*, closed at one end and filled with mercury. These tubes can slide in and out of T, and one is shown enlarged in Fig. 59, contact being made between the

[1] See *B. A. Report*, 1886, p. 328.

mercury in the tubes and the liquid by means of the platinum wire P fused through the glass. The wires dipping into the mercury are taken to a key, K (see Fig. 60), by means of which

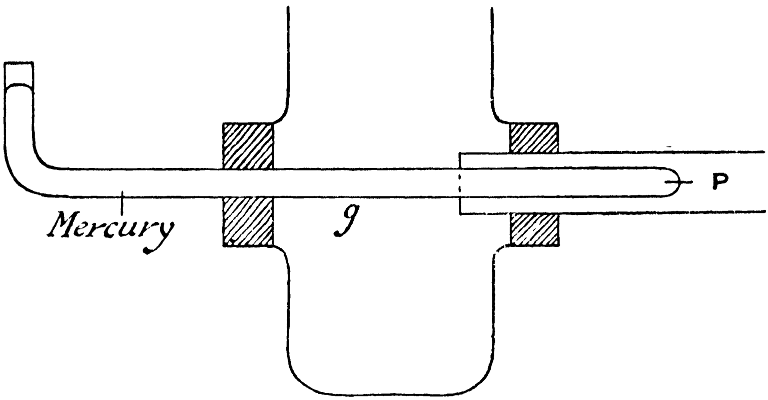


FIG. 59

they may be connected to the quadrants of the electrometer E. The electrodes are connected to a battery, B, in series with a

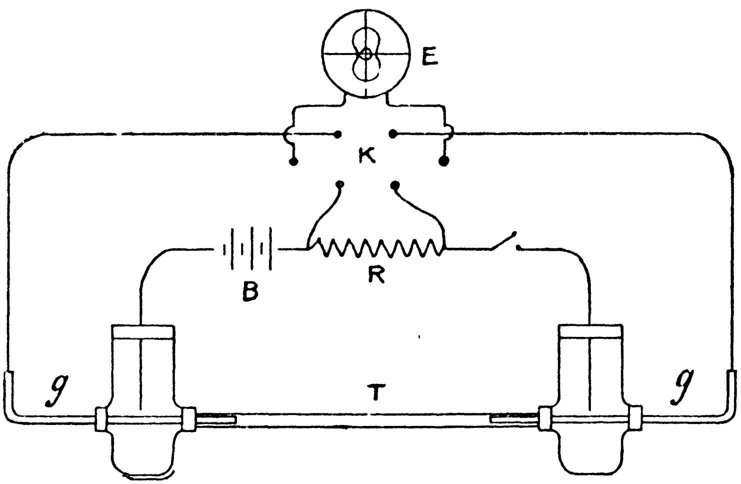


FIG. 60.

known resistance, R (the ends of which are also connected to the key K), and a break-circuit key.

On completing the battery circuit, a steady current may be

maintained in the tube T. The glass tubes *g*, *g* are now adjusted with their ends at a measured distance apart, and on connecting them to the electrometer E by means of the key K, a deflection is obtained which is proportional to the difference of potential between them. The key K is then altered so that the P.D. at the ends of R is measured on the electrometer, the ratio of the deflections in the two cases being proportional to the ratio of the resistances of the liquid between the tubes *g g*, and of R. The resistance R being known, that of the liquid may be calculated. Since no current is taken from the platinum electrodes of *g*, *g*, there will be no polarization. The tube T during the measurement should be placed in a water bath or thermostat and its temperature recorded.

Battery Resistance.

86. The measurement of the internal resistance of a battery is a problem closely related to the measurement of liquid resistance. It is, however, much more difficult to get consistent results here than in the previous measurements.

The resistance of a battery has no real meaning unless the current which the battery is supplying is also specified, since it is a well-known fact that the resistance varies with the current supplied, decreasing as the latter increases. As in illustration of this Professor Carhart found[1] that in the case of a Gassnier cell the resistance fell from 10 ohms to 2·5 ohms for an increase of current from 0·02 ampères to 0·2 ampères.

Then, again, when a current is taken from a battery, the effect of polarization is to lower the E.M.F. of the cell, thus increasing the calculated value of the resistance in methods which depend on the measurement of the relative values of the P.D. at the terminals of the cell when supplying different currents. Consequently, the resistance of a cell not in use is very different from that of the same cell when supplying a current. We must therefore, in measuring the resistance, carefully specify the conditions under which the battery is working when the measurement is made.[2]

[1] See *Physical Review*, vol. ii. No. 5; also *Electrician*, vol. xxxv. p. 18.
[2] See *Electrician*, vol. xxxi. p. 262.

The most satisfactory method of measuring battery resistance is that known as the condenser method. A condenser in series with a ballistic galvanometer is charged from the terminals of the battery when the latter is on open circuit. The first throw of the ballistic needle (δ_1) is, therefore, proportional to the E.M.F. (E) of the cell. The battery terminals are now connected by a resistance (r), and the condenser again charged from the battery terminals. Let the second ballistic throw be δ_2. Then we have—

$$(1) \quad \delta_1 \propto E$$
$$(2) \quad \delta_2 \propto \text{P.D.}$$
$$\propto \frac{r}{r+b} E$$

where b = battery resistance, therefore from (1) and (2) we get—

$$b = \frac{r(\delta_1 - \delta_2)}{\delta_2}$$

87. In practice it is preferable to take the P.D. reading first, and then the open circuit reading immediately on breaking the circuit r, so as to get the value of the E.M.F. as reduced

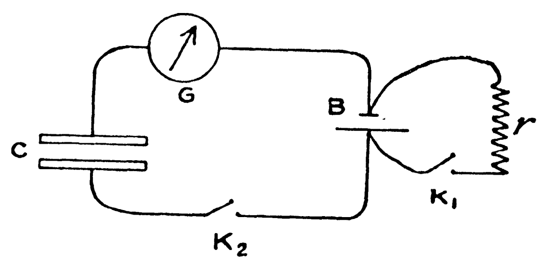


FIG. 61.

by polarization, and therefore the actual E.M.F. in the circuit during the passage of the current. Fig. 61 shows the arrangement of apparatus. G represents the ballistic galvanometer in series with the condenser C, the key K_2, and the battery B. A separate circuit from the battery goes to the resistance r in series with the key K_1. In making a measurement, K_1 is first depressed and then K_2, the throw on the ballistic galvanometer

being recorded. K_2 is then opened, and the condenser short-circuited for an instant by means of its short-circuit plug. K_1 is opened and K_2 again depressed, thus getting the throw due to the condenser being charged from B on open circuit. The cell B is now removed, and replaced by a standard cell, the condenser being charged from this cell on open circuit only; from the galvanometer throw due to the standard cell the scale of the galvanometer may be calibrated to read in volts, assuming the throws are proportional to the E.M.F.'s. In this way, knowing the P.D. in volts, and the value of r in ohms, the current in ampères supplied by the cell during the experiment may be estimated.

The results are then tabulated thus—

Temperature of cell.	δ_1	δ_2	r	Current in ampères.	b (ohms)

The numbers in the last column but one being $= \dfrac{\delta_2(\text{in volts})}{r}$

A curve should now be drawn, showing the connection between the resistance of the cell and the current supplied by it.

A separate set of experiments should be made to determine the effect on the throw δ_2, of keeping the key K_1 depressed for various intervals of time before taking the P.D. reading, and also on the rate of recovery of the E.M.F. after the circuit K_1 has been broken.

88. The following data were obtained from the measurement of the resistance of a freshly made-up Daniell cell.

The E.M.F. of the cell was 1·10 volts.

Temperature of cell.		δ_2	r	b (ohms)	Current in ampères.
16 C	120	23	1	4·2	0·21
16°	120	35	2	4·8	0·16
16°	120	45	3	5·0	0·14
16°	120	50	4	5·6	0·11
16°	130	60	5	5·8	0·10
16°	120	65	9	7·6	0·066
16°	130	70	10	8·5	0·06
16°	130	75	15	11·0	0·04

The accompanying curve (Fig. 62) shows the relation between the internal resistance of the cell and the current flowing through it.

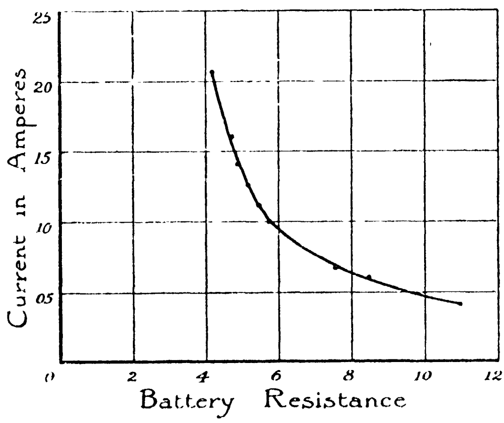


FIG. 62.

GALVANOMETER RESISTANCE.

89. To measure this important quantity various methods have been devised, the principal object being to use the deflections of the galvanometer itself, to furnish the data from which to calculate its resistance, so as to avoid the necessity of having to employ a second galvanometer. In general, however, it will be found that these methods do not give consistent results, the only satisfactory method being to measure it as an ordinary wire resistance on a Wheatstone bridge. Before

making the measurement it is necessary to first remove the suspended needle in the case of a needle galvanometer, or to fix rigidly the suspended coil in the case of a D'Arsonval galvanometer, as in the first case the magnetic effect of the testing current might be sufficient to reverse the polarity of the needle, and in both cases a very severe strain would be thrown on the suspension, due to the relatively large testing current. The temperature of the coils must be determined at the time of the test. If the galvanometer is supplied with a shunt box, the resistances of the shunt coils should also be determined. To insure the temperature of the shunt coils and galvanometer coils being the same, they should be placed together under a glass shade for some hours before a measurement is made. Measurements of this kind should be made on different days, when the temperature is different, in order to determine the temperature coefficient of the galvanometer and shunt coils (see par. 98), but in every case the galvanometer and shunt coils must be kept under the glass shade for at least an hour, at a constant temperature before the measurement is made.

All the precautions already given in connection with the use of the Wheatstone bridge apply in this case.

The results may then be tabulated as follows:—

Temperature.	Galvanometer resistance.	Shunt resistance.			Date.	Time.
		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$		

and from these the following table calculated:—

Mean temperature coefficient of galvanometer.	Mean temperature coefficient of shunt coils.			Ratios temperature coefficient of galvanometer to temperature coefficient of shunt coils.		
	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

If the material employed in winding the shunt coils is the same as that on the galvanometer, the ratios will be unity; if they are of different materials a knowledge of the temperature coefficients is necessary in order to calculate the value of any particular shunt at any stated temperature.

90. The following data have been calculated from measurements of resistance of a 10,000-ohm galvanometer and shunt box on different occasions at different temperatures.

Temperature.	Galvanometer resistance.	Shunt resistance.		
		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
	ohms.			
10° C.	10389·6	1154·4	104·93	10·40
12°	10459·4	1163·3	105·73	10·48
15°	10589·4	1176·6	106·94	10·60
20°	10789·2	1198·8	108·96	10·80

from which data we obtain the following:—

Mean temperature coefficient of galvanometer per cent. per 1° C.	Mean temperature coefficient of shunt coils per cent. per 1° C.			Ratios temperature coefficient of galvanometer to temperature coefficient of shunt coils.		
	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0·40	0·40	0·40	0·40	1	1	1

The above shows that the copper of which the galvano-

meter and the shunt were wound had the same electrical properties.

SPECIFIC RESISTANCE.

91. The resistance (R) of a substance in the form of a rod or wire is directly proportional to its length (l), inversely as its cross-sectional area (A), and directly as some constant (ρ), the value of which depends upon the nature and physical condition of the material. This constant ρ is known as the specific resistance of the material, and may be defined as the ohmic resistance, between the opposite faces of a centimetre cube of the substance. The above relations may be expressed algebraically thus—

$$R = \rho \frac{l}{A}$$

$$\text{and } \rho = \frac{RA}{l}$$

In order to determine the constant ρ of any material, we measure the resistance of a specimen of known length and constant known area, and calculate as above. The measurement of resistance may be made by means of a Wheatstone bridge, all the ordinary precautions necessary for accuracy being taken, and the temperature at which the measurement was made also being specified. The specimen of material if insulated wire must first have the insulation removed from it; this had better be done by hand, as any scraping is apt to injure the uniformity of the wire; the wire should then be cleaned by rubbing with a dry cloth before being connected to the bridge. The length must be measured between the points where the bridge terminals clamp it. To do this a fine scratch should be made at these points, and the wire removed from the bridge after the resistance has been measured, and stretched over a scale, in order to measure the exact length between the scratches, care being taken in the case of fine wires not to lengthen it by stretching. The ends are then cut off at the scratches. The determination of the area may now be made in two ways—either by measuring the diameter (d), and calculating the area from the relation $A = \frac{\pi}{4}d^2$, or else it may be got

from the length, weight, and specific gravity of the wire. This latter method should be adopted, the first being, if necessary, used to check it, as there is a connection between the specific gravity of a substance and its specific resistance, so that the specific gravity would have to be determined in any case, this being done by weighing in air and water. The wire is coiled up into a loose spiral, and suspended from the beam of a balance by means of a fine glass fibre, the weight of which has been counterbalanced, and the weight of the wire taken. The coil of wire is then plunged into a vessel of boiling distilled water, which, after all air bubbles have come off, is allowed to cool down. When near the temperature of the room the water vessel is supported under the beam of the balance, so as to allow the wire to be weighed when immersed in the water: the temperature of the water must be taken when the weight in water is determined, and this should be as near as possible to the temperature at which the resistance measurement was made, the weight of the partially immersed glass fibre being counterbalanced in a preliminary experiment.

The specific gravity is then calculated from—

$$\text{specific gravity} = \frac{\text{weight in air}}{\text{weight in air} - \text{weight in water}}$$

Should the temperature of the water not be that of its maximum density point, 4° C., the above calculation must be multiplied by the density of water at the temperature at which the weighing was made. Corrections for the different air displacements of the weights and wire are very small, and except in cases where very high accuracy is required, may be neglected.

Then if W = weight of wire, V = its volume, and Δ = its specific gravity—

$$\begin{aligned} W &= V\Delta \\ &= lA\Delta \end{aligned}$$

$$\therefore A = \frac{W}{l\Delta}$$

Combining this equation with the resistance equation, we get for the specific resistance—

$$\rho = \frac{RW}{l^2\Delta}$$

92. To measure the diameter of a wire directly, the most satisfactory method is by means of the micrometer microscope. A small scale engraved on glass is introduced in front of the eye-piece, so that a slightly magnified image of it is seen, and the relative size of any object looked at through the microscope may be expressed as so many divisions of this magnified scale. To get the absolute size of the object the magnified scale-divisions have to be calibrated, this being done by placing on the stage of the microscope a piece of glass on which a number of lines at a known distance apart, say $\frac{1}{100}$ or $\frac{1}{1000}$ cm., have been ruled by a dividing engine, and seeing how many of these divisions correspond to a division on the eye-piece scale, the magnifying power of the microscope being kept the same.

In a piece of wire 100 cm. long, readings of the diameter should be taken every 2 or 3 cm. along its length, and the mean diameter used in the calculation for the area.

93. The following measurement of specific resistance of a piece of manganin wire was made:—

A piece of No. 20 S.W.G. silk-covered manganin wire, about 100 cm. long, was cut from a large coil, the insulation was carefully removed, and the wire soldered to thick copper leads. The exact length was then carefully measured, and found to be 97 cm. The resistance measured on a Wheatstone bridge, after making all necessary corrections, was found to be 0·627 ohms @ 9·8° C. The specific gravity was determined by weighing in air and water, and was equal to 8·50, the weight of the wire being 5·150 gms. in air. This gave a mean area of 0·062 sq. cm. for the cross-section of the wire, and a specific resistance of 0·00040.

94. *Specific Dielectric Resistance.*—In the measurement of specific dielectric resistance, the calculation would be similar to that for a wire, if a rod or slab of material is employed in the measurement, but in the case of a wire covered with insulation, the calculation becomes more difficult. The

insulation may be considered as a cylinder surrounding the wire. Let the circle A represent the external circumference of the ring (see Fig. 63), *i.e.* the outside of the insulated wire, and let r_2 be its radius (B represents the conductor of radius r_1), and let c be any layer of insulation of infinitely small thickness $= dr$, and radius r, then, calling ρ the specific resistance, and l the length of the cable, since—

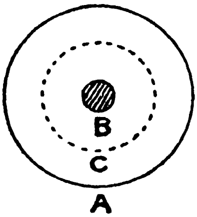


Fig. 63.

$$R = \rho \frac{l}{A}$$

we have for the resistance of the layer c—

$$\mathrm{r} = \frac{\rho dr}{A}$$

$$= \frac{\rho dr}{2\pi r l}$$

but the total resistance of the dielectric is made up of an infinite number of such layers in series, therefore—

$$R = \frac{\rho}{2\pi l} \int_{r_1}^{r_2} \frac{dr}{r}$$

$$= \frac{\rho}{2\pi l} \log_\epsilon \frac{r_2}{r_1}$$

$$\text{or } \rho = \frac{2\pi l R}{\log_\epsilon \frac{r_2}{r_1}}$$

95. In the case of the measurement of the dielectric resistance of the insulation of a length of copper wire, see par. 81. The length of the specimen was 78 cm. and the internal and external radii of the insulating covering were 0·0635 cm. and 0·127 cm. respectively. The dielectric resistance was found to be 4318 megohms at 23° C.

$$\text{Hence } \rho = \frac{2\pi l R}{\log_\epsilon \frac{r_2}{r_1}}$$

$$= \frac{2 \times 3{\cdot}142 \times 78 \times 4318 \times 10^6}{\log_\epsilon 2}$$

$$= 3 \times 10^{12} \text{ ohms per cub. cm. at } 23^\circ \text{ C.}$$

96. *Specific Resistance of Liquids.*—In the determination of the specific resistance of liquids, the form of apparatus and method employed will be one of those described in pars. 82–85. If the liquid is contained in a glass tube, such as that shown in Fig. 58, the length between the platinum points at the ends of the tubes *g*, *g* must be measured, also the mean area of the tube between these points must be found. If, as suggested, the tube is a part of a burette, then the mean area may be found from the graduations, but if not, or if it is desirable to measure it directly, the tube T is disconnected from the end vessels, one end is corked up, and a little mercury poured into the tube, which is then weighed. The tube is placed vertically, and the level of the mercury noted; more mercury is now poured in, care being taken to avoid air bubbles and to keep the temperature constant; the increase in level (l) is measured and the increase in weight (w) found. Calling Δ the specific gravity of mercury at the temperature of the experiment, the mean area A of the part of the tube occupied by l cm. of mercury is $A = \frac{W}{l\Delta}$. Proceeding in this way until the tube is completely filled, the mean area may be determined for various parts of the tube.

In all cases of the measurement of specific resistance of liquids, full particulars regarding the preparation of the solution should be given—as, for instance, whether chemically pure or commercial salts were employed, ordinary or distilled water as solvent, and the percentage composition of the solution by weight or volume, also the density of the solution and the temperature at which the measurement was made.

97. In the following measurement of the specific resistance of a solution of potassium chloride, the strength of which was 0·2 molecular equivalent per 1000 cub. cm. distilled water, the burette graduations for 1 cub. cm. were 1 cm. apart, so that the mean area of the tube was 1 sq. cm. The electrodes connected to the electrometer were placed 20 cm. apart, and the resistance in series with the liquid was 900 ohms. A Hellesen dry battery consisting of four cells in series was employed.

The circuit was completed, and the electrometer deflection, when placed across the ends of the movable electrodes, was 279 scale-divisions; when placed across the 900-ohm resistance, it was 270 scale-divisions. On repeating the first deflection it was found to have remained constant.

$$\text{Hence } \frac{x}{900} = \frac{279}{270}$$
$$x = 930 \text{ ohms at } 18^\circ \text{ C.}$$
$$\therefore \rho = \frac{\text{RA}}{l}$$
$$= \frac{930 \times 1}{20}$$
$$= 46{\cdot}5 \text{ ohms per cub. cm. at } 18^\circ \text{ C.}$$

The temperature was maintained constant by placing the tube containing the liquid in a thermostat regulated to 18° C. The 900-ohm coil was of manganin, and had no temperature correction.

Variation of Specific Resistance with Temperature.

98. In general, when the temperature of a conductor is altered, there is an alteration in its specific resistance, along with alterations of various other physical properties. In order to determine the variation of the specific resistance with temperature, we measure the resistance of a substance at various temperatures, and from these data we can calculate the various values of the specific resistance. In cases where very great accuracy is required, we must take into account in the calculation the fact that both l and A are varying with temperature, since our specific resistance is expressed in terms of a constant volume, 1 cub. cm. of the substance, whereas the mass only is constant. To do this we would require a knowledge of the coefficient of expansion of the substance with heat, or else we would require to determine the dimensions of the specimen at each different temperature. In the majority of cases, however, on account of the exceeding smallness of the change of volume

with temperature within the usual ranges of temperature likely to be met with, this correction is unnecessary.[1]

In making the measurement of resistance, the Wheatstone bridge method had best be employed. The chief difficulties, in addition to those ordinarily met with in bridge work, are (1) the accurate measurement of the temperature of the coil under test, and (2) in allowing for the variation of resistance of the unequally heated leads connecting the coil with the bridge.

99. In designing the coil of wire to be tested, certain points have to be borne in mind. The mass of wire employed in the coil should be as small as possible, so that it may rapidly arrive at the temperature of the surrounding space; the wire should not be wound close on a massive bobbin, but should be in an open spiral, or wound into the form of a ring of large diameter, tied loosely together with silk thread, the winding of course to be non-inductive. The advantage of a shallow ring over a long spiral is that in the former every part of the wire is nearly at the same level in the heating vessel, this being important, since the temperature often varies considerably at different depths.

The wire is cut about 2 cm. longer than is required, and 1 cm. at each end is soldered to the ends of thick copper leads (No. 12 S.W.G.). The exact length of the wire between the soldered joints is then very carefully measured, and the bared part of the wire at the joint coated over with shellac varnish or other insulating covering which will not be affected by the highest temperature to be used. A second pair of thick copper leads, of exactly the same length, are cut from the same material and soldered together at one end, these being used as compensating resistances; the method of connecting up will be shown presently.

100. The heater consists of a vessel containing paraffin oil, placed inside and jacketed by a much larger vessel of water, which can be heated by means of a Bunsen burner, to which a sensitive regulator is attached. The large mass of water insures slow variation of temperature, and it is therefore possible

[1] See Dewar and Fleming on "Measurement of Resistance at Very Low Temperatures," *Electrician*, Oct. 7, 1892.

to maintain the temperature steady for long periods of time. The heating vessel must be placed sufficiently far away from the rest of the apparatus to prevent the latter from being heated by radiation.

The following is the diagram of the connections (see Fig. 64). S represents the standard coil, X the test coil, and C the compensating leads; these are the same length as the leads going to X, and dip into the heater to the same depth, consequently being on the opposite arm of the bridge, and being under exactly the same physical conditions, they compensate for the leads at X at all temperatures. This is similar to the method employed by Callendar in his platinum thermometer. If it is

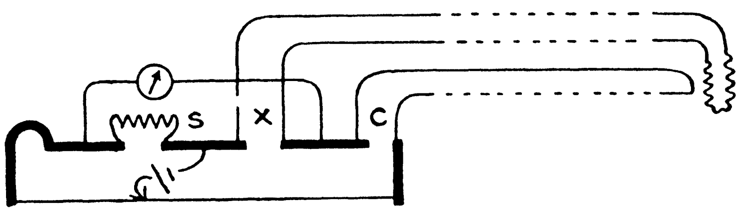


FIG. 64.

desired to interchange S and X, the compensating leads must also be changed to the opposite end of the bridge.

In all measurements the temperature must be kept constant for at least a quarter of an hour before a measurement is made, and no two consecutive measurements should be made within this interval of each other.

101. In obtaining the variation of resistance by this method, it is assumed that the resistance of the bridge wire is known, and its calibration curve has been obtained. Then, if S is kept at a constant temperature, various balancing-points will be found on the wire corresponding to different temperatures of the coil in the heating vessel; from the known resistance of the wire, the calibration curve, and the measured displacement of the tapping-contact, the increase in resistance for a given increase of temperature can be calculated. The resistance of the coil and leads is taken at the temperature of the room, and also the resistance of the leads alone at the same temperature;

by subtracting this latter value from the former, we obtain the resistance of the coil alone at the temperature of the room, and by adding on the increments of resistance at the different temperatures as determined above, the actual resistance of the coil at the different temperatures can be found, from which the specific resistance is calculated.

A curve should now be drawn with values of ρ for ordinates and temperatures for abscissæ; from this curve the law of variation of specific resistance with temperature may be deduced. This law may be expressed in general by the following relation:—

$$\rho_{t^\circ} = \rho_0 (1 \pm \alpha t^\circ \pm \beta t^2)$$

where ρ_t = specific resistance at temperature t° C., ρ_0 = specific resistance at 0° C., α and β are coefficients which depend on the material of the wire, and may be either positive or negative according to the nature of the material, β being always an exceedingly small quantity, and representing the variation of α with temperature. Provided β is large enough, it may be deduced from a very carefully plotted curve, thus—

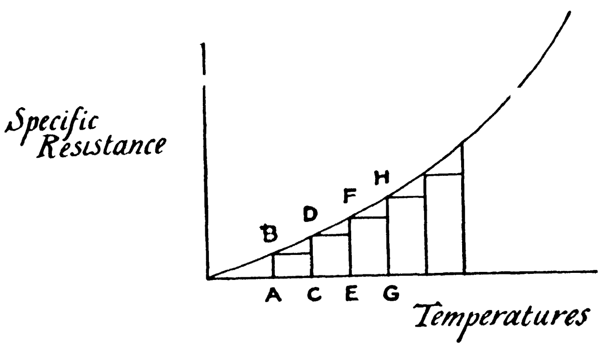


Fig. 65.

From the temperature scale erect a number of perpendiculars, AB, CD, EF, etc. (see Fig. 65), equidistant from one another. Measure the differences (CD − AB), (EF − CD), etc., and plot these against the mean temperature between A and C, C and E, etc. From this curve, which represents the variation of α with temperature, and which will be practically a straight

line, determine β. Putting this value of β into the equation $\rho_t = \rho_0 (1 \pm at \pm \beta t^2)$, from two values of ρ_t we can calculate a. The value of a is positive if the specific resistance increases, and negative if it decreases with temperature. Similarly, β is positive if a increases with temperature, and negative if it decreases.

102. For most substances it is sufficient to calculate a only, from two values of ρ.

$$\frac{\rho_{t_1^\circ}}{\rho_{t_2^\circ}} = \frac{1 + at_1^\circ}{1 + at_2^\circ}$$

$$\therefore a = \frac{\rho_{t_2} - \rho_{t_1}}{t_2^\circ \rho_{t_1} - t_1^\circ \rho_{t_2}}$$

It has been shown by Mattheissen[1] that for all pure metals, except iron and thallium, a has a value about 0·00366.

103. More recently, Dewar and Fleming,[2] in a research on the variation of specific resistance of metals and alloys at low temperatures, in which they employed temperatures down to $-200°$ C., have shown that in all pure metals the curves of temperature variation of specific resistance tend to meet at some point near the absolute zero of temperature; also that three distinct kinds of curves exist—those given by iron, tin, nickle, and copper, in which β is positive; those by gold, platinum, palladium, and silver, in which β is negative; and that by aluminium, in which $\beta = 0$.

In the case of alloys they found the curves to be in general straight lines, but differing from those of the pure metals in that they did not run towards the absolute zero on the temperature scale, the direction depending on the metals composing the alloy. If, for instance, the alloy consisted of metals in the same group, such as platinum-iridium or platinum-rhodium, its behaviour more nearly corresponded to that of a pure metal than in the case of an alloy such as platinoid, where the component metals are dissimilar. A very small trace of

[1] *Phil. Trans.*, 1862.

[2] See Dewar and Fleming on "Resistance of Metals and Alloys at Low Temperature," *Phil. Mag.*, vol. xxxiv., Oct., 1892; also vol. xxxvi., Sept., 1893.

impurity was found to produce a marked effect on the direction of the curve.

104. The following experiment illustrates the method of measuring the coefficients of resistance variation of a copper wire. The wire was made up into a loose coil and placed in a thermostat, the temperature of which could be very accurately regulated; heavy copper leads connected it to a metre bridge, the wire of which had been calibrated and all the corrections determined, compensating leads were also taken to the bridge. The coil was allowed to stand for half an hour in the bath at each temperature before a reading was taken. The temperatures were taken by means of thermometers reading to $\frac{1}{10}°$ C., on which $\frac{1}{100}°$ could be estimated. The thermometers had all been compared with one another, and with a standard thermometer. The standard coils were of manganin. After making all corrections, the following resistances were obtained for the coil at various temperatures:—

Temperature.	Resistance.
	ohms.
10°	103·892
20°	107·810
30°	111·753
40°	115·721
50°	119·715
60°	123·753
70°	127·777
80°	131·846
90°	135·940

From these readings a curve may be plotted, and the values of α and β deduced, or they may be calculated as follows.

Assuming the law to be $R_t = R_0 (1 + \alpha t + \beta t^2)$, we have taking the first and second readings—

$$103·892 = R_0 (1 + 10\alpha + 100\beta)$$
$$107·810 = R_0 (1 + 20\alpha + 400\beta)$$

from which we get—

$$\alpha = 0·00382$$
$$\text{and } \beta = 0·0000012$$

From similar equations, we get—

Temperature.	Coefficients.	
	α	β
10—20	0·00382	0·0000012
20—30	0·00384	0·0000012
30—40	0·00387	0·0000013
40—50	0·00389	0·0000012
50—60	0·00392	0·0000013
60—70	0·00394	0·0000012
70—80	0·00397	0·0000012
80—90	0·00400	0·0000013
Mean value ...	0·00390	0·0000012

105. *Effect of Molecular Change.*—It will be seen from the table of specific resistances on pp. 398, 399, that the values obtained by different experimenters are by no means identical, even when the chemical composition and method of manufacture is as nearly as possible the same. These divergences must therefore be put down to some difference in molecular arrangement, due to some slight difference in handling or in manufacture.

In the case of copper, it has been shown by Fitzpatrick[1] that there is some connection between the specific gravity of the material and its specific resistance, an increase in the former causing a decrease in the latter; this decrease, however, is not proportional to the increase of specific gravity, but changes more rapidly. He also showed that even in wires of the same metal there may be a slight difference in density due to some difference in drawing.

106. The permanent effect of heat on the resistance of wires is very marked, in consequence of the wire becoming annealed, the specific resistance of all metals being diminished by annealing, although in the case of alloys the effect is very much less than in that of pure metals. Thus Matthiessen[2]

[1] *B. A. Report*, 1890. [2] *Phil. Trans.*, 1862

found that the resistance of copper wire diminished by about 2% on annealing by heating to redness and then cooling slowly; whereas the platinum-silver alloy (66% Ag., 33% Pt.) did not alter appreciably.

Partial annealing may, however, occur in a material due to age: this has been found in the case of both hard-drawn silver and copper wires; but those conductors that do not alter in resistance by baking at a temperature of 100° C. for several days, are found not to alter much with age.

107. All mechanical operations, such as winding, etc., set up mechanical strains in wires, which harden them and raise their resistance; this, however, gradually returns to its original value, but the change may be hastened by heating and annealing. This change, for a wire of any given guage, is always greater the smaller the diameter of the spiral into which the wire is coiled. It is therefore important, in the case of standard resistance coils, that the diameter of the bobbin should be large compared with the guage of the wire, in order to prevent excessive strains being set up in the material due to winding.

108. A frequent cause of alteration of resistance in the case of alloys is the presence of zinc in the alloy. This has been found to slowly crystallize out, and cause a permanent alteration and rotting of the material in the case of German silver.

The whole question of molecular change and consequent alteration of specific resistance in metals and alloys is exceedingly important, and would form a good line of investigation for more advanced students.

FAULT TESTING.

109. In cables or coils of wire which may be immersed in a conducting liquid, faults are liable to occur the accurate determination of the position of which is of great importance.

The faults likely to occur in the case of such an insulated cable are the following:—

(*a*) Complete fracture of the wire and insulation, the broken end of the wire making a good "earth."

(*b*) Local breakdown of insulation, causing partial earth, but no fracture of wire.

(*c*) Complete fracture of wire, but no breakdown of insulation.

The first case is that of a cable which has snapped in two, the bare copper end making contact with the earth, as shown diagrammatically in Fig. 66, where ABC represents the submerged cable, fractured at B. To find the distance from A to the fracture we measure the resistance between A and earth on

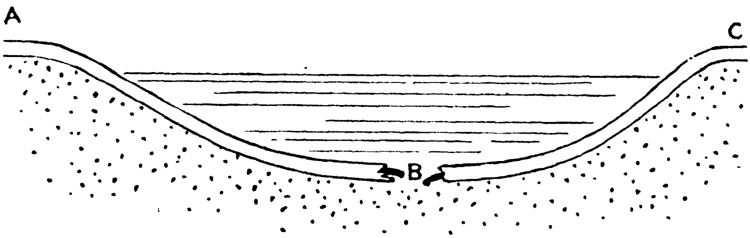


FIG. 66.

a post-office bridge, and thus get the resistance of the part of wire AB (the resistance of the earth being negligible). Consequently, knowing from the specification of the wire its resistance per mile, we can calculate what length this measured resistance corresponds to.

110. The second case, which is more important, as occurring more frequently in practice, could be determined in the same

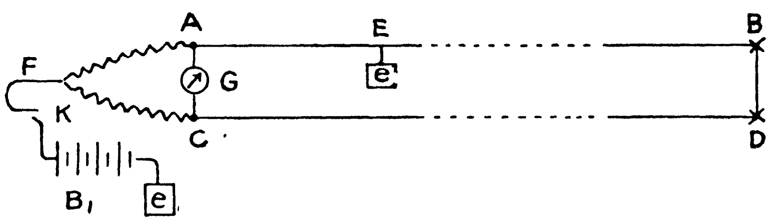


FIG. 67.

way, provided the earthing of the wire had been good; but if there is only a partial earth, the resistance and polarization introduced there would interfere with the results. Of the various methods which may be employed in this case, perhaps that known as the "loop test" is the best; it however requires a second cable (see Fig. 67). Let AB and CD represent the

two cables, one of which, AB, has a partial earth at E. The ends B D are joined or "looped" together by a wire of small resistance; A C are joined by a bridge galvanometer G, and by two resistances, AF and CF, the point F being connected through the key K with the battery B′, the other pole of which is earthed at *e*.

It will now be seen that this arrangement of conductors constitutes a Wheatstone bridge, FA and FC being two arms, whilst AE and CDBE are the others.

The resistances FA and FC are adjusted until, on depressing the key K, and making the galvanometer circuit, there is no deflection. Then—

$$\frac{FC}{FA} = \frac{CE}{AE}$$

But CE + AE = R (where R = the resistance of the two cables in series);

$$\therefore CE = R - AE$$

$$\text{and } \frac{FC}{FA} = \frac{R - AE}{AE}$$

$$\text{or } AE = \frac{R(FA)}{FC + FA}$$

So that, knowing the resistance per mile of the cable, we can calculate the distance corresponding to the resistance AE.

The advantage of this method is that resistance and polarization at E do not affect the measurement. The battery B, however, must have sufficient E.M.F. to overcome any back E.M.F. due to polarization at E.

111. The third type of fault is the most difficult to localize. Two methods are available, and it is advisable to use one as a check on the other. The first is the measurement of the dielectric resistance of the cable, and the second is the measurement of its electrostatic capacity from one end to the break, the other end of the cable being insulated.

The first method assumes (which may not always be the case) that the dielectric resistance is uniform along the cable.

In par. 94 we have already shown that in a cable the dielectric resistance is—

$$R = \frac{\rho}{2\pi l} \log_e \frac{r_2}{r_1}$$

where ρ = specific resistance of the insulating material, l = the length of the copper conductor, r_2 and r_1, the external and internal radii respectively of the insulating covering. The value of R may be found by some of the methods described in the section dealing with high resistance measurement, and ρ may be found from previous tests, or by testing a known length of the cable. From these measurements l may be calculated approximately.

112. As a check on this measurement, the following capacity measurement should be made, since it has been shown in par. 262 that the capacity of a cable of length l and S.I.C. = σ, whose external and internal insulation radii are r_2 and r_1, is—

$$K = \frac{\sigma l}{2 \log_e \frac{r_2}{r_1}}$$

the capacity being measured between the free end of the cable and earth by some of the methods described in Chapter V., σ being determined either from previous tests or from an actual test of a measured length cut from the cable, and the length l calculated. These two measurements should be made from each end of the cable, and the mean value of l taken for each end.

113. In order to practise these tests an artificial cable may be constructed as follows. A number of coils of insulated wire,

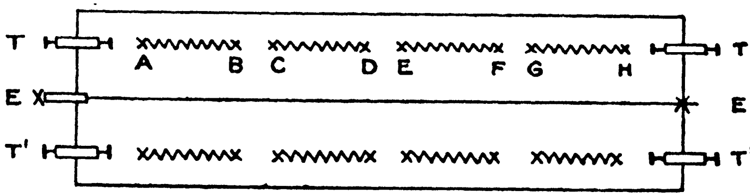


FIG. 68.

such as No. 40 manganin, are soldered to separate terminals, AB, CD, EF, etc. (see Fig. 68), so that they can be connected

up in any manner to the terminals T, T, T′, T′, which represent the ends of the lines; a wire, E, of low resistance stretched between them represents the "earth." The coils are arranged so that they can be locked up in a box with only the terminals T, T, T′, T′, E, E projecting out. The demonstrator can thus arrange any fault on the artificial cables—such as a "dead earth," by connecting one of the coil terminals to the wire E; or a "partial earth," by connecting one terminal of a coil to E through a small battery—and then lock up the box, leaving the student to localize it; the total length and resistance of the wire on the coils being known.

Absolute Measurement of Resistance.

114. In making comparisons and measurements of resistance, it is necessary that we should have some standard of resistance in terms of which we are to express the values of the resistances we measure. Several such standards have been proposed from time to time; the most important, however, was that due to Werner Siemens (1860), who proposed, as a standard of resistance, that of a column of pure mercury, 100 cm. long, and 1 sq. mm. in cross-sectional area, at a temperature of 0° C. This standard was quite empirical, and simply represented a value which was found to be approximately of the order of the resistances which were most commonly measured at that time. Later investigations showed that a resistance could be expressed in terms of the units of length, time, and mass. When this was done for the Siemens unit it was found to be proportional to a velocity of $0{\cdot}98 \times 10^9$ cm. per second; it was then decided to call that resistance which had a value proportional to a velocity of 10^9 cm. per second, the practical unit of resistance.

Starting from this definition, fresh determinations of resistance in absolute measure had now to be made, in order to determine the exact length of a mercury column which would represent the unit of resistance, since the legal definition of the ohm was expressed in terms of a mercury column. Mercury being a substance which, whilst being a good conductor, is a liquid, and is therefore free from internal strains which might alter its physical

properties, and which are liable to occur in a solid conductor. On account of the difficulty of working with mercury resistances, it is usual to determine in absolute measure the resistance of a wire conductor, which is immediately afterwards compared on a wire bridge with a column of mercury.

115. We cannot here enter into the full discussion of the various methods of determining resistance in absolute measure, as that would be of little use, since very few physical laboratories can afford to provide the apparatus necessary for such a purpose; yet the main principles involved can be illustrated by very simple apparatus, from which the student may be able to grasp the difficulties to be met with in more accurate work. It must, however, be strictly borne in mind that we do not presume to call the measurements made in this way "determinations."

In order to obtain the resistance of a wire directly in absolute measure, we have the choice of two fundamental principles upon which to base our methods, the first being Joule's law of heating, which establishes a relation between the current, the resistance, and the heat produced in the wire, and the other, some application of Ohm's law, which defines the resistance of a wire as the ratio between the potential difference at its ends and the current flowing through it. Methods based on both of these principles will be described, but the student is recommended to study the original papers, references to which are given, in order to get the full discussion of these experiments, since we only propose to describe the method sufficiently for measurements being made to a first approximation.

116. *Joule's Method.*—This method of determining the value of the resistance of a wire in absolute measure, in terms of the amount of heat generated in it when a current of known strength flows through it, cannot rank along with the other methods for accuracy, both on account of the difficulty of making the measurement of the heat evolved, and also since an accurate knowledge of the mechanical equivalent of heat is required, this quantity not being known to the degree of accuracy with which we can determine resistance by the methods based on Ohm's law. The practice of the method, however, forms a splendid test of the ability of a careful student. The apparatus required

for this experiment consists of a thin copper calorimeter, which is suspended inside a water-jacketed space; inside the calorimeter is placed the wire resistance which is to be measured, and which should consist of a coil of bare manganin wire, which has been varnished over to make it insulating. The coil should, if possible, take the form of an open spiral, so that the liquid in the calorimeter is in contact with every part of it, a bobbin being objectionable, on account of the difficulty of ascertaining its exact temperature and calculating its heat capacity. The wire is soldered to two copper leads, the dimensions of which may be so chosen that their rise of temperature shall be approximately equal to that of the liquid, thus avoiding a correction for heat lost by conduction through the leads.[1] In series with the manganin resistance is a voltameter or standard galvanometer, a secondary battery, regulating resistance, and break-circuit key. The liquid in the calorimeter may be water or, better still, aniline,[2] the specific heat of which is accurately known, and remains more constant with alteration of temperature than that of water.

If we call R the resistance of the manganin coil in absolute units, C the current in absolute units, t the time in seconds during which the current flows, H the quantity of heat given out in gramme degrees Centigrade, and J the mechanical equivalent of heat, *i.e.* $4{\cdot}2 \times 10^7$ ergs, then—

Heat produced in coil = heat given out

$$C^2Rt = JH$$

$$\text{and } R = \frac{JH}{C^2t}$$

The accurate measurement of H, the heat given out by the coil, is somewhat difficult, since, in addition to the heat gained by the liquid surrounding the wire, we have to take into account the heat gained by the calorimeter, stirrer, thermometer, and the wire coil itself, besides estimating the heat lost by radiation. The heat gained by the apparatus is calculated from its water

[1] See Ayrton and Haycraft, *Proc. Phys. Soc.*, Dec. 14, 1894.

[2] Griffiths, "Specific Heat of Aniline," *Phil. Mag.*, vol. xxxix, Jan., 1895.

equivalent, which may be experimentally determined, as described in vol. i. p. 76; whilst the heat radiated may be found by an experiment similar to that described in vol. i. p. 77. The water equivalent of the apparatus may also be calculated, since it is equal to the product of the weight of each part immersed into its specific heat. The correction for radiation may be made negligible by arranging the current so that the temperature at the end of the experiment is as much above the temperature of the air as it was below it at the commencement.

A known weight of liquid is placed in the calorimeter, and the temperature taken by means of a delicate thermometer graduated to $\frac{1}{10}°$ C., on which the temperature may be estimated to $\frac{1}{100}°$ C.; a second thermometer is placed in the jacketed space surrounding the calorimeter.

The current is now started and maintained constant until the liquid, which has been kept continuously stirred, has risen a few degrees in temperature; the current is then stopped, and the time during which it has flowed noted accurately on a stop-watch. The thermometer in the calorimeter should, if the stirring has been performed properly, cease rising almost at the same instant that the current was stopped; this temperature, $T_2°$, is now noted. Then, if $T_1°$ was the initial temperature of the liquid, m its mass in grammes, s its specific heat, W_ϵ the water-equivalent of the apparatus, and r the number of units of heat lost by radiation during the time t, and which has been determined by a separate experiment, the total quantity of heat given out by the coil is—

$$H = ms(T_2° - T_1°) + W_\epsilon(T_2° - T_1°) + r$$

The current C is now obtained in absolute units from the current-measuring apparatus employed (see par. 130), and the value of R calculated in absolute units. The student should then proceed to check this result by measuring the resistance of the manganin coil on a wire bridge against a standard resistance, when the resistance in ohms $\times 10^9$ should give the same result as that obtained in the above experiment.

117. In the second class of methods, the principle of which depends on Ohm's law, the resistance is expressed in terms of an electromotive force and a current, the values of which must be known in absolute measure.

One of the best-known methods of this class is that proposed by Lord Kelvin,[1] and usually known as the British Association or B. A. method.

In this method a circular coil of wire is mounted in a frame, so as to be free to rotate about a vertical axis in the earth's magnetic field. At the centre of the coil, and protected, by a closed glass vessel, from air currents, a magnetic needle, with mirror attached, is suspended by a long silk or spider line.

As the coil rotates in the earth's field, the flux of lines of force through it is continually altering with the position of the coil with respect to the magnetic meridian; an E.M.F. will therefore be induced in it, the magnitude of which may be calculated from the known strength of the field, and the rate of change of lines in the coil. If the coil has its circuit completed by joining the ends together, this E.M.F. will induce a current in it, which will be alternately in opposite directions with respect to the coil itself, but will always be in the same direction with respect to the needle at its centre; this current produces a deflection of the needle, from which its magnitude may be calculated in absolute measure, and hence, by dividing the E.M.F. in absolute measure by the current in absolute measure, we obtain the resistance of the coil also in absolute measure.

118. The following calculation may be regarded as giving the resistance of the coil to a first approximation.

Let E = E.M.F. set up in the coil in absolute measure;
C = current in the coil in absolute measure;
R = resistance of the coil in absolute measure;
H = horizontal intensity of the earth's magnetic field;
A = effective area of the coil, *i.e.* the sum of the areas of each turn;
N = total number of lines passing through the coil;
n = speed of rotation in revolutions per second;

[1] See Appendix D., *B. A. Report*, 1863.

then, when the coil is inclined to the magnetic equator at an angle θ—

$$N = HA \cos\theta$$

$$\text{and } E = HA \sin\theta \frac{d\theta}{dt}$$

$$\text{But } \frac{d\theta}{dt} = 2\pi n$$

$$\therefore E = 2\pi n HA \sin\theta$$

This, by Ohm's law, neglecting the effect of self-induction, is = CR. Hence—

$$C = \frac{2\pi n HA \sin\theta}{R}$$

Now, the magnetic force f, at right angles to the plane of the coil, acting on a needle of pole strength, m, at the centre of the coil, is—

$$f = \frac{2\pi Csm}{r}$$

where s is the number of spirals on the coil, and r is the mean radius of the coil in centimetres;

$$\therefore f = \frac{4\pi^2 smnHA \sin\theta}{Rr}$$

and, for simplicity, call this—

$$f = \Delta \sin\theta$$

Now, f can be resolved into two components at right angles to one another, one, f_1, acting at right angles to the magnetic meridian, and the other, f_2, acting along the meridian; and if θ is the angle which f makes with the meridian, then we have—

$$\begin{aligned} f_1 &= f \sin\theta \\ &= \Delta \sin^2\theta \\ \text{and } f_2 &= f \cos\theta \\ &= \Delta \sin\theta \cos\theta \end{aligned}$$

Integrating the values of f_1 and f_2 for a complete revolution of the coil, we get—

$$\text{mean value of } f_1 = \frac{\Delta}{2}$$

$$\text{mean value of } f_2 = 0$$

Hence the deflection of the magnet at the centre of the coil is due to the component f_1, and, calling this deflection δ, we have—

$$f_1 = m\text{H} \tan \delta$$

$$\text{therefore } \frac{4\pi^2 smn\text{HA}}{2\text{R}r} = m\text{H} \tan \delta$$

$$\text{and R} = \frac{2\pi^2 sn\text{A}}{r \tan \delta}$$

$$\text{or, since A} = \pi r^2 s$$

$$\text{R} = \frac{2\pi^3 s^2 nr}{\tan \delta}$$

which expresses the resistance of the coil in absolute measure It must be noted that the deflection δ is the angular deflection of the needle, and not that of the spot of light. The speed of rotation of the coil may be measured by the stroboscopic disc method, described in par. 120. For the full details of the method, the student is referred to the *Report of the Electrical Standards Committee of the British Association*, 1863.

119. Another method of determining resistance absolutely, which has the additional advantage of being a very good method of comparing low resistances, is that due to Lorenz, and modified by Professor V. Jones.[1]

In this method of determining resistance, the fall of potential down the resistance carrying a current is balanced against the E.M.F of a simple dynamo, so arranged that the E.M.F. may be calculated from its dimensions, and whose magnetic field is produced by the current flowing through the resistance.

The dynamo consists of a copper disc rotating inside a coil of wire and coaxial with it, as is diagrammatically shown in Fig. 69. The coil is in series with a battery, B, a regulating resistance, r, and the resistance R to be determined absolutely. From the ends of R potential wires are taken through the galvanometer G, one to the axis, and the other to the periphery of the copper disc, which is rotated so as to produce an E.M.F. in opposition to the P.D. at the ends of R, the speed of

[1] *Phil. Trans.*, 1891

rotation being altered until no deflection is obtained on the galvanometer G. Then, calling C the current in the coil and R its resistance in absolute units, M the coefficient of mutual

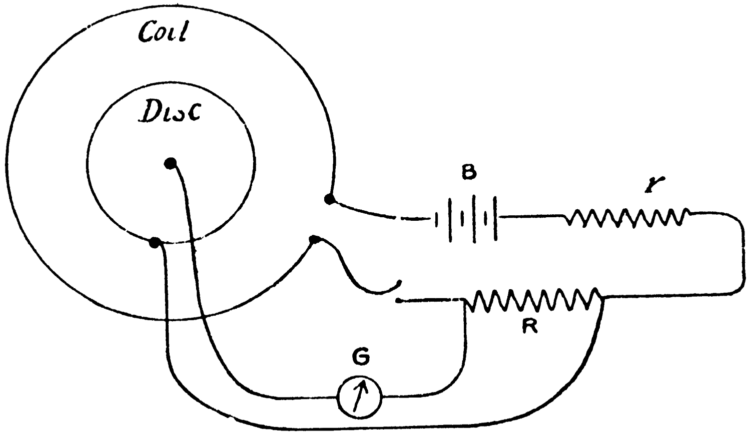


FIG. 69.

induction between the coil and the disc, and n the speed of rotation of the disc in revolutions per second—

The E.M.F. produced by the disc is $E = MnC$

But this, by Ohm's law, is equal to CR; hence—

$$R = Mn$$

The value of M may be calculated from the dimensions of the disc and coil,[1] but the calculation is very complicated; M, however, does not require to be known if it is only desired to *compare* two resistances. In this case one of the resistances, say R_1, is placed in the position of R in the diagram, and the disc rotated until there is no deflection on the galvanometer G. Let the speed be n_1 revolutions per second. The coil R_1 is now replaced by the other coil, R_2, and the disc again rotated till the galvanometer needle shows no deflection. Let the speed now be n_2 revolutions per second, then—

$$(1) \quad R_1 = Mn_1$$

$$\text{and } (2) \quad R_2 = Mn_2$$

$$\text{hence } \frac{R_1}{R_2} = \frac{n_1}{n_2}$$

[1] See *Phil. Mag.*, Jan., 1889, also July, 1896; and *B. A. Report*, 1894.

If R_2 is a standard resistance, the value of R_1 may thus be calculated.

120. In order to be able to regulate the speed of the disc, and in cases where the highest accuracy is not required, also to measure it, the apparatus known as the stroboscopic disc may be employed.

This consists of a cardboard disc attached to the same axle that carries the copper disc, and therefore rotating at the same speed. On this are drawn a number of concentric rings, which are divided up into a number of segments, painted alternately white and black, or white and red, the number of segments differing in each ring, say 60 in the first, 34 in the second, 20 in the third, 16 in the fourth, etc. In front of this disc is placed a tuning-fork, to the prongs of which are attached thin aluminium plates, overlapping one another and with a long narrow slit cut in each, parallel to the prongs, so that when the fork is at rest the slits coincide, and when it is vibrating they coincide twice in every double vibration. When the disc is rotating, say 10 times a second, the first ring of segments will pass the slits at the rate of 600 per second, the second ring at 340 per second, and so on. Now, if the fork vibrates 300 times per second, a person looking through the slits at the rings of segments would see them 600 times a second, and consequently the first ring of 60 segments would appear to be at rest whilst the others would appear to be moving in the opposite direction to that of the disc. The speed of rotation of the disc is calculated from the number of teeth on the segment which appears to be at rest, and from the rate of vibration of the tuning-fork. Should none of the segments be absolutely at rest, the speed may be calculated by timing the apparent rate of rotation of the slowest moving one, and from the direction of its motion. For instance, if in the above case the 60-segment ring appeared to move past the slits at the rate of 1 segment per second, the speed of rotation with the above fork would be 600×1, according as the apparent direction of rotation was with or against that of the disc.

Some difficulty may be experienced from thermal currents set up at the rubbing contacts; these, if constant, may be either

allowed for, by observing the galvanometer deflection which they produce, and using that as the true zero, or else they may be annulled by introducing an E.M.F. in opposition to them, which will just balance their effect. In order to obtain good contact at the periphery of the disc, more than one brush may be employed. In Jones's modification of the Lorenz method the brushes consisted of thin metal tubes, through which a stream of mercury was kept flowing, the edge of the copper disc being amalgamated; this was found to give exceedingly satisfactory results.

For further details of the method the student is referred to the original papers.[1]

RESISTANCE STANDARDS.

121. The results of many careful researches have given us data from which the dimensions of the standard ohm may be calculated. This quantity is now legally defined as follows:—[2]

"The ohm, which has the value 10^9 in terms of the centimetre and the second of time, is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14·4521 grammes in mass, of a constant cross-sectional area, and of a length of 106·3 centimetres."

Practically, it is found that a mercury column is not a convenient form in which to embody the unit, the requirements of a practical unit being briefly as follows: (1) it must be easy to construct; (2) it ought to be made of some substance of high specific resistance, in order to avoid having a large bulk; (3) its temperature variation of resistance should be as small as possible; (4) it must not be liable to change its value with time, either from oxidation or from molecular change; (5) it must be able to stand handling without injury, and be of such a form that its temperature may be accurately determined.

122. After many experiments on various substances,[3] the most satisfactory, both as regards permanency, low temperature

[1] *Phil. Trans.*, 1891; *Electrician*, vol. xxxi. p. 620; vol. xxv. pp. 543, 562; vol. xxxv. p. 351.

[2] See *London Gazette*, Aug. 24, 1894.

[3] See *B. A. Report*, 1862–65.

variation, and high specific resistance, was found to be an alloy of 66·6 per cent. silver and 33·4 per cent. platinum, known as platinum-silver, having a specific resistance equal to 14 microhms, and a temperature variation of resistance of 0·031 per cent. per 1° C.

123. The accompanying figure (Fig. 70) shows the form taken by the standard. A double silk-covered, well-paraffined platinum-silver wire is wound non-inductively in the space between two brass cylinders, the free ends being soldered to thick copper wires, and the space round the wire filled up with paraffin wax. The case is immersed to the depth of the narrow part in melting ice, and the temperature taken by a

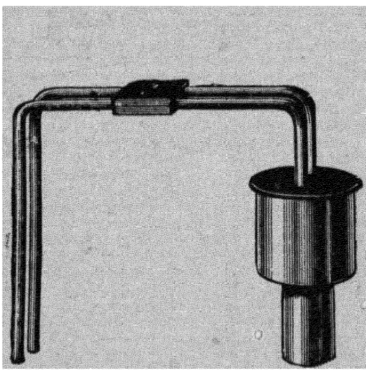


FIG. 70.

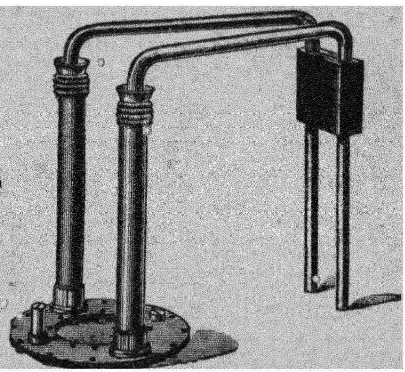


FIG. 71.

thermometer which passes into the inner tube. The chief objections to this form are that it is difficult to insure that every part of the coil is at the same temperature, and that the thermometer registers this temperature. It also takes a long time to settle down to a steady temperature, on account of the large mass of paraffin wax surrounding the wires, and it has been suspected that the strains set up in the wire, due to expansion and contraction of the paraffin wax, may cause a permanent alteration in the resistance of the wire.[1]

124. A modification of this form of standard, which gets over some of these difficulties, is that due to Dr. Fleming[2] (see Fig. 71). The coil is wound in the form of a flat spiral, and

[1] *Electrician*, vol. xxix. p. 277; *B.A. Report*, 1890.
[2] *Electrician*, vol. xxii. p. 74.

its ends soldered to thick copper leads, which pass up from the coil inside ebonite tubes, insulated from them by air, except at the ends, where they are held by ebonite cups containing paraffin oil. The coil is placed between two annular discs of brass, with flat grooves in their opposite faces, which, when screwed together, form a closed space of rectangular section. The coil is embedded in paraffin wax in the lower groove, but the upper groove is empty, an opening being made into it through which air may be forced when the apparatus is immersed in water, to test for leakage at the joints. The advantages claimed are, better insulation and more uniform temperature throughout

125. A description of the standards would, however, be incomplete without mention of the standards employed at the Berlin Reichsanstalt,[1] the general form taken by the 1-ohm standard being shown in Fig. 72.

The wire is No. 18 B.W.G. manganin double white silk covered, the ends being soldered to copper washers, which are then screwed and soldered with silver solder to the ends of thick copper leads, the resistance of which together amounts to about 140 microhms. The bobbin on which the wire is wound is a hollow brass cylinder 4 cm. diameter, covered first with a shellaced silk tape dried at 140° C.; the wire is also shellac varnished and dried for 10 hours at 140° C. The insulation resistance of the wire is of the order of a million megohms; the wire when wound on the cylinder being held by a dry cloth and not in the bare hand. The axial length of the coil is 4 cm. The coil hangs by the leads, in a vessel of paraffin oil 11 cm. broad, 48 cm. long, and filled to a depth of 12 cm., provided with a fan to keep the liquid in circulation. The radiating surface of such a coil is about 100 sq. cms. With such standards a maximum current of 1 ampère could be used.

126. *Relations between the Various Standards.*—Since from time to time, on account of new and more accurate determinations of the ohm having been made in terms of a certain length

[1] See translation of a paper by Feussner and St. Lindeck, *Electrician*, vol. xxxvi. p. 509.

of mercury column, fresh standards have been issued, and different experimenters have used different standards, we here enumerate the more important standards, and give a table

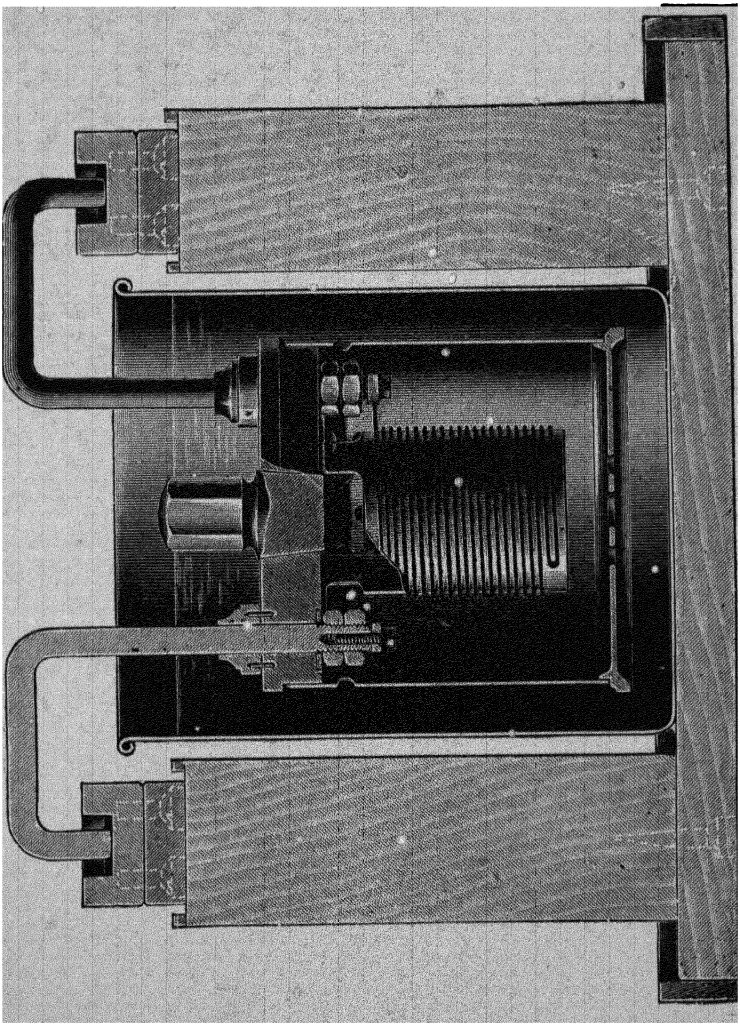


Fig. 72.

showing the relation of each to the present legal standard, which is defined in terms of a mercury column 106·3 cm. long.

Standards.	Centimetres of mercury at 0° C and 1 sq. mm. section.	In terms of present legal ohm.
Siemen's unit	100·00	0·9407
B.A. unit (1864)	104·88	0·9866
Legal ohm (1884)	106·00	0·9972
Present legal ohm (1892) ...	106·30	1·0000

In connection with the various standards of resistance that have from time to time been employed, the student must be careful, when studying any published paper or research, to note the particular unit of resistance employed in the measurements. If the unit employed was one of the older ones, the measurements must be reduced in terms of the 1892 standard before they can be compared with recent experiments. Thus, if the standard employed was the legal ohm of 1884, any measurement of resistance made in terms of it must be reduced in the ratio of $\frac{106·3}{106}$ in order to compare it with measurements made in terms of the 1892 standard. The same remark applies to the determination of E.M.F., which involves the product of a current into a resistance. Determinations made in terms of the 1884 ohm will, therefore, be about 0·3 per cent. too high as compared with those made from the 1892 standard.

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On Sensitive Galvanometers	Threlfall	*Phil. Mag.*, vol. 29, June, 1890.
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Resistance of Amalgams	Willows	*Phil. Mag.*, vol. 48, p. 433.
Dielectric Strength of Oils	Gray	*Amer. Assoc. Pro.*, vol. 48, p. 122.
Insulation Tests	Ayrton and Mather	*Phil. Mag.*, vol. 49, p. 343.
Differential Galvanometer	Crawley	*Inst. Elect. Eng. Jour.*, vol. 30, p. 908.
Measurement of Standard Resistances	Glazebrook	*Phil. Mag.*, vol. 50, p. 410.
Insulation Resistance	Ashton	*Ibid.*, vol. 2, p. 501.
The Electric Thermostat	Darwin	*Elect.*, vol. 52, p. 256.
Electrical Methods of Measuring Temperature	Callendar	*Ibid.*, vol. 52, p. 767.
Temperature Experiments with Insulating Materials	Rayner	*Ibid.*, vol. 54, p. 884.
Comparison of Resistances	Smith	*Ibid.*, vol. 57, p. 976.
Constancy of Manganin Resistances	Jaeger and Lindeck	*Ibid.*, vol. 57, p. 930.
The Temperature Coefficients of Guttapercha	Winnertz	*Ibid.*, vol. 58, p. 453.
Resistance in a Magnetic Field	Grunmach	*Ibid.*, vol. 58, p. 256.
Variation of Manganin Resistance with Atmospheric Humidity	Rosa and Babcock	*Ibid.*, vol. 59, p. 339.
Self-Testing Bridge	Callendar and Griffiths	*Ibid.*, vol. 60, p. 477.
Dielectric Strength of Insulating Materials	Russell	*Ibid.*, vol. 60, p. 160.
Resistance Coils and Comparisons	Drysdale	*Ibid.*, vol. 60, p. 20.
Measurement of Electrolytic Resistance	Franklin	*Ibid.*, vol. 60, p. 138.
Contact with Dielectrics	Appleyard	*Phil. Mag.*, No. 58, Oct., 1905.
The Dielectric Strength of Air	Russell	*Ibid.*, No. 62, Feb., 1906.
Electrical Resistance of Alloys	Willows	*Ibid.*, No. 72, Dec., 1906.

II.

CURRENT.

129. THE measurement of current should really have been dealt with before the measurement of resistance, since the absolute measurement of the latter quantity involves an absolute measurement of current. We have, however, taken resistance measurement first, chiefly on account of its very great importance, and also since "measurements" of resistance are in general only comparisons of the resistances of coils with that of some standard coil, and do not involve a measurement of current.

The absolute unit of current is defined in terms of the magnetic force which it produces, and is that current which, if flowing in a circuit of 1 cm. length, bent into an arc of 1 cm. radius, would exert unit force (one dyne) on a unit magnetic pole placed at the centre of the arc.

The practical unit of current is the ampère, and has a value $\frac{1}{10}$ absolute unit.

In making absolute measurements of current the magnetic effect alone is employed, but for ordinary measurements either of the two other effects of the current may be used, viz. the heating effect or the chemical effect, provided they have been once for all standardized in terms of the magnetic effect.

130. *Absolute Determinations of Current.*—In accordance with the definition of unit current, we must measure the magnetic force which the current exerts at a certain point; it is, however, impossible to construct an apparatus for this purpose in terms of the definition, and therefore it is usual to employ circular coils, and calculate the total effect which a current flowing in them will produce at their centre.

In general two classes of instruments are employed in making absolute determinations of current strength, these being—

(1) Standard galvanometers.

(2) Standard electro-dynamometers.

In the first class of instrument the magnetic force set up by the current flowing in a circular coil of known dimensions, acts on a magnetic needle suspended at its centre, under the influence of a magnetic-controlling force of known strength, whilst in the second class the galvanometer needle described above is replaced by a coil of wire of known dimensions through which the current to be measured also passes, its value being deduced from the magnetic force with which the one coil acts on the other.

131. In constructing an apparatus for the absolute measurement of current, there are certain conditions which must be fulfilled in order to insure accuracy, and which it will be as well to enumerate.

(*a*) The "constants" of the apparatus must be of a very permanent character, and should not be affected to any appreciable extent by slight relative displacements of the various parts, such as might occur due to expansion, contraction, or warping.

(*b*) There should be no variable quantity introduced into the measurement over which the experimenter has not got complete control.

(*c*) The number of measurements which involve readings of the highest possible accuracy, should be as few as possible.

In the various methods for determining current in absolute measure to be described, we shall point out wherein certain methods have an advantage over others in respect to the above conditions.

STANDARD GALVANOMETERS.

132. *Tangent Galvanometer.*—This instrument is so called because the tangents of the angles of deflection of the needle are proportional to the currents producing them, and is one of the simplest and easiest to use of the various standard

current-measuring instruments, although in point of accuracy it may be inferior to some of the others.

In its simplest form the tangent galvanometer consists of a single coil of wire, at the centre of which a magnetic needle is suspended, so that when under the directive influence of the earth's magnetism only, its magnetic axis is at right angles to the axis of the coil. When a current flows through the coil the needle is deflected out of the meridian, until the moment of the deflecting couple is balanced by the moment of the controlling couple due to the earth. Calling C the value of the current in absolute (C.G.S.) measure, m the strength of the magnetic pole, l the length of the magnet, r the radius of the coil, δ the angular deflection of the needle, and H the horizontal intensity of the earth's magnetic force, we have—

$$\frac{2\pi Cml}{r}\cos\delta = \text{deflecting couple}$$

$$\text{and } ml\text{H}\sin\delta = \text{controlling couple}$$

$$\text{therefore } \frac{2\pi Cml}{r}\cos\delta = ml\text{H}\sin\delta$$

$$\text{and } C = \frac{r}{2\pi}\text{H}\tan\delta$$

If instead of there being only one turn in the coil there are n turns, and the radius is sufficiently great for them to be assumed to be all at the same distance from the centre of the coil, then—

$$C = \frac{r}{2\pi n}\text{H}\tan\delta$$

The quantity $\frac{2\pi n}{r}$ is, of course, invariable, once the coil is wound, and is known as the "constant" of the coil, being usually denoted by the letter G. We may therefore write the above equation—

$$C = \frac{\text{H}}{\text{G}}\tan\delta$$

Also, for a given place, assuming the earth's magnetism to be constant at the centre of the coil, $\frac{\text{H}}{\text{G}}$ becomes a constant, and the formula may be written—

$$C = K\tan\delta$$

The constant G of the coil is usually determined once for all when the coil is wound, although it may be determined electrically by comparing it with a coil of known constant at any time, without unwinding it; the method of doing this will be described later on.

133. The main objection to the tangent galvanometer as a standard current-measurer is that the calculation involves a knowledge of **H**, the horizontal intensity of the earth's magnetic force, this being a somewhat variable and difficult quantity to measure with accuracy.

In some cases the galvanometer-control is supplied by a permanent magnet, in addition to the earth's control, and the field due to the permanent magnet being much greater than that due to the earth, small variations in the latter do not become so important. Such an arrangement, although better than the previous one, is always open to the objections to the use of permanent magnets, which alter with time and temperature, making it necessary to redetermine the value of H from time to time.

134. In actual practice, it is found necessary to have considerably more than one turn in the galvanometer coil, and since the turns cannot all be at the same distance from the centre, a correction must be applied. If Fig. 73 represents a

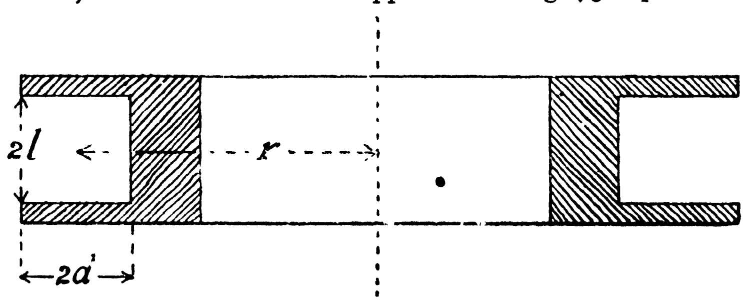

FIG. 73.

section through the coil, the depth of the windings being $2d$, and their breadth $2l$, the mean radius of the coils being r, then it can be shown [1] that—

[1] *Electrician*, vol. xxvii. p. 716; also Mascart and Joubert's "Electricity and Magnetism," vol. ii. p. 104.

$$G = \frac{2\pi n}{r}\left\{1 + \frac{1}{3}\frac{d^2}{r^2} - \frac{1}{2}\frac{l^2}{r^2}\right\}$$

and therefore—

$$C = \frac{r}{2\pi n\left\{1 + \frac{1}{3}\frac{d^2}{r^2} - \frac{1}{2}\frac{l^2}{r^2}\right\}} H \tan \delta$$

The values of d and l should be small compared with r the mean radius of the coils, which latter dimension should not be less than 15 cm. The best proportion between d and l being—

$$\frac{d}{l} = \frac{\sqrt{3}}{\sqrt{2}}$$

whilst in any case d should not be greater than $\frac{1}{10}r$.

135. The needle of the tangent galvanometer must be small compared with the radius of the coil, in order that when deflected from its zero position it may move in a uniform field. It should not be more than 1 cm. long, otherwise a correction must be made for its length, this correction, however, disappearing when the deflection is 27°. For a needle of length one-tenth the radius of the coil, the divergence from the tangent law is about 0·5 per cent.

136. *Construction.*—The bobbin on which the wire is to be wound should be made of some material which, while being an insulator and absolutely non-magnetic, will not be liable to change of shape due to warping, etc. In this latter respect, wood, ebonite, etc., are unsatisfactory; wood well seasoned, however, may be suitable, provided the coil be built up of six or eight pieces inclined to one another, so that at different parts of the bobbin the grain of the wood will run in different directions. Metals, on the other hand, such as brass, aluminium, etc., whilst being easy to turn up accurately, are liable to contain iron, which renders them quite useless. The best material of all would probably be white marble; the expense of manufacture, however, is somewhat prohibitive. Having got a suitable coil properly "turned up," the circumference must be accurately measured by means of a steel tape; this having been recorded, the wire is carefully wound on, the mean diameter inside and outside the insulation having

been previously carefully measured by a micrometer gauge, as it is required in order to calculate the mean diameter of the coil when wound, this being the diameter of the bobbin plus the depth of the windings. The number of turns wound on is also recorded.

137. The needle must be suspended accurately in the centre of the coil (both radially and axially) by means of a spider's-web or quartz-fibre suspension, and may be arranged either to indicate its deflections by means of a light pointer moving over a graduated circular scale, or else more accurately by a mirror, lamp, and scale.

Should the latter method be adopted, the needle may be attached to the back of the mirror, and in reading the deflections it must be borne in mind that the angular deflection of the spot of light is twice that of the needle. If a straight scale is used, the deflection in scale-divisions, divided by the distance from the mirror to the scale, also expressed in scale-divisions, gives the tangent of twice the angle of deflection, and the tangent of half that angle may easily be obtained from a table of tangents. For approximate calculation, one-half the tangent of twice the angle may be taken as equal to the tangent of half the angle.

138. *Correction for Torsion.*—If the suspension is not too short, and consists of a very fine quartz or spider's thread, it will not be necessary to make any correction for torsion. Should the suspension, however, be stout or short, then a correction will be necessary, and may be made as follows. The needle is allowed to come to rest at zero free from torsion; it is then, by means of a magnet, caused to rotate once round, and the scale reading when it comes to rest is noted. If there is any torsion, the spot of light will come to rest a little to the opposite side of the zero from the direction of rotation. If half this angle[1] is θ, then the coefficient of torsion, τ, is—

$$\tau = \frac{\theta}{360 - \theta}$$

so that instead of a deflection, δ, we write $\delta(1 + \tau)$.

[1] Since the angle of rotation of the mirror is half the angle of rotation of the spot of light.

139. *Adjustment.*—In setting up the tangent galvanometer (see Fig. 74), the coil must be set so as to lie in the plane of the meridian, and then be carefully levelled till the needle hangs exactly in the centre of the coil (radially and axially). The lamp and scale is then set up at the proper distance from the mirror, so that the distance from the centre of the mirror to each end of the scale is the same, and the image of the cross wire in the spot of light is at the zero on the scale.

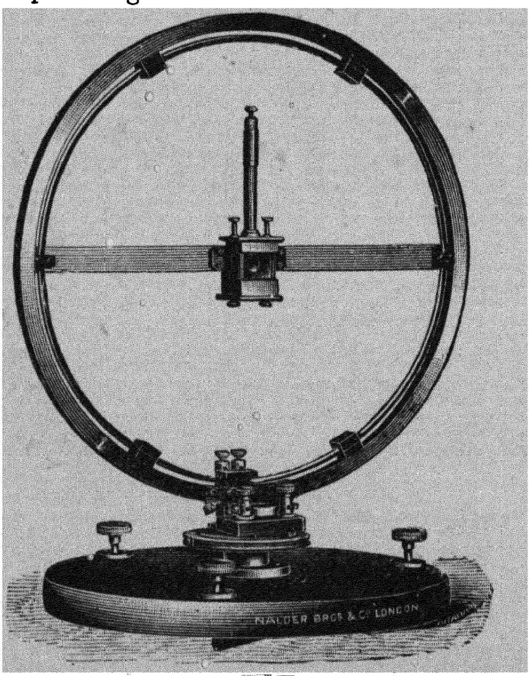

FIG. 74.

A current sufficient to give a moderate deflection on the scale is now sent through the instrument, and the deflection noted; the current is then reversed, and the deflection to the opposite side of the zero noted. If this is not the same as the previous deflection (the current having been kept constant), it means that the mirror and needle are not parallel to one another, or, in the case of a pointer instrument, that the pointer is not at right angles to the needle.

The mirror, or pointer and needle, must then be adjusted relatively to one another until, on reversing the current, the same deflection is obtained on both sides of zero.

140. *Sensitiveness of a Tangent Galvanometer.*—The position of maximum sensitiveness in a tangent galvanometer is when the needle is in the plane of the controlling field, as in that position the moment of the deflecting force is a maximum, whilst the controlling moment is a minimum. The minimum error, however, introduced into the calculation by a given error in reading the deflection, occurs when the deflection is between 40° and 50°, an error at that portion of the scale of $\frac{1}{10}°$ producing an error of 0·35 per cent. in the result.

141. *Determination of the Controlling Force at the Needle.*—The determination of the strength of the controlling force acting on the needle may be made by the method of Gauss, which will be fully described later (see par. 268), and which is applicable both when it is due to the earth's control, or to the earth and a permanent magnet; we will therefore refer the student to the section dealing with the determination of the horizontal intensity of the earth's magnetic force. After having determined H, the time of swing (1 D.V.) of the galvanometer needle should be noted and recorded: let it be T_1; then, should the control be at any time altered, it can always be brought back to the original value by adjusting the permanent magnet, so that the needle has again a time of swing, T_1, or, should it be necessary to reduce or increase the controlling force, the new value of H may be calculated from the new time of swing, for, if H and H_1 represent the controlling forces which give times of swing T_1 and T_2, then—

$$\frac{H}{H_1} = \frac{T_2^2}{T_1^2}$$

142. *Helmholtz Tangent Galvanometer.*—In order to have a more uniform field round the needle, Helmholtz designed a tangent galvanometer having two equal coils, with their axes on the same line and placed parallel to one another, the distance apart of the coils being equal to their mean radius.

The needle is hung on the line joining the centres of the coils, midway between them.

This arrangement (see Fig. 75) eliminates the terms of the second order from the calculation, the current in absolute measure being obtained from the relation [1]—

$$C = \frac{(r^2 + x^2)^{\frac{3}{2}}}{4\pi r^2 n} H \tan \delta$$

where n = number of turns on each coil;
r = mean radius of each coil;
$2x$ = mean distance apart of the two coils.

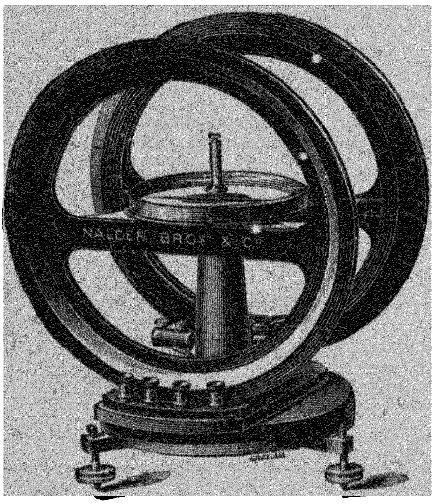

Fig. 75.

In order to see the effect of the two coils on the field round the needle, the student is referred to Maxwell's "Electricity and Magnetism," vol. ii., where plates showing the distribution of the field for one and two coils are given. In this galvanometer, when the depth of the windings is small compared with the mean radius of the coils, the correcting factor for the length of the needle practically vanishes.

143. In a single-coil tangent galvanometer the following were the dimensions:—

Mean radius of coils, r = 9·67 cm.
Radial depth of coils, $2d$ = 0·20 ,,
Axial length of coil, $2l$ = 2·40 ,,
Number of turns, n = 27

Hence the coil-constant G is—

[1] See "Elements of the Mathematical Theory of Electricity and Magnetism" (J. J. Thomson), par. 211.

$$G = \frac{2\pi n}{r}\left\{1 + \frac{1}{3}\frac{d^2}{r^2} - \frac{1}{2}\frac{l^2}{r^2}\right\}$$

$$= \frac{2 \times 3\cdot142 \times 27}{9\cdot67}\left\{1 + \frac{0\cdot10^2}{3 \times 9\cdot67^2} - \frac{1\cdot2^2}{2 \times 9\cdot67^2}\right\}$$

$$= 17\cdot55 \times 0\cdot992$$

$$= 17\cdot42$$

The torsion coefficient of the silk-thread suspension was determined by setting the spot of light on the scale to zero; the mirror and needle were then turned through one complete revolution, when the spot of light was found to come to rest ten scale divisions from the zero. The scale was 1000 mm. from the mirror, and the scale divisions were half-millimetres.

Hence the angular displacement of the light was—

$$\frac{10}{2 \times 1000} = 0\cdot005 = \tan 0\cdot3^\circ$$

and bearing in mind that the angular displacement of the spot of light will be twice that of the needle, the latter is—

$$\theta = \frac{0\cdot3^\circ}{2} = 0\cdot15^\circ$$

and τ, the torsion coefficient, is—

$$\tau = \frac{\theta}{360^\circ - \theta} = \frac{0\cdot15}{360 - 0\cdot15} = 0\cdot0004$$

144. *Sine Galvanometer.*—The standard sine galvanometer is very similar to the standard tangent galvanometer in construction (see Fig. 76), and the remarks which apply to the construction of one apply equally to that of the other, with this difference, however, that in the sine galvanometer the bobbin carrying the coils is capable of rotation about a vertical axis, a separate scale and pointer frequently being attached so that the angular rotation may be measured.

The adjustment of the sine galvanometer is similar to that of the tangent. In using the instrument after the pointer has been set to zero under the influence of the controlling force, the current is sent through it and a deflection of the needle obtained, the coils are then rotated so as to follow up the

motion of the needle, the current being kept constant; this has the effect of making the needle deflect still further, but eventually, provided the current is not too strong, the coils will gain upon the needle, and the zero on the scale may be brought

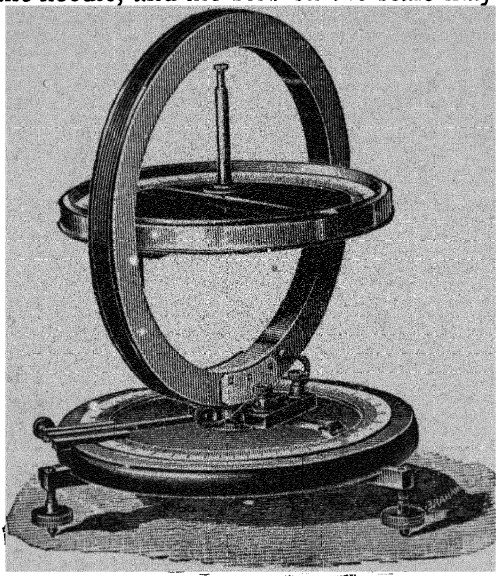

FIG. 76.

under the pointer in its new position; we have then equilibrium between the controlling and deflecting moments, the deflecting force acting at right angles to the needle, and therefore exerting its maximum turning moment.

If we call, as before, C = current in C.G.S. units,
r = radius of the coil,
n = number of turns on coil,
θ = angular rotation of coil,
m = pole strength of needle,
l = length of needle,
H = strength of controlling field,

we have—

$$\text{deflecting moment} = \frac{2\pi n C m l}{r}$$

$$\text{and controlling moment} = H m l \sin\theta$$

therefore—

$$\frac{2\pi n C m l}{r} = \text{H} m l \sin \theta$$

$$\text{and C} = \frac{r}{2\pi n} \text{H} \sin \theta$$

$$\text{or C} = \frac{\text{H}}{\text{G}} \sin \theta$$

G being $\frac{2\pi n}{r}$, the galvanometer constant; or, as before (see par. 134), if there are several layers on the galvanometer coil—

$$\text{G} = \frac{2\pi n}{r} \left\{ 1 + \frac{1}{3} \frac{d^2}{r^2} - \frac{1}{2} \frac{l^2}{r^2} \right\}$$

145. For a given controlling field the sine galvanometer does not admit of a very large range of current measurement, since, if the deflection is at all large, on rotating the coils the position of instability of the needle is soon reached, when it turns right round; so that, if required to measure currents of widely differing values, an adjustable controlling field must be provided, or else the galvanometer must be shunted.

The sine galvanometer is, however, more sensitive than the tangent, its maximum sensitiveness being reached just at the point of instability of the needle

146. The great advantage of the sine law instrument over the tangent instrument is in the case where the relative values of two or more currents are required to be measured, or where the constant of the instrument is obtained by comparison with a standard measuring instrument and not calculated from the dimensions of the coils, because all galvanometers used in the above manner follow the sine law independently of the shape of the coil, whilst only circular coils will follow the tangent law We may therefore obtain the relative calibration curve of any galvanometer by plotting the deflections of the needle when the current is sent through it before the coils are rotated, against the sine of the angular rotation of the coils required to bring the scale zero underneath the pointer.

Should the galvanometer coil not be provided with a special scale and pointer to register its angular rotation, that may be

easily measured in the following way. After the current has been sent through the instrument, and the deflection δ of the needle measured, the coils are rotated round to follow up the motion of the needle, until the zero on the scale stands underneath the pointer. The current is then broken, and the scale-reading, ϕ, where the pointer comes to rest noted; then ϕ represents the angle through which the coils have been rotated, and $\sin \phi$ is proportional to the current; therefore, by plotting values of δ against $\sin \phi$ we get the relative calibration curve of the instrument.

147. *Gray's Standard Sine Galvanometer.*—One of the objections to the ordinary form of standard sine galvanometer is the difficulty of measuring the coil constants accurately; this difficulty has been overcome in a modified form of sine galvanometer due to Professor T. Gray,[1] in which a long solenoid

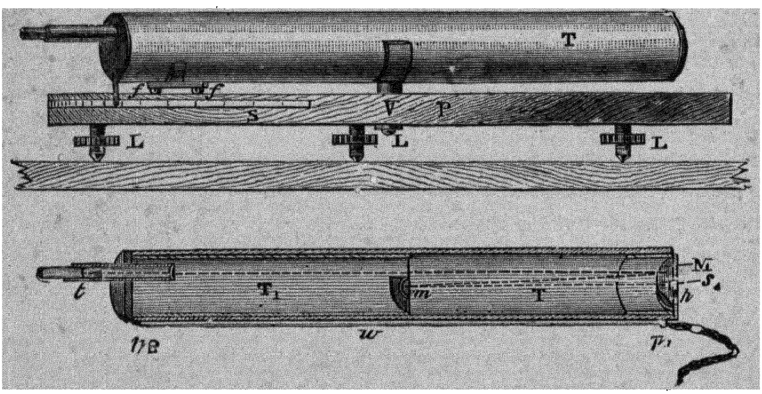

FIG. 77.

is employed instead of a ring-shaped coil. In this way, by using a solenoid whose length is from eight to ten times its radius, the field produced in the mean plane of the coil is very uniform, and may be calculated with great accuracy.

The coil of about 10 cm. diameter is mounted in a tube, T (see Fig. 77), which is free to rotate about a vertical axis, V, attached to the base P, the latter being mounted on three levelling screws, L, L, L. A pointer is attached to one end of

[1] *Phil. Mag.*, vol. xxii. Oct., 1886.

the tube, and moves over the scale S, so that the angular rotation of the coil may be measured, the end of the coil at S being supported on the small feet f, f.

The needle which is attached to the back of the small mirror m is suspended at the centre in the mean plane of the coil. At the end of the tube opposite the mirror there is a small slit, s, supplied with cross wires, and above it a plane mirror, M. When the cross wires are illuminated their image is reflected from m to M, and to the telescope t at the other end of the tube.

The coil is levelled and turned round until the image of the cross wires at s coincides with those in the telescope; the needle is then in the meridian, and at right angles to the axis of the coil. On sending a current through the instrument, the needle deflects; and in order to again make the cross wires coincide the coil must be rotated through an angle, which may be read off on the scale S. Let this angle be θ; then, when unit current flows in the coil, the magnetic force at its centre is—

$$f = \frac{4\pi n l}{\sqrt{r^2 + l^2}},$$

where $2l$ = length of the coil;

n = numbers of turns per centimetre;

r = radius of coil.

Expanding this, we get—

$$f = 4\pi n\left(1 - \tfrac{1}{2}\frac{r^2}{l^2} + \tfrac{1}{8}\frac{r^4}{l^4} - \text{etc.}\right)$$

Taking as far as the second term in this expression, we get, for the current C in absolute measure—

$$C = \frac{H \sin\theta}{4\pi n\left(1 - \tfrac{1}{2}\frac{r^2}{l^2}\right)}$$

H being the value of the magnetic controlling force at the needle.

From this formula it will be seen that when the length of the coil is great compared with its diameter, a small error in the determination of r will produce an extremely small effect in the calculation of the current.

Standard Electro-Dynamometer.

148. This form of current measurer differs from those previously mentioned in that the permanent magnet system is replaced by a small coil connected in series with the large coil, at the centre of which it is suspended by means of a bifilar suspension, which in addition to supplying the controlling force, conveys the current into and out of the small coil.

The normal position of the small coil, when no current is flowing, is with its axis at right angles to that of the large coil, and in the magnetic meridian.

When a current is sent through the instrument, the small coil tends to set itself coaxially with the larger, this tendency being balanced by the controlling couple due to the earth and that due to the suspension.

If we call C = current in absolute measure;
G = constant of the large coil;
g = constant of the small coil;
H = strength of the earth's controlling field;
θ = angular deflection of the small coil;
K = constant of the bifilar suspension;

then, when the deflecting and controlling moments are in equilibrium—

$$C^2Gg \cos \theta = CgH \sin \theta + K \sin \theta$$

from this we have—

$$\tan \theta = \frac{C^2Gg}{CgH + K}$$

CgH is always small compared with K, so that, on expanding the above, we get—

$$\tan \theta = \frac{\dfrac{C^2Gg}{K}}{1 + \dfrac{CHg}{K}}$$

$$= \frac{C^2Gg}{K} - \frac{C^3HGg^2}{K^2}$$

If we now reverse the current in the coils, we get a slightly different deflection, θ_1, and —

$$\tan \theta_1 = \frac{C^2Gg}{K} + \frac{C^3HGg^2}{K^2}$$

whence, by addition, we get—

$$\tan \theta + \tan \theta_1 = \frac{2C^2Gg}{K}$$

$$\text{and } C^2 = \frac{K}{2Gg}(\tan \theta + \tan \theta_1)$$ [1]

149. We have next to determine K, the constant of the bifilar suspension, and in order to do this we proceed as follows. The constant depends on the mass (M) of the suspended coil, and varies proportionally with it; hence we may write—

$$K = M\tau$$

where τ is a constant depending only on the suspension. To determine τ we set the coil vibrating about a vertical axis, and determine its time of double vibration, T. From this we get the well-known relationship—

$$T = 2\pi\sqrt{\frac{I}{K}}$$

$$= 2\pi\sqrt{\frac{I}{\tau M}}$$

I being the moment of inertia of the oscillating system. To the coil is now attached a bar of non-magnetic material of known mass, M′, and moment of inertia, I′, and the time of one double vibration is again taken: let it be T′. Then—

$$T' = 2\pi\sqrt{\frac{I + I'}{\tau(M + M')}}$$

from the equations for T and T′ we get—

$$K = \frac{4\pi^2 I'M}{M(T_1^2 - T^2) + M_1T_1^2}$$

150. In order to insure greater uniformity of field at the

[1] See *Electrician*, vol. xxviii. p. 272.

suspended coil, the large coil, in some forms of dynamometer, is replaced by two coils arranged af:er the manner of the coils in the Helmholtz tangent galvanometer, the small suspended

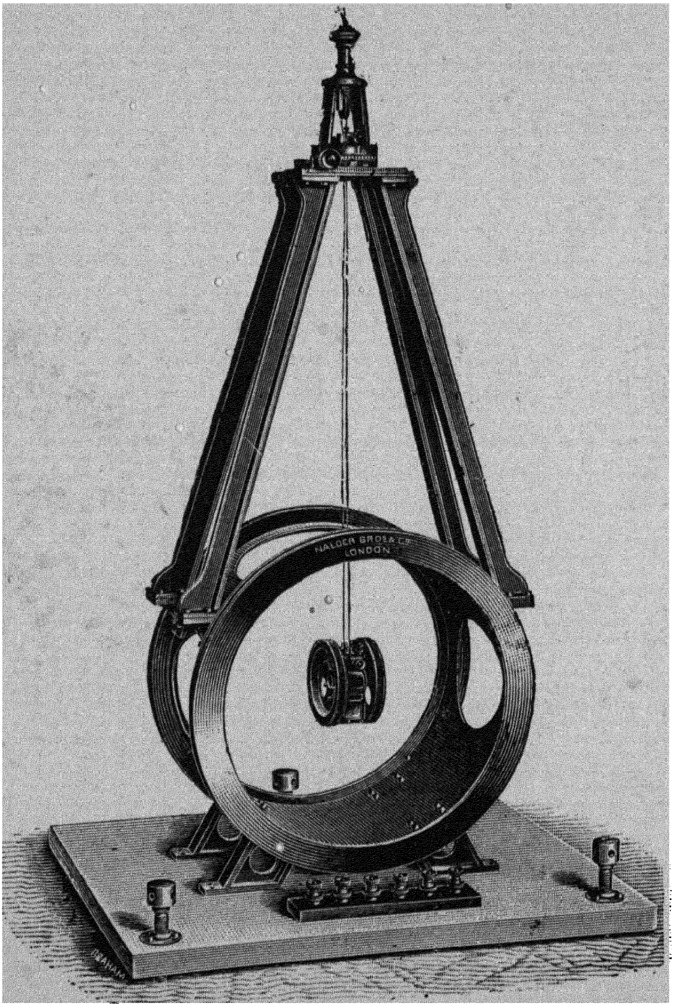

FIG. 78.

coil taking the place of the needle in that instrument; such an arrangement is shown in Fig. 78.

The chief objections to the absolute electrc-dynamometer

are the uncertainty introduced by the bifilar suspension, unless its constant is redetermined for each measurement of current, and the difficulty of accurately measuring the mean radii of the two coils and the angular deflection of the suspended coil.

151. *Current Balance.*—Of all the methods proposed for the absolute measurement of current, the current-balance method is perhaps the most accurate. It has the great advantage over the other instruments previously described, that the final calculation for the current does not involve so many measurements requiring first-class accuracy.

In its simplest form the current balance consists of two coils, one of large and one of small diameter, the coils being placed with their planes horizontal; the smaller one being suspended coaxially above the larger from the beam of a delicate balance. The coils are arranged in series, the current being calculated from the dimensions and the attraction of one for the other.

If we call A = mean radius of the large coil;
a = mean radius of the small coil;
x = distance between the mean planes of the coils;
C = current in absolute measure;
nn' = the number of turns on the coils;
m = mass in grammes which counterbalances the force between the coils;
g = acceleration of gravity;

then the force acting between the coils can be shown to be[1]—

$$f = \frac{6\pi^2 a^2 A^2 C^2 x n n'}{(A^2 + x^2)^{\frac{5}{2}}} \text{ dynes}$$

$$= mg$$

$$\text{Hence } C^2 = \frac{mg(A^2 + x^2)^{\frac{5}{2}}}{6\pi^2 a^2 A^2 x n n'}.$$

It can also be shown that the best conditions obtain when $x = \frac{A}{2}$. Putting this value into the equation, we get—

$$C^2 = \frac{A^2}{a^2} \times \frac{mg}{nn'} \times \frac{1}{16 \cdot 97}.$$

[1] *Electrician*, vol. xxviii. p. 250; also "Mathematical Theory of Electricity and Magnetism" (J. J. Thomson), par. 216.

From this result it will be noted that we do not require to know the exact value of the radius of either coil, but only the ratio of the squares of the radii. This is a very important point, since the exact determination of the mean radius of a coil is a difficult matter, whilst the ratio of the radii of the coils may be determined by an electrical method with great accuracy. All the other quantities in the calculation are capable of measurement with great accuracy.

In making a measurement, the current may be reversed in both coils, and then first in one and then in the other, the mean of four values of m, two attractions and two repulsions, being taken.

152. In order to obtain greater sensitiveness and uniformity of field, Lord Rayleigh has constructed a balance having two fixed coils, one placed coaxially above the other, parallel to it, the small coil being suspended midway between the two large coils. For further particulars the student is referred to Lord Rayleigh's paper on the determination of the electro-chemical equivalent of silver,[1] for which research the balance was constructed.

153. *Determination of the Ratio of the Radii of Two Coils by Bosscha's Method.*—In a coil of wire the cross section of which is rectangular, it is easy to show[2] that if $2l$ represents the axial length of the coil, $2d$ the radial depth, n the number of turns, and r the mean radius, that the constant G is—

$$G = \frac{2\pi n}{r}\left\{1 + \frac{1}{3}\frac{d^2}{r^2} - \frac{1}{2}\frac{l^2}{r^2}\right\}$$

Consequently, if G_1, n_1, r_1, d_1, and l_1 represent these values for one coil, and G_2, n_2, r_2, d_2, l_2 the values for another coil then—

$$\frac{G_1}{G_2} = \frac{n_1 r_2}{n_2 r_1}\left\{\frac{1 + \frac{1}{3}\frac{d_1^2}{r_1^2} - \frac{1}{2}\frac{l_1^2}{r_1^2}}{1 + \frac{1}{3}\frac{d_2^2}{r_2^2} - \frac{1}{2}\frac{l_2^2}{r_2^2}}\right\}$$

[1] *Phil. Trans*, Part II., 1884.
[2] Mascart and Joubert, "Electricity and Magnetism," vol. ii. p. 104.

and therefore—

$$\frac{r_1}{r_2} = \frac{n_1 G_2}{n_2 G_1} \left\{ \frac{1 + \frac{1}{3}\frac{d_1^2}{r_1^2} - \frac{1}{2}\frac{l_1^2}{r_1^2}}{1 + \frac{1}{3}\frac{d_2^2}{r_2^2} - \frac{1}{2}\frac{l_2^2}{r_2^2}} \right\}$$

Now, since the ratios $\frac{d_1}{r_1}$, $\frac{l_1}{r_1}$, $\frac{d_2}{r_2}$, $\frac{l_2}{r_2}$ are small, their squares may either be neglected or the values calculated from approximate data with sufficient accuracy; we may therefore write—

$$\frac{r_1}{r_2} = \frac{n_1 G_2}{n_2 G_1}$$

In order to get the ratio $\frac{G_2}{G_1}$, the coils are placed with their planes vertical, one being inside the other and coaxial with it, the planes of the coils being parallel to the magnetic meridian. A small error in the adjustment of the two coils affects the ratio as the square of the displacement.[1]

A very small magnet attached to the back of a mirror is then suspended by a fibre at the common centre of the two coils. The coils are connected in parallel, but in such a way that if the current goes round one in a clockwise direction, it goes in a counter-clockwise direction round the other. Resistances are inserted in the circuit of each coil, the connections being shown diagrammatically in Fig. 79, where, for the sake of clearness, the coils are shown as if they were lying apart from one another. A represents the large and B the small coil, C the battery, K the key, and ρ_A and ρ_B the resistances in series with the coils A and B.

The resistances in the circuits of the coils are then adjusted until, on closing the circuit at K, no deflection is observed on the small needle suspended at the common centre of the two coils, this being observed with a lamp and scale in the usual way, the whole arrangement being similar to a differential galvanometer. When an exact balance is obtained, if c_1 and c_2 represent the currents in A and B respectively, and G_1 and

[1] *Phil. Trans. Roy. Soc.*, 1885.

G_2 are the constants of A and B, R_A and R_B being the resistances of the two coils, then—

$$G_1 c_1 = G_2 c_2$$

But $R_1 c_1 = R_2 c_2$, where $R_1 = (\rho_A + R_A)$ and $R_2 = (\rho_B + R_B)$;

$$\text{therefore } \frac{G_2}{G_1} = \frac{R_2}{R_1}$$

$$\text{and } \frac{r_1}{r_2} = \frac{n_1 R_2}{n_2 R_1}$$

It will be noticed that we do not require to know the exact value of the resistance of either coil, but only the ratio of the two resistances, this being capable of measurement with very great accuracy, as will be seen by referring back to the chapter dealing with resistance measurement.

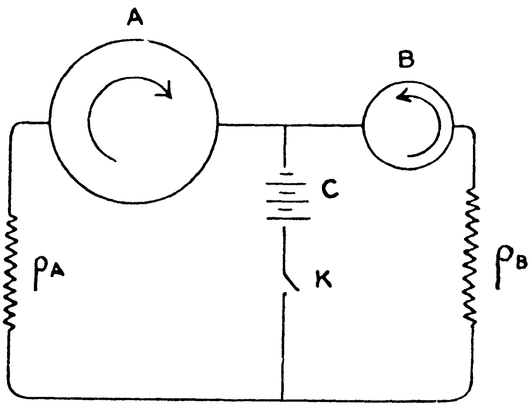

FIG. 79.

154. A neat method of comparing the resistances of the two coils whilst in position is due to Lord Rayleigh. In it the two coils, one inside the other, are connected so as to form the two arms of a Wheatstone bridge, which we will diagrammatically represent as follows (Fig. 80). A and B represent the two coils—really one inside the other, but shown separated for the sake of clearness; the common terminal *a* is connected to the battery through a reversing key, K; the other ends of the coils

are connected to the mercury cups *b* and *d*, the coil B being connected to *d* through a known resistance, R_1; two resistance coils, R_2 and R_3, in parallel form the third, and a coil, R_4, the fourth arm of the bridge; the bridge galvanometer, G, is placed across *bd*. A thick copper strap connected to the battery, and shown in dotted lines, can also connect *b* to *d*.

When the ratio of the constants of the coils is to be determined, the copper strap is placed across *bd*, and R_1 is adjusted,

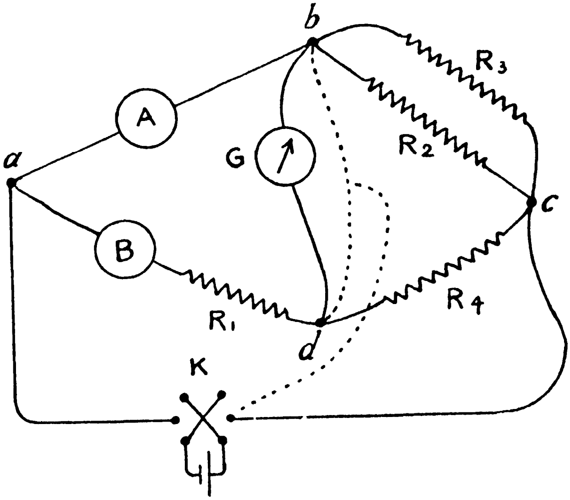

FIG. 80.

until, on completing the battery circuit, there is no deflection obtained on the little needle suspended at the common centre of the two coils. The strap is then removed, and the battery connected to C; the coil R_3 is then adjusted, till, on completing the battery circuit, the bridge galvanometer G shows no deflection. Then—

$$\frac{G_1}{G_2} = \frac{\left\langle\frac{R_2}{R_3}\right\rangle}{R_4}$$

$\left\langle\frac{R_2}{R_3}\right\rangle$ being the resistance of the two coils R_2 and R_3 in parallel, which may be calculated from their separate values.

155. Fig. 81 shows a form of current balance designed by the author for use in the laboratory. It consists of a large coil mounted on a brass stand supplied with levelling screws of such a size that it will go inside the case of a chemical balance. The smaller coil is wound on an ebonite bobbin, and so arranged that it can be suspended from one arm of the balance when the scale-pan is removed. The brass rods supporting it have adjustable screws at their ends, to admit of the coil being

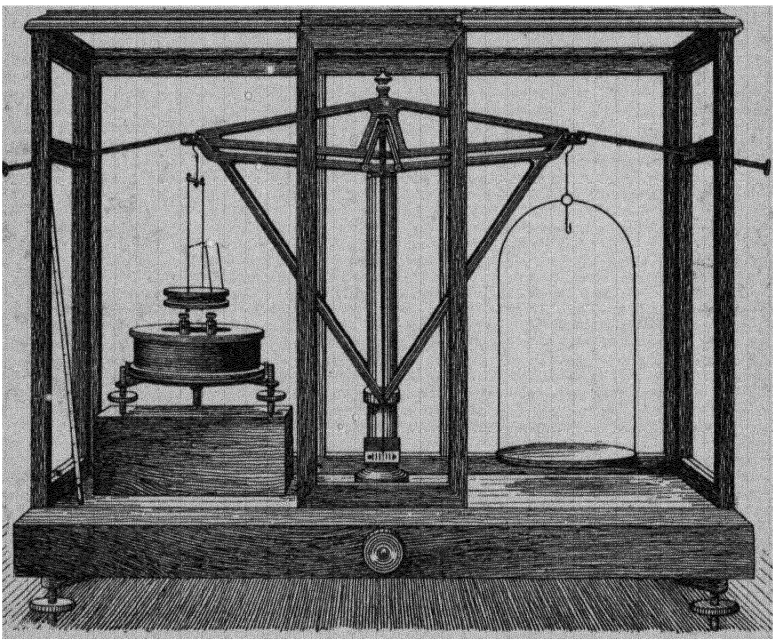

FIG. 81.

levelled properly. Inside the ebonite bobbin fits a brass ring, from which a galvanometer mirror and needle may be suspended, when it is required to make a measurement of the ratio of the radii by the method of Bosscha. In order to be able to adjust the suspended coil accurately, relatively to the large coil both as regards centering and distance apart of the mean planes, brass templates were made of the exact distance between the outer circumference of the ebonite coil and the inner

circumference of the large coil, and also of the exact distance between the upper flanges of the two coils when their mean planes were the proper distance apart. The following are the data of dimensions and winding of the coils :—

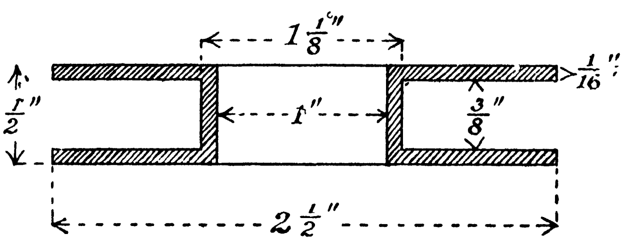

FIG. 82.—Small Coil.

Small Coil.—The external circumference of the windings was carefully measured by a steel tape, and was $6\frac{19}{32}''$, the total number of turns was 60, being 10 layers each of 6 turns. The resistance was 1·5 ohms.

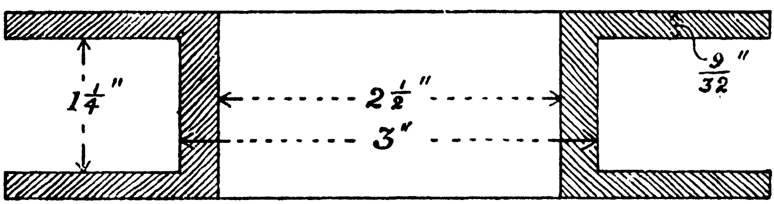

FIG. 83.—Large Coil.

Large Coil.—The external circumference of the windings, as measured by a steel tape, was $15\frac{13}{64}''$, the total number of turns was 460, being 20 layers of 23 turns each. The resistance was 4 ohms.

In setting up the apparatus, the scale-pan is removed from one side of a chemical balance, and the small coil hung in its place. Very thin insulated copper wires are led from the coil to the standard of the balance where they are attached, and then led to the rest of the apparatus. The large coil is now adjusted in position, and both it and the small coil levelled carefully by means of a spirit-level and the templates until they are at the required distance apart. Weights are placed in the other

scale-pan, to counterbalance the coil, and the final adjustment made with sand.

156. The most convenient way to use the balance is to place known weights in the scale-pan, and then adjust the current in the coils until the attraction between them exactly counterbalances the weight, and brings the pointer of the balance back to the scale zero. It will be found that the balance is extremely sensitive to small variations of current, and it is better to arrange two stops, one on either side of the pointer, to prevent it swinging too far from the zero position. A liquid or carbon resistance should be placed in series with the balance.

157. The following comparison was made between a copper voltameter and the above current balance. An accuracy of more than 1 per cent. was not aimed at, on account of the unsensibility of the balance used, which was an old one, and not that for which the coil had been designed to be used. The two coils were connected in series with each other and with a copper voltameter, carbon resistance, and six secondary cells, and the following data obtained :—

Weight of cathode before deposition	60·240 gm.
,, ,, after ,, ...	60·903 ,,
Gain in weight	0·663 ,,
Time of deposit	22 min. 12 sec.
Temperature	12° C.
Weight required to counterbalance the attraction of the coils	2·153 gm.

Taking ϵ for copper as 0·0003279, we get—

$$C = \frac{0{\cdot}663}{0{\cdot}0003279 \times 1332}$$
$$= 1{\cdot}518 \text{ ampères.}$$

The corrected ratio for the radii of the coils was determined by the method of Bosscha, and $\frac{A}{a} = 2{\cdot}28$.

$$\text{Hence } C = 2{\cdot}28\sqrt{\frac{2{\cdot}153 \times 981{\cdot}3}{460 \times 60 \times 16{\cdot}97}}$$
$$= 0{\cdot}153 \text{ absolute units.}$$
$$= 1{\cdot}53 \text{ ampères.}$$

This shows a very close agreement between the measurements, and if the balance had been more sensitive, the determination of current would have been much more accurate.

158. In determining the ratio of the radii of the two coils by the method of Bosscha, we have—

$$\frac{r_1}{r_2} = \frac{n_1 G_2}{n_2 G_1} = \frac{n_1 R_2}{n_2 R_1} \left\{ \frac{1 + \frac{1}{3}\dfrac{d_1^2}{r_1^2} - \frac{1}{2}\dfrac{l_1^2}{r_1^2}}{1 + \frac{1}{3}\dfrac{d_2^2}{r_2^2} - \frac{1}{2}\dfrac{l_2^2}{r_2^2}} \right\}$$

The resistances required in series with the coils, in order to obtain a balance when a current was sent through the two in parallel, were 1000 ohms in series with the smaller coil of 60 turns, and 3392 ohms in series with the coil of 460 turns. The axial lengths and radial depths of the coils were measured, and he following data obtained: –

$$2l_1 = 0{\cdot}375'' \qquad 2d_1 = 0{\cdot}488'' \qquad r_1 = 1{\cdot}050''$$
$$2l_2 = 1{\cdot}250' \qquad 2d_1 = 1{\cdot}921'' \qquad r_2 = 1{\cdot}960''$$

From this we get—

$$\frac{1 + \frac{1}{3}\dfrac{(0{\cdot}244)^2}{(1{\cdot}05)^2} - \frac{1}{2}\dfrac{(0{\cdot}187)^2}{(1{\cdot}05)^2}}{1 + \frac{1}{3}\dfrac{(0{\cdot}960)^2}{(1{\cdot}960)^2} - \frac{1}{2}\dfrac{(0{\cdot}625)^2}{(1{\cdot}960)^2}} = 0{\cdot}997$$

$$\text{Therefore } \frac{r_1}{r_2} = \frac{60 \times 3392}{460 \times 1000} \times 0{\cdot}997 \quad = \frac{1}{2{\cdot}28}$$

VOLTAMETERS.

159. We have seen in the preceding sections how an electric current may be determined in absolute measure by means of the magnetic effect. For ordinary use in the laboratory, in the standardization of instruments, etc., it is found more convenient to measure current by means of its chemical effect. The value of the current cannot be calculated from the chemical decomposition produced in the same way that it can from the deflection of a standard galvanometer, or from the balancing weights required in a current balance, but the constant depending on the nature of the decomposition, and called the electrochemical equivalent, is determined experimentally by comparison

with one of the standard current measurers. We are therefore able to employ the chemical decomposition produced by a current as a sort of secondary standard. The object of thus setting up a secondary standard, instead of using one of the standard methods, is that the chemical method of measuring current is much easier to carry out, and requires less complicated apparatus than the other methods, whilst being very accurate.

Faraday was one of the first to investigate the chemical effects produced by an electric current, quantitatively, and the results of his researches may be briefly summed up in his own words.

"For a constant quantity of electricity, whatever the decomposing conductor may be, whether water, saline solutions, acids, fused bodies, etc., the amount of electro-chemical action is also a constant quantity, *i.e.* would always be equivalent to a standard chemical effect founded upon ordinary chemical affinity."[1]

It will therefore be seen that if we can once for all determine the "amount of chemical action" produced by a known quantity of electricity, we can at any time measure a quantity of electricity by finding the amount of chemical action which it produces, and comparing it with that produced by a known quantity of electricity. Also, since the quantity of electricity is equal to the current flowing, multiplied by the time during which it flows, we can, by dividing the quantity of electricity by the time, calculate the current strength.

The determination of the "amount of chemical action" in a substance produced by a known quantity of electricity is usually called the determination of the electro-chemical equivalent of the substance, this being expressed as the number of grammes of substance electrolysed per coulomb of electricity, a coulomb being the quantity of electricity which passes when one ampère flows for one second.

160. Numbers of researches have been made to determine the electro-chemical equivalents of different substances; some of these results are given in the table at the end of the book. It is found, however, that comparatively few substances fulfil the

[1] Faraday's "Experimental Researches," vol. i. par. 505.

conditions necessary for secondary standard current measurers; of these silver stands out as better than the others, and for this reason it has been adopted as the substance to be used in current measurement, the legal definition of the ampère—the practical unit of current—being expressed as follows: The ampère is that current "which has the value of one-tenth in terms of the centimetre, the gramme, and the second of time, and which is represented by the unvarying electric current, which, when passed through a solution of silver nitrate in water, in accordance to the specification appended hereto and marked A, deposits silver at the rate of 0·001118 gramme per second."[1]

161. The determination of the electro-chemical equivalent of silver was made by Lord Rayleigh,[2] to whose paper the student is referred for details of the measurement.

Although the silver voltameter affords the most accurate method of measuring a current by chemical means, yet in ordinary laboratory work the copper voltameter is generally employed, on account of greater cheapness and ease of manipulation, although there are various sources of error which must be guarded against, and which will be treated of later.

162. In regard to the silver voltameter, we cannot do better than reproduce the specification for its preparation referred to in the legal definition of the ampère, since this represents the result of long and patient investigation with varying conditions.

"In the following specification the term silver voltameter means the arrangement of apparatus by means of which an electric current is passed through a solution of nitrate of silver in water. The silver voltameter measures the total electrical quantity which has passed during the time of the experiment, and by noting this time the time average of the current or, if the current has been kept constant, the current itself can be deduced.

"In employing the silver voltameter to measure currents of about one ampère, the following arrangements should be adopted. The cathode on which the silver is to be deposited

[1] See *London Gazette*, Friday, Aug. 24, 1894.
[2] *Phil. Trans. Roy. Soc.*, 1884.

should take the form of a platinum bowl not less than 10 cm. in diameter, and from 4 cm. to 5 cm. in depth. The anode should be a plate of pure silver some 30 sq. cm. in area and 2 mm. or 3 mm. in thickness.

"This is supported horizontally in the liquid near the top of the solution by a platinum wire passing through holes in the plate at opposite corners. To prevent the disintegrated silver which is formed on the anode from falling on to the cathode, the anode should be wrapped round with pure filter paper secured at the back with sealing wax.

"The liquid should consist of a neutral solution of pure silver nitrate, containing about 15 parts by weight of the nitrate to 85 parts of water.

"The resistance of the voltameter changes somewhat as the current passes. To prevent these changes having too great an effect on the current, some resistance besides that of the voltameter should be inserted in the circuit. The total metallic resistance of the circuit should not be less than 10 ohms.

"The platinum bowl is washed with nitric acid and distilled water, dried by heat, and then left to cool in a desiccator. When thoroughly dry it is weighed carefully.

"It is nearly filled with the solution, and connected to the rest of the circuit by being placed on a clean copper support, to which a binding screw is attached.

"This copper support must be insulated. The anode is then immersed in the solution so as to be well covered by it, and supported in that position; the connections to the rest of the circuit are made.

"Contact is made at the key noting the time of contact. The current is allowed to pass for not less than half an hour, and the time at which the current is broken is observed. Care must be taken that the clock used is keeping correct time during this interval.

"The solution is now removed from the bowl, and the deposit is washed with distilled water and absolute alcohol, and dried in a hot-air bath at a temperature of 160° C. After cooling in a desiccator it is weighed again. The gain in weight gives the silver deposited.

"To find the current in ampères, this weight, expressed in grammes, must be divided by the number of seconds during which the current has passed, and by 0·001118.

"The result will be the time average of the current, if during the interval the current has varied.

"In determining by this method the constant of an instrument, the current should be kept as nearly constant as possible, and the readings of the instrument observed at frequent intervals of time. These observations give a curve from which the reading corresponding to the mean current (time average of the current) can be found. The current, as calculated by the voltameter, corresponds to this reading."

163. In connection with the silver voltameter, it is worthy of note that Professor T. Gray,[1] who has had large experience with it, advocates the use of silver plates for both cathode and anode, instead of a silver anode and a platinum bowl cathode, the advantages claimed being that, on account of the lightness of the silver cathode, a more delicate balance may be used on which to weigh it; also it is much easier to clean the silver plates before the experiment than the platinum bowl. From a large number of experiments he found the best results were obtained from a solution of silver nitrate containing from 5% to 10% by weight of silver nitrate, and working at a current density of 200 to 600 sq. cm. per ampère. If these limits were exceeded, it was found that the silver deposit became unsatisfactory and difficult to wash, on account of its not adhering firmly to the cathode.

164. *Copper Voltameter.*—For most purposes not requiring the highest accuracy it is usual to employ the copper voltameter in preference to the silver one, since the manipulation of the plates does not require the same skill, nor are the conditions for accurate work so limited. The copper voltameter has, however, certain peculiarities which, on account of its frequent use in the laboratory, merit special attention.

Many experimental researches have been undertaken from time to time with a view to perfecting this instrument, but of these perhaps the most important is the work done by Professor

[1] *Phil. Mag.*, Nov., 1886.

T. Gray[1] in determining the value of the electro-chemical equivalent of copper, and we will largely follow his suggestions in connection with this part of the subject.

The voltameter consists of a glass vessel containing a solution of copper sulphate, into which, suspended from suitable clips, the copper electrodes dip. The plates should be made of good electrolytic copper, the cathodes being as thin as is consistent with strength, in order that the gain in weight due to the deposit may represent a larger fraction of the total weight of the plate than it otherwise would if the plates were heavy; also, by having the plates light a more delicate balance may be employed, and the weight determined with greater accuracy. The anodes may be made of considerably thicker copper plate, since they will be gradually dissolved away.

The plates should be cut with a lug at the top for attachment to the clips, all the corners being rounded off, and no sharp edges left, since it is found that the deposit tends to form in a crystalline manner at such places. The clips for holding the plates should be of brass, copper, or platinoid springs, so that a good electrical contact may be obtained, whilst the plates may easily be removed for cleaning and weighing. A convenient form of clip is shown in Fig. 84, consisting of a copper spring pressing on a copper plate fastened to a backing of ebonite. The plates should be about 1 cm. apart and perfectly parallel to one another, otherwise the deposit will not be uniform over the surface.

165. *Treatment of the Plates.*—The plates, before use, must be thoroughly cleaned and polished with sand-paper, the sand being afterwards removed by placing them in running water and rubbing with a clean rag or brush. *On no account are the fingers to be placed on that part of the plate which is to receive the deposit.*

If the plates are oxidized they may be cleaned by dipping them into a bath of potassium cyanide, care being taken to have excess of cyanide present. After removal from the cyanide they must be thoroughly washed with water. After cleaning,

[1] *Phil. Mag.*, vol. xxii., Nov., 1886; also *Phil. Mag.*, vol. xxv., Mar., 1888.

the plates are placed in the voltameter, each cathode being between two anodes, the number of cathodes required, depending on the current to be measured and the size of the plates; tables of data will be given later.

In the case of the standardization of an instrument, the circuit will consist of the voltameter, the instrument to be calibrated (if necessary so arranged that the current in *it* may be reversed), a

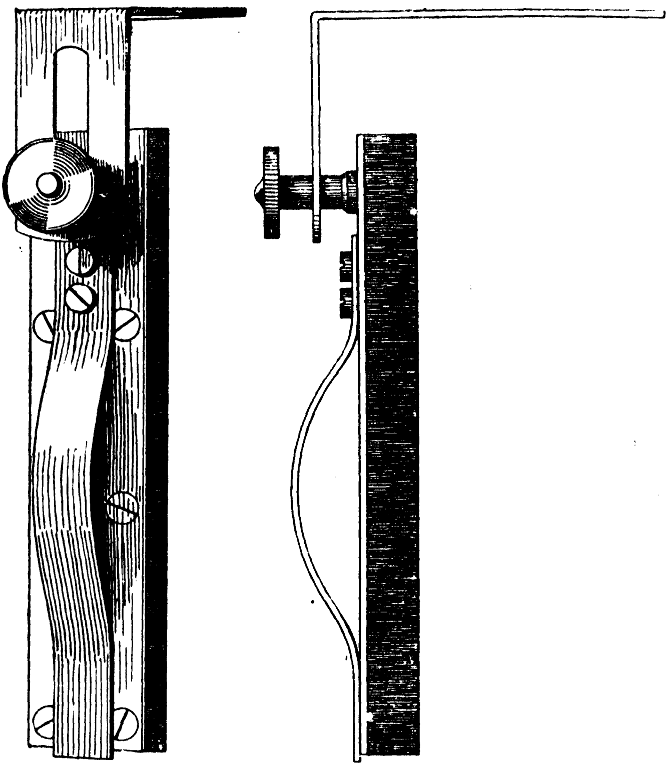

FIG. 84.

variable resistance, break-circuit key, and secondary battery. The resistance is adjusted until the required current is obtained, as indicated roughly by the reading on the instrument; this is allowed to flow for fifteen minutes, the cathode is removed and *at once* plunged into a bath of slightly acidulated water, to remove the copper sulphate solution from the surface before

the plate can oxidize; it is then thoroughly washed in tap-water and dried in slightly heated clean white blotting-paper; after allowing to cool in a dessicator, it is weighed, the weight being taken to $\frac{1}{10}$ mg. The plate should be lifted by means of forceps, and, if possible, the fingers should never touch it. The plate is now carefully replaced in the voltameter, and the current started, the time being taken on a stop-watch. During the deposition, which should last one hour, the current must be kept constant, by means of the variable resistance in circuit, so that the reading on the instrument remains the same. When the current is stopped, and the time taken, the cathode is removed and subjected to exactly the same treatment as before, and then weighed. If W represents the gain in weight of the cathode in grammes, T the time in seconds during which the deposition has lasted, ϵ the electro-chemical equivalent of copper, and C the current in ampères, then—

$$C = \frac{W}{\epsilon T}$$

166. Having described the method of using the voltameter, we will briefly state the causes which effect the value of ϵ. These are found to be—

(*a*) The size of the plates, *i.e.* the current density at the electrodes.

(*b*) The density of the solution.

(*c*) The chemical action of the solution on the plates.

(*d*) The temperature.

These points have been investigated more or less fully by Gray and others,[1] and Gray gives a table of *apparent* electro-chemical equivalents, for various current densities at various temperatures, which may be assumed to include any errors that may arise from cause (*c*).

The solution for which this table is constructed is made as follows. Pure recrystallized copper sulphate is dissolved in tap-water (distilled water is not necessary) until a density of 1·18 is reached, and to this one per cent. by volume of strong

[1] Gannon, *Electrician*, vol. xxxii. p. 216; Schuster, *Electrician*, vol. xxxii. p. 216; Gore, *Nature*, vol. xxv. p. 472; Vanni, *Wied. Ann.*, vol. 44, p. 214.

sulphuric acid is added. The quantity of this used should be about 3 cub. cm. of solution per square centimetre of plate-area immersed, the solution being used for an aggregate time of ten hours and then replaced by fresh. Measurements made in accordance with the above instructions may have a probable error of not more than 0·05 %.

167. The following table of values of ϵ for copper are given by Gray, on the assumption that ϵ for silver is 0·001118:—

Area of cathode in square centimetres per ampère.	Values of ϵ.				
	2° C.	12° C.	23° C.	28° C.	35° C.
50	0·0003288	0·0003287	0·0003286	0·0003286	0·0003282
100	0·0003288	0·0003284	0·0003283	0·0003281	0·0003274
150	0·0003287	0·0003281	0·0003280	0·0003278	0·0003267
200	0·0003285	0·0003279	0·0003277	0·0003274	0·0003259
250	0·0003283	0·0003278	0·0003275	0·0003268	0·0003252
300	0·0003282	0·0003278	0·0003272	0·0003262	0·0003245

It will be seen from the above table that the apparent electro-chemical equivalent increases with the current density, the increase being, however, small. The effect of change of temperature is also small until a temperature of about 30° C. is reached, when it becomes important. Alterations in the density of the electrolyte produce very little effect between the limits of 1·15 and 1·18, there being a slight decrease in the value of ϵ as the density increases.

168. One of the most difficult points to determine is the chemical action of the solution on the cathode. The dissolving action that appears to go on is attributed by Schuster to the action of the dissolved oxygen in the solution, and on electrolyzing in a vacuum the gain in weight of the cathode is slightly greater. It was also found to be necessary to have the solution distinctly acid before consistent results could be obtained. Gray has shown that the rate of solution of the cathode is very irregular but never exceeds $\frac{1}{200}$ mg. per square centimetre per hour, and it is a minimum for solution densities between 1·10 to 1·15.

Various attempts have been made to obtain a solution which will not act on the cathode, and Vanni gives one made by adding 0·005 gramme of sulphuric acid per litre to a perfectly neutral solution of copper sulphate, this giving a value for ϵ of 0·0003287 between 50 and 200 sq. cm. per ampère plate-area. The same result is claimed by Oettel [1] using alcoholic solutions of copper sulphate.

169. *Iodine Voltameter.*—For the measurement of very small currents of electricity, such as those that are employed in ordinary reflecting galvanometers, the usual forms of silver or copper voltameters are hardly applicable, the quantity of electricity passing in an electrolysis, lasting several hours even, being very small, so that the gain in weight of the cathode is very slight, and the effect of the dissolved oxygen in the solution on the deposit may be serious. In order to measure very small currents such as these, an ingenious voltameter has been constructed by Mr. Herroun,[2] which depends on the estimation of the amount of iodine liberated from one of its salts during electrolysis.

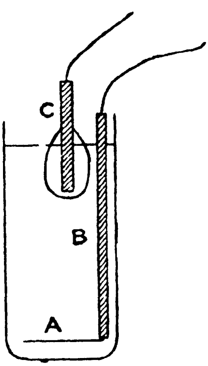

FIG. 85.

The voltameter consists of a tall narrow beaker, at the bottom of which is a platinum anode, A (see Fig. 85), connection to which is made by means of a platinum wire fused into a glass tube containing mercury, B. The cathode consists of a rod of pure zinc, wrapped in filter-paper, in order to prevent pieces of zinc which may become detached from falling to the bottom of the vessel; this rod only dips a few centimetres into the solution, which is a 10–15 per cent. solution of neutral zinc iodide. The zinc iodide solution should be kept in a dark place, in contact with a little pure zinc, in order to prevent its decomposing. Should zinc iodide not be obtainable, the electrolyte may be made up of a solution containing 5 per cent. zinc chloride, to which 15 per cent. potassium iodide has been added.

[1] Oettel, *Electrician*, vol. xxxi. p. 59.
[2] *Phil. Mag.*, vol. xl., July, 1895.

When a current is sent through the voltameter, iodine separates out at the platinum anode, but, on account of its great density, remains at the bottom of the glass vessel. The current density at the cathode should not exceed 1 ampère per 200 sq. cm. surface, otherwise insoluble periodate is liable to be formed.

After the electrolysis is stopped, the zinc cathode is at once removed, and the solution stirred up; it will have a reddish-brown colour, due to the liberated iodine. The amount of iodine liberated is 0·001314 gramme per coulomb, and in order to determine the amount liberated by the electrolysis, the brown solution is titrated with a standard solution of pure sodium thiosulphate, a convenient strength being found to be 12·845 grammes of pure sodium thiosulphate to 1000 cub. cm. water; 1 cub. cm. of this solution being equivalent to 0·00657 gramme iodine or 5 coulombs of electricity per cubic centimetre of thiosulphate required in the titration.

The vessel containing the brown solution should be placed on a sheet of white paper, and a burette filled with the sodium thiosulphate solution fixed in a retort stand, so as to be able to run it into the iodine solution. The thiosulphate is run in slowly, and, when the brown solution is nearly decolorized, drop by drop. When the last trace of colour vanishes the burette is closed, and the amount of liquid that has been run out noted. Multiplying the number of cubic centimetres of liquid used by five, and dividing by the time of the electrolysis in seconds, the current in ampères is obtained.

If greater accuracy is desired the burette should be weighed before and after the experiment, and the volume of thiosulphate solution used calculated from its specific gravity and the weight used.

Also, when the titration has been nearly completed, a little clear starch solution may be added to the iodine solution, and the exact point when the titration has been completed judged by the vanishing of the blue coloration produced by the free iodine on the starch.

170. When very small currents are to be measured, such as may be employed in the calibration of high resistance sensitive

galvanometers, the time of electrolysis must be considerable, and the form of the voltameter must be altered to prevent the diffusion of the iodine through the solution, the most convenient form being a U-tube with an asbestos plug at the bend in the tube.

If the thiosulphate solution has been standing for any length of time it is liable to decompose, and should be tested with a standard iodine solution.

171. The following experiment will illustrate the accuracy of the above method of measuring current.

A current from three secondary cells was sent through an iodine voltameter in series with a 50-ohm standard coil and an adjustable resistance. The current was regulated by the adjustable resistance, so that the potential difference at the terminals of the 50-ohm coil just balanced the E.M.F. of a Clark standard cell of 1·434 volts. The current was allowed to flow for 33 min. 20 sec. The zinc was then removed, and the solution stirred up, and titrated with the standard solution of sodium thiosulphate (5 coulombs per cubic centimetre). It was found to require 11·47 cub. cm.

$$\text{Current as determined by Clark cell} = \frac{1{\cdot}434}{50} = 0{\cdot}0286 \text{ amp.}$$

$$\text{Current as determined by voltameter} = \frac{11{\cdot}47 \times 5}{2000} = 0{\cdot}0286 \text{ amp.}$$

Lord Kelvin's Current Balances.

172. We have already described the principle of the absolute current balance, and from the nature of the calculation for the current passing through it, it will be seen that its indications are independent of the magnetic force of the earth, the controlling moment being due to a weight in the scale-pan of the balance.

This principle has been employed by Lord Kelvin in a series of instruments for the measurement of currents, the instruments being called current balances. These instruments, however, differ from the absolute current balance in this respect: that whilst in the former the current is calculated from the dimensions of the apparatus and the balancing weight employed; in

the latter, the constant of the instrument is determined by the aid of a copper voltameter. On account of the invariability of this constant, the balances may be employed as secondary standard current measuring instruments.

In each of the balance instruments, except the kilo-ampère balance, each movable ring is actuated by two fixed rings—all three approximately horizontal (see Fig. 86). There are two such groups of three rings—two movable rings attached to the two ends of a horizontal balance arm pulled, one of them up and the other down, by a pair of fixed rings in its neighbourhood. The current is in opposite directions through the two movable rings to practically annul disturbance due to horizontal components of terrestrial or local magnetic forces. In all the instruments the balance arm is supported by two trunnions, each hung by an elastic ligament of fine wire, through which the current passes into and out of the circuit of the movable rings. In all the balance instruments in which the movable ring is between the two fixed rings, the mid-range position of each movable ring is in the horizontal plane nearly midway between the two fixed rings which act on it. The current goes in opposite directions through the two fixed rings, so that the movable ring is attracted by one and repelled by the other. The position of the movable ring equidistant from the two fixed rings, is a position of minimum force, and the sighted position, for the sake of stability, is above it at one end of the beam and below it at the other, in each case being nearer to the repelling than to the attracting ring by such an amount as to give about 0·2 per cent. more than the minimum force.

The balancing is performed by means of a weight which slides on an approximately horizontal graduated arm attached to the balance; and there is a trough fixed on the right-hand end of the balance, into which a proper counterpoise weight is placed, according to the particular one of the sliding weights in use at any time. For fine adjustment of the zero, a small metal flag is provided, as in an ordinary chemical balance. This flag is actuated by a fork, having a handle below the case outside. To set the zero, the left-hand weight is placed with its pointer at the zero of the scale, and the flag is turned to one side or

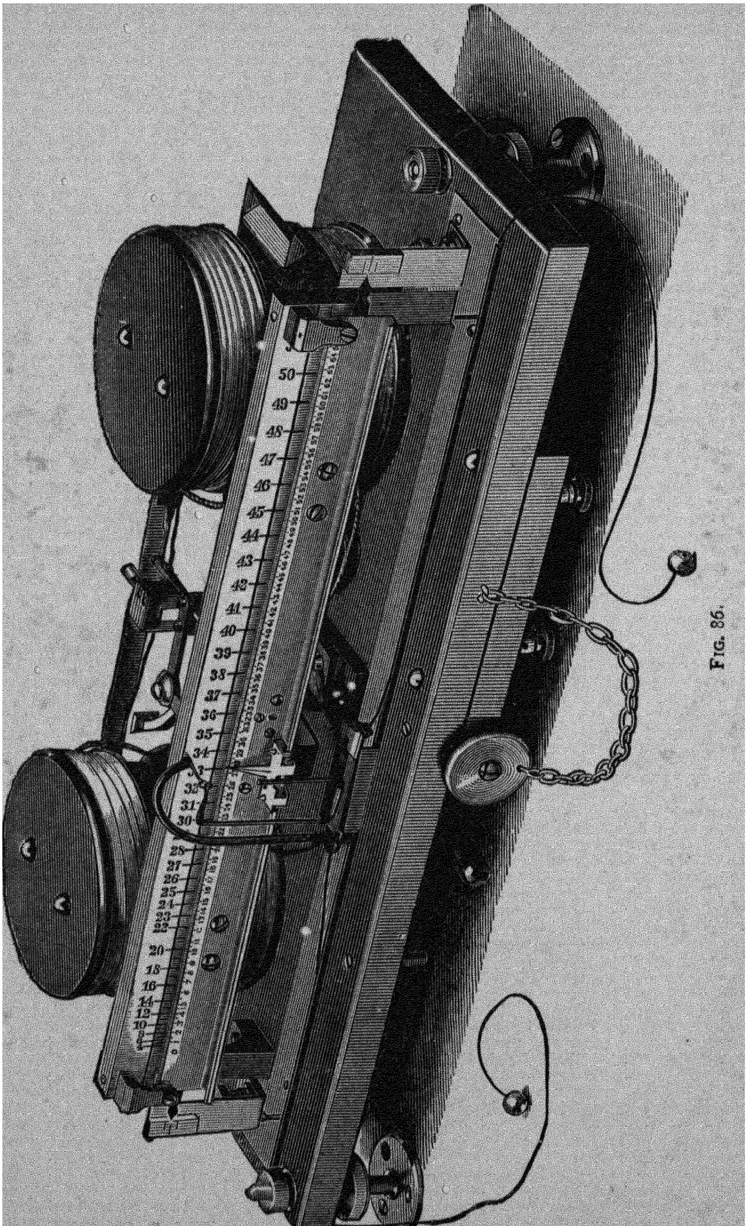

FIG. 86.

the other until it is found that, with no current in the rings, the balance rests in its sighted position.

To measure a current, the weight is slipped along the scale until the balance rests in its sighted position. The strength of the current is then read off approximately on the fixed scale (called the inspectional scale) with the aid of the finely divided scale for more minute accuracy, according to the explanations given below. Each number on the inspectional scale is twice the square root of the corresponding number on the fine scale of equal divisions.

The slipping of the weight into its proper position is performed by means of a self-releasing pendant, hanging from a hook carried by a sliding platform, which is pulled in two directions by two silk threads passing through holes to the outside of the glass case. Four pairs of weights, sliding and counterpoise, of which the sledge and its counterpoise constitute the first pair, are supplied with each instrument. These weights are adjusted in the ratios of 1 : 4 : 16 : 64, so that each pair gives a round number of ampères, half-ampères, quarter-ampères, or of decimal sub-divisions or multiples of these magnitudes of current, on the inspectional scale. The useful range of each instrument is from 1 to 100 of the smallest current for which its sensibility suffices, these ranges in the centi-ampère, deci-ampère, and deka-ampère balances being from 1 to 100 centi-ampères, deci-ampères, and ampères, respectively. The balances are designed to carry 75 per cent. of their maximum current continuously, and their maximum current long enough for all standard purposes. The following table gives for each type of instrument the value per division of the inspectional scale corresponding to each of the four pairs of weights:—

Weights.	I. Centi-ampères per division.	II. Deci-ampères per division.	III. Ampères per division.
1st pair	0·25	0·25	0·25
2nd "	0·50	0·50	0·50
3rd "	1·0	1·0	1·0
4th "	2·0	2·0	2·0

The fixed inspectional scale shows, approximately enough for most purposes, the strength of the current; the notches in the top of the aluminium scale show the precise position of the weight corresponding to each of the numbered divisions on the fixed scale, which practically annuls error of parallax due to the position of the eye. When the pointer is not exactly below one of the notches corresponding to integral divisions of the inspectional scale, the proportion of the space on each side, to the space between two divisions, may be estimated inspectionally with accuracy enough for almost all practical purposes. Thus we may readily read off 34·2 or 34·7 by estimation, with little chance of being wrong by one in the decimal place. But when the utmost accuracy is required, the reading on the fine scale of equal divisions must be taken, and the strength of the current calculated by the aid of a table of doubled square roots. Thus, for example, if the reading is 292, we find 34·18, or say 34·2, as the true scale reading for the strength of current; or, again, if the balancing position of the pointer be 301 on the fine scale, we find 34·70 as the true reading of the inspectional scale. The centi-ampère balance, with a thermometer to test the temperature of the rings, and with platinoid resistances up to 1600 ohms, serves to measure potentials from 10 to 400 volts, the following being the constants when so used :—

Weights.	Resistance in circuit.[1]	Volts per division of fixed scale.
First pair	400	1·0
,,	800	2·0
,,	1200	3·0
,,	1600	4·0

If the second pair of weights is used, the constants will be double those given above.

173. *Adjustment of the Balances.*—The instrument should be levelled, in accordance with the indications of the attached spirit-level, by means of the levelling screws on which the sole plate of the instrument stands.

[1] Including the resistance of the instrument, which is about 50 ohms.

In the centi- and deci-ampère balances, the beam can be lifted off its supporting ligaments by turning a handle attached to a shaft passing under the sole-plate of the instrument.

This shaft carries an eccentric, on the edge of which rests the lower end of a vertical rod, which is fixed at its upper end to a tripod lifter. When the instrument is to be removed by hand from place to place, the lifter should be raised; but when it is fixed up for regular use, it is advisable to keep the beam always hanging on the ligaments.

A set of four sliding weights, of which the carriage constitutes one, is supplied with each instrument. The carriage is fitted with an index to point to the movable scale, and is intended to remain always on the rail. One or other of the weights is to be placed on the carriage in such a way that the small hole and slot in the weights pass over the conical pins. The weights are moved by means of a slider, which slides on a rail fixed to the sole-plate of the instrument, and carries a pendant with a vertical arm intended to pass up through the rectangular recess in front of the weight and carriage. The slider and weight are shown in position in the figure. The slider is moved by silk cords, which pass out at the ends of the glass case. When the cords are not being pulled for shifting the weight, their ends should be left free so that the pendant may hang clear of the weight.'

When a weight is to be placed on or removed from the carriage, the slider should be drawn forward at the top until it is clear of the weight, and then pushed to one side until the weight is adjusted, when it may be replaced in position in a similar manner.

Cylindrical counterpoise weights, with a cross-bar passed through them, are supplied for the purpose of balancing the sliding weights when they are placed at the zero of the scale. The sliding weight should be placed so that the index of the carriage points to the zero of the scale, and the proper counterpoise weight should be placed in the trough, fixed to the right-hand end of the beam, with its cross-bar passing through the hole in the bottom of the trough. The flag, which is attached to the cross trunnion of the beam, should then be turned by

means of the handle projecting from under the sole-plate until the index on the end of the movable scale points to the middle one of the fine black lines on the fixed scale opposite to it. Care must be taken when making this adjustment that the fork which moves the flag is not left in contact with it, as this would impede the free swing of the beam. The fork should be turned back a little after each adjustment of the flag, and, when the flag is being adjusted, it is better to watch the flag itself, and make successive small adjustments until the beam stands at zero, than to make successive trials by pushing the handle round while watching the position of the index.

If the ligament has stretched since the instrument was standardized, the index at one end of the movable scale will be found to be below the middle line on its vertical scale, when the index at the other end is correctly pointing to its zero position. The error so introduced would be a small one, but it may easily be put right by slightly loosening the movable beam to the base-plate, and raising it by slipping one or two thicknesses of paper below it until the indexes simultaneously point to their zero position.

In using the centi-ampère balance as a voltmeter when great accuracy is required, care must be taken that the effect of change of temperature in changing the resistance of the coils of the instrument, and of the external resistance coils is allowed for; and in this use of the instrument it is advisable to employ currents such as can be measured by the lightest weight on the beam. When the instrument is to be used as a voltmeter, four resistances are provided, three of which are 400 ohms, and the fourth is less than 400 ohms, by the resistance of the coils of the instrument at a certain specified temperature. The smallest resistance is intended to be included by itself in the circuit when the lowest potentials are being measured, and in series with one or more of the others when the potential is so high as to give a stronger current than can be measured with the lightest weight on the beam. The correction for temperature is, for the copper coils of the balance, about 0·38% per 1° C., and for the platinoid resistances, about 0·024% per 1° C.

SIEMENS' ELECTRO-DYNAMOMETER.

174. A most useful piece of apparatus in the laboratory for the measurement of currents is the Siemens' electro-dynamometer. It also partakes of the nature of a secondary standard current measurer, inasmuch as it is independent of variations of the earth's magnetism or of the magnetism of permanent magnets, the controlling force being supplied by the torsion of a spiral spring, the constant of which is determined once for all, and, provided the instrument is carefully used, will remain unaltered for years. The instrument is shown in Fig. 87, and consists essentially of two coils—one fixed to the framework of the instrument, and the other suspended by a silk thread so that it hangs with its axis at right angles to that of the other coil. To the upper end of the suspended coil is also fixed one end of the spiral spring, the other end of which is attached to a torsion head, which has a pointer moving over a circular scale divided into degrees. A pointer is also attached to the movable coil, which is free to move between two stops on either side of the scale zero. The free ends of the suspended coil dip into mercury cups, the connections being made so that the fixed and movable coils are in series. In most forms of this instrument there are two fixed coils—one consisting of a few turns of thick wire, and the other of many turns of finer wire, thus enabling two degrees of sensibility to be obtained. The central terminal (3) is common to both coils, terminals 1 and 3 are those of the suspended coil in series with the thin fixed coil, while terminals 2 and 3 are those of the suspended coil in series with the thick fixed coil. A spring not shown in the figure, actuated by a screw at the back of the instrument, can be released when the instrument is not in use, and supports the weight of the suspended coil.

In setting the instrument up, it must be placed so that when the suspended coil swings freely its axis is in the magnetic meridian, so that the earth's magnetism will not produce any deflecting effect on it. The instrument is levelled until the coil swings freely, and its pointer is at the zero on the scale, the torsion head pointer also being at zero. Thy winding of

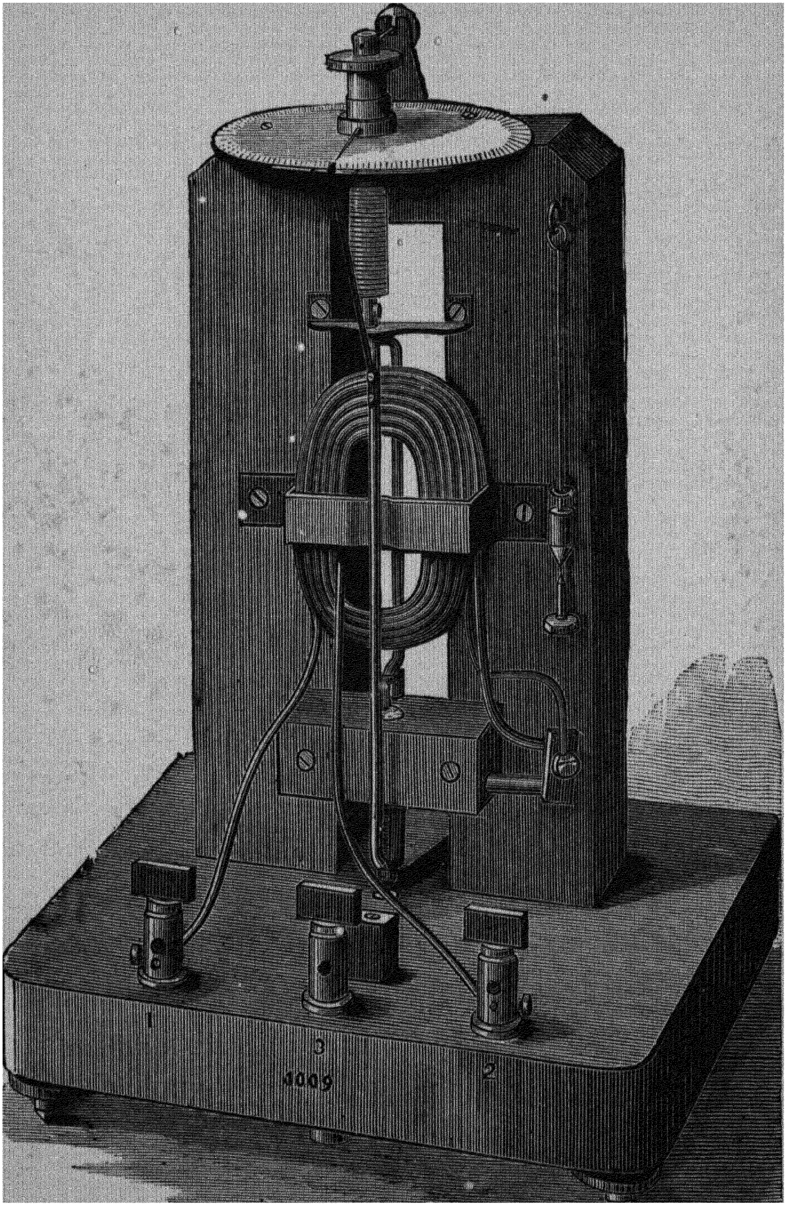

Fig. 87.

the instrument is such that when a current traverses the two coils, the suspended coil rotates in a counter-clockwise direction, the distance through which it can move being limited by the stops. The torsion head is now rotated in a clockwise direction, until the pointer attached to the coil is brought back to zero. Since the same current flows through both coils, the resultant magnetic effect between them must be proportional to the product of the currents in each—that is, to the square of the current. Also, since in the torsion control the controlling force is proportional to the angle of torsion ϕ, we have—

$$C^2 \propto \phi$$
$$\text{and } C = K\sqrt{\phi}$$

where K is a constant depending on the torsion control and winding of the instrument, and must be determined experimentally for the dynamometer.

175. *Determination of the Constant of the Dynamometer.*—To determine the constant K, we must send a known current through the instrument, and find the angle of torsion ϕ, required to bring the suspended coil back to zero. The current may be measured by means of a current balance, standard galvanometer, or voltameter. The dynamometer, after having been levelled as described, is connected in series with a variable resistance, secondary battery, voltameter, and break-circuit key. The current is first adjusted until the deflection of the torsion head required to produce equilibrium is pretty large, the instrument being much more sensitive at high than at low readings, since the squares of the readings increase less rapidly at high than at low values. The voltameter plates are then removed, and after being prepared as described previously, (par. 165), are weighed, and replaced in the voltameter; the electrolysis should be allowed to go on for about one hour, the current being kept constant by means of the variable resistance.

The current C is calculated from the gain in weight of the cathode, and then the constant calculated, since—

$$K = \frac{C}{\sqrt{\phi}}$$

ϕ being the angle of torsion as read off on the dial. Two or

three such determinations may be made at various parts of the scale, and the mean of the values thus obtained taken as the constant of the instrument.

176. *Calibration of a Direct Current Reading Instrument.*—The Siemens' electro-dynamometer affords a very simple means of absolutely calibrating a direct-reading ammeter, the ammeter being connected in series with the dynamometer, and with a variable resistance, battery, and break-circuit or reversing key. Simultaneous readings are taken on both instruments with various currents, the currents being calculated from the reading on the Siemens' electro-dynamometer, by multiplying the square root of the reading of the torsion angle by the dynamometer constant. Thus a calibration curve may be plotted for the direct-reading instrument, or a table of corrections made out.

In using the electro-dynamometer in connection with other apparatus, care must be taken that the magnetic effect of the current in the other apparatus does not affect it.

177. In order to determine the constant of a Siemens' electro-dynamometer, it was connected in series with a copper voltameter, carbon resistance, break-circuit key, and six secondary cells. The copper cathode was carefully weighed before the deposit.

Weight of cathode before deposit	58·654 gm.
" " after "	60·244 "
Gain in weight	1·590 "

The time of deposit was one hour, and the temperature of the bath was 11° C. The dynamometer reading was kept constant at 59·3.

Taking ϵ for copper as equal to 0·0003279—

$$\text{current} = \frac{1{\cdot}590}{0{\cdot}0003279 \times 3600}$$
$$= 1{\cdot}347 \text{ ampères}$$

$$\text{Hence, since } C = K\sqrt{\phi}$$
$$1{\cdot}347 = K\sqrt{59{\cdot}3}$$
$$\text{and } K = 0{\cdot}175$$

178. REFERENCES TO SCIENTIFIC PAPERS.

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On an Electrodynamic Balance	Helmholtz	*Pro. Roy. Soc.*, vol. 32, p. 39.
Absolute Determination of a Current by a Balance	Blyth	*Pro. R.S.E.*, vol. 13, p. 650.
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Calculation of the Constants of the Absolute Electro-Dynamometer	Gray	*Ibid.*, vol. 33, Jan., 1892.
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Copper Electrolysis in Vacuo	Gannon	*Pro. Roy. Soc.*, vol. 55, p. 66.
Action of $CuSO_4$ and H_2SO_4 on Metallic Copper	Schuster	*Ibid.*, vol. 55, p. 84.
On the Electrolysis of Copper and Silver	T. Gray	*Phil. Mag.*, vol. 22, Nov., 1886.
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On the Apparent Variation of the Electro-Chemical Equivalent of Copper	Vanni	*Ibid.*, vol. 28, p. 12.
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Heating Effects of Electric Currents	Preece	*Pro. Roy. Soc.*, vol. 36, p. 464; vol. 43, p. 280; vol. 44, p. 109; vol. 48, p. 68.

Title of Paper.	Author.	Reference.
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Standards of Measurement	Carhart	*Science*, vol. 8, p. 326.
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Galvanometers	Ayrton and Mather	*Phil. Mag.*, vol. 46, p. 349.
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Commercial Electrical Measuring Instruments	Shoults	*Northern Soc. Elect. Eng. Pro.*, vol. 5, p. 31.
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Electro-Chemical Equivalents of Oxygen and Hydrogen	Lehfeldt	*Phil. Mag.*, No. 89, May, 1908.
The Lead Voltameter	Betts and Kern	*Elect.*, vol. 54, p. 16.
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A New Current Weigher	Ayrton, Mather, Smith	*Ibid.*, vol. 60, p. 751.
The Silver Voltameter	Smith, Mather, Lowry	*Ibid.*, vol. 60, p. 403.

III.

ELECTRO-MOTIVE FORCE.

179. THE absolute determination of an electro-motive force in electro-magnetic measure is made from the relations which subsist between electro-motive force, current, and resistance, the product of current and resistance being equal to electro-motive force. This being so, if current and resistance are expressed in absolute units, the electro-motive force will be in absolute units; if current is in ampères and resistance in ohms, electro-motive force will be in volts, the volt being the practical unit of electro-motive force, and being that necessary to send a current of one ampère through a resistance of one ohm.

STANDARDS OF E.M.F.

180. A standard of E.M.F. might be arranged by sending a known current through a known resistance, and using the E.M.F. at the ends of the resistance as the standard. This is done in some cases, but it is found much more convenient to employ some form of voltaic cell whose E.M.F. is a known multiple of the unit.

If such secondary standards are to be employed, they must fulfil certain conditions.

(*a*) They must be easily made and reproduced.

(*b*) They must remain absolutely constant under constant physical conditions.

(*c*) Their alteration with the alteration of conditions must be accurately known.

(*d*) They must always return to their original value when the original conditions are reproduced.

STANDARD CELLS.

181. Very few, of numbers of primary batteries that have been brought out from time to time, fulfil the above conditions. One or two cells, however, stand out from the others as being suitable for use in this direction. Of these we propose to describe the three best known—

(1) The Clark cell.
(2) The Daniell cell.
(3) The Weston cell.

Of these the first is by far the most important, both on account of its constancy, and on account of the exact knowledge we possess of its behaviour under varying conditions, since its action has been the subject of numerous researches. It is also important because the legal definition of the volt is expressed in terms of the E.M.F. of one of its modifications, the definition being as follows:—

182. "The volt which has the value 10^8 in terms of the centimetre, the gramme, and the second of time, being the electrical pressure that, if steadily applied to a conductor whose resistance is one ohm, will produce a current of one ampère, and which is represented by 0·6974 $(\frac{1000}{1434})$ of the electrical pressure at a temperature of $15°$ C. between the poles of the voltaic cell known as Clark's cell, set up in accordance with the specification appended and marked B." [1]

183. The particular form of cell mentioned in the specification marked B. is usually known as the Board of Trade or B.O.T. Clark cell, and differs somewhat from the original form; as, however, there are a great many modifications of this cell, and more especially as some of the most accurate information we possess on Clark cells was obtained from some of these other forms, it will be necessary to describe some of the more important modifications, and to discuss their relative merits and deficiencies.

184. *The Rayleigh H Type of Clark Cell.*—This modification was adopted by Lord Rayleigh after long and careful investigation; [2] it consists of an H-shaped glass vessel (see Fig. 88).

[1] *London Gazette*, Aug. 24, 1894.
[2] *Phil. Trans. Roy. Soc.*, 1884, 1885, 1886.

In one limb is placed some mercury which has been purified by distilling in vacuo, and to which contact is made by means of the platinum wire P_1 fused through the glass. On

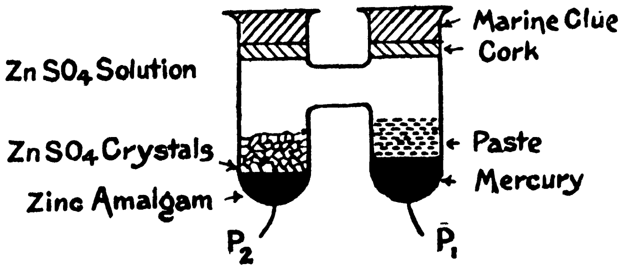


FIG. 88.

the top of the mercury is placed a layer of paste composed of mercury, mercurous sulphate (Hg_2SO_4) and zinc sulphate. In the other limb is placed some zinc amalgam (1 Zn to 9 Hg), a platinum wire, P_2, fused through the glass, making contact with it. A few crystals of $ZnSO_4$ are placed on the top of the amalgam, and the remainder of the glass vessel is filled with a solution of $ZnSO_4$, saturated at 30° C. The open ends are closed with corks and marine glue. Slight modifications of this cell have been made by Kahle[1] and Cooper,[2] the form due to the latter being specially adopted for immersion in a water bath.

185. *The Board of Trade Clark Cell.*[3]—The shape and construction of the B.O.T. form of Clark cell is designed with a view to make the cell more portable than the Rayleigh form. The cell is contained in a glass tube (see Fig. 89).

Pure redistilled mercury is placed at the bottom of the tube, contact with which is made by means of a platinum wire, the upper part of which is insulated from the rest of the cell by a glass tube. The paste above the mercury is similar in composition to that in the Rayleigh cell, the zinc sulphate solution also being saturated at 30° C. The negative electrode,

[1] *Elect.*, vol. 29, p. 516, and vol. 31, p. 265.
[2] *Pro. Roy. Soc.*, vol. 59, p. 368.
[3] *Elect.*, vol. 27, p. 98.

however, consists of pure unamalgamated zinc, which passes through the solution and enters a little way into the paste.

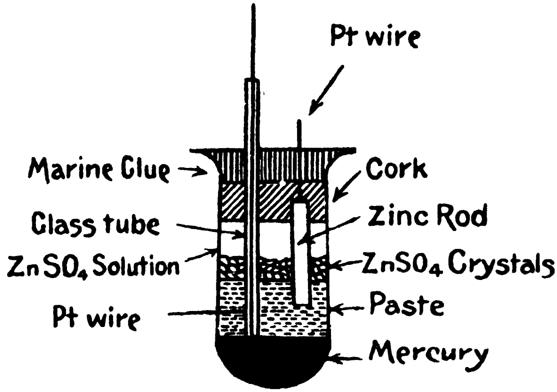


FIG. 89.

The cell is sealed up with a cork and marine glue. If Clark cells are made up in accordance with the specification B, the E.M.F. is stated not to vary more than 0·001 volt from 1·434 volt at 15° C.

186. *The Carhart Clark Cell.*—This form was introduced by Professor Carhart[1] to overcome some of the difficulties met

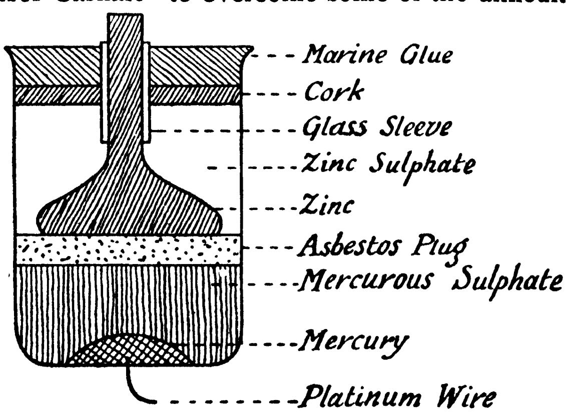


FIG. 89*a*.

with in the Clark cell; the arrangement of materials is shown in Fig. 89*a*.

[1] *Phil. Mag.*, vol. 28, Nov.. 1889.

The main points of difference between this and the previous cell are (*a*) the pad of asbestos separating the paste from the zinc, (*b*) the protecting glass sleeve round the upper part of the zinc rod, and (*c*) the saturation of the zinc sulphate solution at 0° C. instead of 30° C. The E.M.F. of this cell is about 0·4 % higher than that of the B.O.T. form, being 1·438 volts at 15° C.

187. *Callendar's Crystal Clark Cell.*—This is a modification of the B.O.T. cell introduced by Professor Callendar,[1] in which the zinc sulphate solution is replaced by moist crystals of $ZnSO_4$, packed closely round the zinc rod. The glass containing-tube is smaller in bore and considerably longer, thus allowing the complete immersion of the essential parts of the cell in a water bath.

187*a*. *Behaviour of Clark Cells.*—One of the chief criticisms against the B.O.T. Clark cell during recent years refers to the inconstancy of its E.M.F. This want of constancy may be experienced in two ways: (*a*) cells made up by different people with chemicals emanating from different sources, are found to differ in their E.M.F.'s; and (*b*) cells made up by the same individual which agree amongst themselves when new are found to differ, in some cases very considerably, after having been in use for some time. That the first case of want of agreement should exist is not very surprising, and points to the presence of impurities in the materials of which the cells are composed—different chemists having widely different notions respecting the meaning of the words "chemically pure." This cause of difference is a very serious one, for not only is it impossible for all who intend setting up standard cells to obtain their materials from the same source, but there is no real assurance that the same standard of purity will be maintained for a lengthened period by the same manufacturer. The causes of variation in the second case are, if anything, still more serious, on account of the difficulty of discovering and removing them. In particular, it has been found that specimens of the B.O.T. cell cannot be relied upon to agree amongst themselves more closely than 1 part in 1000 when freshly

[1] *B.A. Report* (Toronto); *Elect.*, vol. 39, p. 638.

made up,[1] and after standing some time even this agreement was not maintained, Cooper[2] having found that cells which, a week after construction, had a mean error of 1 in 7000, after $3\frac{1}{2}$ years differed by 1 in 700, which is considerably more than the limit permitted in the Board of Trade specification.

As regards the causes of inconstancy in the B.O.T. cell, Carhart has shown that the mercury is apt to come in contact with the zinc, which is unamalgamated, and thus alter the E.M.F. Local action may also occur where the zinc enters the paste, resulting in the amalgamation of the zinc. Another defect is that, in consequence of the zinc sulphate solution being saturated at 30° C., ordinary fluctuations in temperature produce crystallizations and resolutions of the $ZnSO_4$, thus causing differences in density to occur at different levels; this affects the E.M.F., and also tends to set up local action at the zinc rod.

It was partly with the object of overcoming this latter difficulty that the Carhart Clark cell was introduced. In it an asbestos plug separates the mercurous paste from the zinc rod, whilst the $ZnSO_4$ solution is saturated at 0° C, so that at all ordinary temperatures there never can be any $ZnSO_4$ crystals present in the cell, and consequently fluctuations in temperature will not set up differences in density in the solution. It has been pointed out recently, however, by Messrs. Spiers, Twyman, and Waters,[3] that if an error of a few tenths of a degree be made in the temperature of saturation of the $ZnSO_4$ solution, there will be a corresponding error of several parts in 10,000 in the value of the E.M.F. of the cell; and also that every time a current is taken from the cell a small quantity of zinc is dissolved, thus causing the solution to become gradually denser.

Another cause of disturbance in the Clark cell has been investigated recently by W. Jaeger.[4] The normal $ZnSO_4$

[1] Dearlove, *Elect.*, vol. 40, p. 387; Cooper, *Elect.*, vol. 40, p. 165; Fisher, *Elect.*, vol. 36, p. 647.

[2] *Elect.*, vol. 40, p. 748.

[3] *Phil. Mag.*, vol. 45, p. 285.

[4] *Ann. Phys. Chem.*, 63. 1, pp. 354, 365.

crystal has the chemical formula $ZnSO_4 + 7H_2O$; this, however, at a temperature of 39° C. loses a molecule of water and becomes $ZnSO_4 + 6H_2O$. These two forms of zinc sulphate possess different solubilities, and cells made up each with a different form would have different E.M.F.'s. It was also shown that when cooled slowly the $ZnSO_4 + 6H_2O$ may not return to its original form, unless in contact with a crystal of the form $ZnSO_4 + 7H_2O$, and at 15° C. the difference in the E.M.F.'s of two such cells would amount to as much as 7 parts in 10,000, the $ZnSO_4 + 6H_2O$ cell having the lower E.M.F.

187*b*. *Temperature Coefficient.*—In all forms of Clark cell the E.M.F. varies considerably with temperature; consequently not only is an accurate knowledge of the temperature coefficient necessary, but also an accurate knowledge of the temperature of the cell. This latter, in the ordinary made up standard cells, is registered by means of a thermometer placed alongside the cell in the brass containing-case, but for accurate work this is not satisfactory, since, with a fluctuating temperature, there may be very considerable differences in the temperature of the thermometer and cell on account of their different thermal capacities. Such cells must be kept in a water-bath, at a constant temperature, for some time before using.

In the Rayleigh H cell the temperature coefficient has been carefully measured by Lord Rayleigh [1] for a number of different cells, and from the mean of these the following correction is obtained :—

$$E_{t^\circ} = 1{\cdot}4345 \{1 - 0{\cdot}00077 (t^\circ - 15^\circ)\}$$

Thus an error of 1° in reading the temperature would give an error of about 8 parts in 10,000 in the E.M.F.

In the Carhart Clark cell the temperature coefficient is about one half of the above, the correction given by Professor Carhart being—

$$E_{t^\circ} = 1{\cdot}438 \{1 - 0{\cdot}000387 (t^\circ - 15^\circ)\}$$

the difference being due to the different saturation temperature of the $ZnSO_4$ solution.

[1] *Phil. Trans. Roy. Soc.*, part ii., 1885.

In the B.O.T. cell Ayrton[1] states, that after having been set up for some time the Hg_2SO_4 settles down, leaving a clear solution above, and should the zinc rod dip only into this solution and not into the paste, the temperature coefficient will only be half its normal value. Small alterations in the saturation temperature of the $ZnSO_4$ solution also affect the temperature coefficient considerably.

The Callendar crystal cell has a temperature coefficient practically the same as that of the Rayleigh and B.O.T. forms.

187*c*. *Time or Diffusion Lag.*—No matter how accurately we may know the temperature of a cell and have determined its temperature coefficient, it does not always follow that we are justified in applying it in order to calculate the E.M.F. of the cell, as it is found that the change in E.M.F. does not immediately follow on the change of temperature, but in general lags behind it. This phenomenon is known as "time lag," and is due to the slowness of the process of diffusion in the cell when the density of a particular layer of solution has been altered by the temperature change.

Many experiments have been made to determine the amount of the time lag in cells, and the limiting rate of fluctuation of temperature that may be permitted whilst using the ordinary temperature coefficient. The most important results in connection with this point are those of Ayrton and Cooper,[2] who have shown that in a cell of the B.O.T. pattern, if kept at a constant temperature for 30 minutes before use, the E.M.F. will not differ by more than 1 part in 10,000 from its calculated value. The time lag must depend largely on the construction of the cell and on its treatment; as if shaken, the rate of diffusion would be much greater than if kept perfectly still. Callendar[3] states that the E.M.F. of a cell kept perfectly still may alter for several weeks, consequent on a sudden change of temperature, before it comes to a steady value. The Callendar crystal cell has a great advantage over the other

[1] Ayrton's "Pract. Electricity," vol. i. p. 475.
[2] *Pro. Roy. Soc.*, vol. 59, p. 368.
[3] *B. A. Report*, Toronto.

forms in this respect, since there is no liquid in the cell in which diffusion currents may be set up; the lag in consequence is very small.[1]

187*d*. *Behaviour of the Cell when supplying Currents.*—Although it is not usual to take any appreciable current from a standard cell in ordinary testing, yet in view of possible accidental short circuits it is advisable to investigate how far the generation of such currents will affect the E.M.F. of the cell. On this point there have been a number of interesting experiments carried out. According to Skinner,[2] the amount of polarization in a Clark cell is directly proportional to the current density, but if the resistance of the external circuit is not less than 10,000 ohms, polarization need not be taken into account. In the Carhart Clark cell,[3] when the external resistance was 30,000 ohms, no polarization was detected, and when it was 1000 ohms it only amounted to 1 part in 10,000 in five minutes.

As regards the production of larger currents, such as those produced by an accidental short circuit, Fisher gives some interesting and remarkable data.[4] In one experiment, after allowing a Clark cell to supply a current on a potentiometer due to a want of balance of $\frac{1}{100}$ volt for 1 minute, the E.M.F. fell 6 parts in 14,000, and the cell required 10 minutes' rest to recover; whilst after an actual short-circuit for 30 seconds through 2 inches of No. 18 copper wire, the E.M.F. fell 1 part in 1400, and required 30 minutes' rest to recover. The rate of recovery could in every case be accelerated by charging the cell from a slightly higher E.M.F. Thus a cell whose E.M.F. had fallen to 0·17 volt by short-circuiting recovered in $2\frac{1}{2}$ hours when charged from an E.M.F. of 1·4 volts as against 24 hours if left to itself. It has also been found that the internal resistance of Clark cells rises greatly after standing for some time without supplying a current; but that it becomes reduced very considerably after a temporary short circuit, so that an occasional short-circuit, far from being injurious to the cells, was, if anything, rather beneficial.

[1] *Phil. Mag.*, vol. 45, p. 294.
[2] *Elect.*, vol. 33, p. 644.
[3] *Phil. Mag.*, vol. 28, Nov., 1889.
[4] *Elect.*, vol. 33, p. 647.

187*e*. *Materials for Clark Cells.*—The mercury and zinc supplied by chemists can, in general, be relied upon for purity, if they have been redistilled. The greatest difficulty will doubtless be experienced in obtaining pure mercurous sulphate. This generally contains mercuric sulphate which, under the action of water, splits up into an acid and a basic salt; the latter can readily be detected by its yellow colour, is insoluble in water, and does not affect the E.M.F.; but the acid salt must be got rid of, and, as it is soluble in water, it is recommended to wash the mercurous sulphate in water repeatedly and grind it up with pure mercury. This treatment, however, will not always be found to give satisfactory results. The zinc sulphate should also be neutral, and the neutralization of the acid sulphate with zinc oxide does not recommend itself as an easy or satisfactory process.

On the whole, the difficulties met with in obtaining pure materials must be largely responsible for the differences in the behaviour of Clark cells, especially since such impurities as are likely to exist exercise a large effect on the E.M.F. of the cell.

Standard Daniell Cell.

188. The Daniell cell, although much simpler to construct than the Clark cell, has the disadvantage that it is not portable. If, however, care is exercised in making it up, it will be found a very convenient form of standard for laboratory work, and has also the great advantage that its temperature coefficient is practically negligible.

The best form in which to embody this standard, and the best conditions for working, have been carefully investigated by Dr. J. A. Fleming.[1]

The cell consists of a glass U-tube fixed to a wooden base. Into one limb of the tube dips the copper, and into the other the zinc plate; the copper plate being surrounded by a solution of copper sulphate, and the zinc by zinc sulphate solution, the liquids being carefully poured into the tube so that the interface

[1] Fleming, "On the use of Daniell's Cell as a Standard of Electro-Motive Force," *Phil. Mag.*, vol. xx., Aug., 1885.

between them is sharp. The interface may be arranged so as to be at the level of an escape-tube provided with a glass tap, so that when it becomes blurred as the liquids diffuse into one another, a little of the solution may be run off and the interface again made sharp. Two additional glass reservoirs, one connected to each limb of the U-tube, may be provided, in which a supply of solution, $CuSO_4$ in one and $ZnSO_4$ in the other, may be kept. The copper plate should consist of a thin copper wire 4″ long, on which copper has been plated until it is about $\frac{1}{4}$″ thick, the rod being always freshly plated before use. The zinc rod, of about the same dimensions, should be of twice-distilled zinc, which has been cast into rods and then amalgamated with pure mercury.

189. *Solutions.*—The solutions may be made of various strengths, but the E.M.F. of the cell varies slightly with the density of the solution. The two sets of solutions most frequently employed are equi-dense solutions of specific gravity 1·2 at 15° C., and $ZnSO_4$ solution specific gravity 1·4 and $CuSO_4$ solution specific gravity 1·1 at 15° C.

Of these the equi-dense solutions of specific gravity 1·2 are made by dissolving 28·25 parts by weight of pure crystallized $CuSO_4$ in 71·75 parts of distilled water, and 32 parts by weight of $ZnSO_4$ in 68 parts of distilled water; whilst the $ZnSO_4$ solution of density 1·4 takes 55·5 parts by weight of $ZnSO_4$ crystals to 44·5 parts of distilled water, and the $CuSO_4$ of density 1·1, 16·5 parts by weight $CuSO_4$ to 83·5 parts by weight of distilled water.

190. *Electro-Motive Force.*[1]—Careful measurements of E.M.F. were made, using the above solutions, the standard of reference being the Clark cell, which was taken as 1·435 volt at 15° C., the results being as follows:—

Using equi-dense solutions of 1·2 sp. gr.—

$$\text{E.M.F.} = 1{\cdot}102 \text{ volt}$$

Using $ZnSO_4 = 1{\cdot}4$ sp. gr. and $CuSO_4 = 1{\cdot}1$ sp. gr.—

$$\text{E.M.F.} = 1{\cdot}072 \text{ volt}$$

[1] See also par. 220.

these representing the values as obtained from a freshly made up cell. After standing about an hour these values increase by about 0·003 volt.

191.—*Causes of Variation of E.M.F. in the Daniell Cell.*—The purity of the electrodes greatly affects the E.M.F. of the cell; if, for instance, the copper rod is cleaned with sand-paper instead of being copper-plated, the E.M.F. increases by about 0·6 %. Oxidation of the copper raises the E.M.F. by uncertain amounts.

The difference between pure zinc and pure zinc amalgamated is very small; oxidation of the zinc lowers the E.M.F., whilst the smallest deposit of copper on it, which will occur if any of the $CuSO_4$ solution diffuses into the $ZnSO_4$, lowers the E.M.F. by from 2 % to 3 %. As regards the solutions, an increase in density of the $ZnSO_4$ lowers the E.M.F., whilst an increase in density of the $CuSO_4$ raises it.

The effect of temperature on the cell is small, the E.M.F. falling by about 0·00015 volt per degree Centigrade rise of temperature.

When not in use the rods should be removed from the solutions and kept in separate vessels.

The Cadmium Cell.

192. Many experiments have been made with various combinations of metals and salts, with a view to obtain a primary cell free from the defects found to exist in the present Clark cell, and one of the most successful of these has resulted in the production of the Cadmium cell.

This cell was first devised by Weston in America in 1891,[1] as the result of endeavours to obtain a cell with a small or negligible temperature coefficient. Weston, recognizing that one of the most important factors determining the variation of the E.M.F. of the Clark cell with temperature was the alteration in density of the $ZnSO_4$ solution, chose as the electrolyte in his cell a salt which was equally soluble in hot or cold

[1] Patent No. 22482, 1891.

water, viz. cadmium sulphate.[1] The electrodes of the cell are mercury and an amalgam of cadmium and mercury. The mercury is covered with a layer of paste consisting of neutral mercurous sulphate, mercury, and cadmium sulphate; the remainder of the cell being filled with a saturated solution of neutral cadmium sulphate. The form of cell adopted by Weston is shown in Fig. 89*b*.

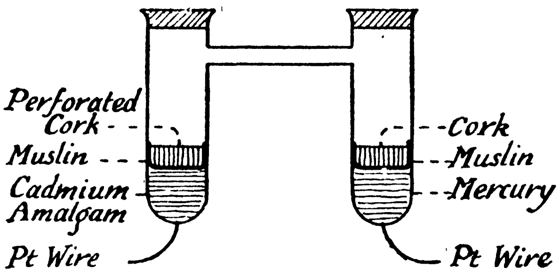


FIG. 89*b*.

The E.M.F. given by Weston is 1·019 volt, and the temperature coefficient 0·01 % per 1° C.

192*a*. In 1893 Dearlove published the results of some of his experiments on cadmium cells of the above form,[2] dealing more particularly with the influence of the amalgam composition on the E.M.F., the results showing an increase in the E.M.F. with the proportion of cadmium, a pure cadmium electrode giving an E.M.F. of 1·08 volt, whilst an amalgam of 1 of Cd to 100 Hg gave an E.M.F. of 1 volt. The results show a rapid rise of E.M.F. with the proportion of cadmium till a proportion of 7 Cd to 100 Hg is reached, when the E.M.F. becomes nearly constant. Dearlove also gives the temperature coefficient as ranging between 0·003 % – 0·007 % per 1° C., the E.M.F. falling as the temperature rose, and the whole variation being due to the action of heat on the amalgam.

[1] *Berichte der Chemischen Gesellschaft*, 30, 824, 1897.
[2] *Elect.*, vol. 31, p. 645.

192*b*. Jaeger and Wachsmuth[1] have investigated the influence of impurities on the cells, and find that the effect of all ordinary impurities is very small; thus with 1 % of zinc present in the cadmium the E.M.F. only increased 0·0019 volt, and an increase of 1 % cadmium in the amalgam raised it by 0·001 volt. The same authors, in a paper published in 1897,[2] give a detailed description of the cadmium standard cell adopted by the Reichsanstalt, which is of the H pattern with an amalgam composition 6 Hg to 1 Cd, the temperature coefficient being 0·004 % per 1° C. at 20°C. and the E.M.F. 1·0186 volts at 20° C.[3] Certain irregularities were found to exist at low temperatures, this being explained by Cohen,[4] and by Callendar and Barnes,[5] as being due to a break in the solubility curve of cadmium sulphate occurring at 13° C., and to the existence of two hydrates of $CdSO_4$ similar to the two hydrated zinc sulphates found in the Clark cell.

192*c*. *Cadmium Crystal Cell.*—From a consideration of the behaviour of the H-shaped cadmium cell it occurred to the

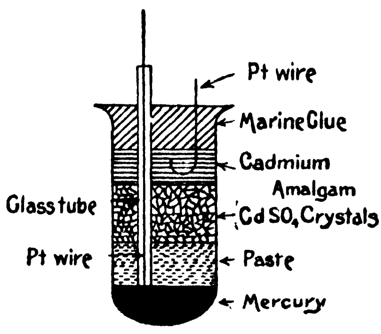


Fig. 89*c*.

author that some of the difficulties might be overcome by replacing the cadmium sulphate solution by moist crystals of cadmium sulphate. Fig. 89*c* illustrates the construction of the cell.

[1] *Electrotechnische Zeitschrift*, No. 37, p. 507, 1894.
[2] *Ibid.*, Oct. 21, 1897.
[3] *Wied. Ann.*, No. 18, 1898.
[4] *Verslag Akad.*, Amsterdam, Nov. 10, 1897
[5] *B.A. Report*, Toronto.

The cell is best made up in a narrow glass tube about $\frac{1}{2}''$ diameter, as if the tube is too wide the materials are apt to get shaken up if the cell is subjected to rough treatment, such as being sent through the post. The paste consists of mercurous sulphate (Hg_2SO_4) made up to the consistency of thick cream with a saturated solution of $CdSO_4$. On the top is placed a layer of powdered cadmium sulphate firmly pressed down and slightly moistened with saturated $CdSO_4$ solution. The cadmium amalgam (1Cd to 6Hg), which is quite liquid at about 60° C., is then poured in and the platinum connecting wire placed in it before it solidifies. The cell is sealed up with marine glue.

The results of a great number of experiments [1] have shown that the cell made up in this way is very reliable, the E.M.F. at 20° C. being 1·0187 volt, and the temperature coefficient 0·003 % per 1° C. An important point brought out by these experiments was that the E.M.F. and temperature coefficient of the cell would hardly be influenced at all by the substitution of distinctly acid Hg_2SO_4 and $CdSO_4$ for absolutely neutral salts, this being a great advantage in view of the difficulty of obtaining absolutely neutral salts.

These crystal cells will stand very severe treatment, quickly recovering from a temporary short circuit, especially if they are charged for a few minutes from a slightly higher E.M.F. It is, indeed, recommended that the cells should be short-circuited for a few seconds and then slightly charged before using them, if they have been lying idle for some time, this being found to greatly reduce their internal resistance. On freshly making up the cells they should be heated in a water bath to 30° or 40° C. two or three times to bring their E.M.F.'s to a steady value, after which they may be standardized and used without any further delay.

Care of Standard Cells.

193. In using standard cells the greatest care must always be exercised to prevent the possibility of a short circuit, and it is

[1] *Phil. Mag.*, vol. 48, p. 152.

advisable to connect in series with a cell whenever it is used, a resistance of the order of 10,000 ohms.

In general it is found convenient to get a case containing two cells (see Fig. 90), one being used for testing, etc., and marked in some suitable manner, the other being only employed to check the first from time to time, and its terminals should be protected so as to prevent the possibility of its being accidentally used for testing purposes.

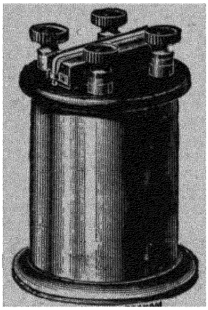


FIG. 90.

In the case of those cells having an appreciable temperature coefficient, it is necessary to keep the cell at some definite temperature for a little while before using. This may be done by packing it in a box with cotton wool, when the temperature will alter very slowly, or, still better, by placing it in a thermostat. The temperature of the cell is usually recorded by a thermometer which dips into it. These thermometers are of two kinds—one with a straight stem, the other with the stem bent so as to lie along the top of the cell. The former, although the more fragile, is by far the better form, as the mercury column is liable to break at the bend in the tube, and the temperature cannot be accurately taken.

COMPARISON OF E.M.F.'S.

194. Any method of comparing E.M.F.'s of cells to be accurate must be a null method, *i.e.* one which does not involve the taking of a current from the cell, thus avoiding polarization which would otherwise occur and vitiate the results.

One of the most accurate methods of comparing two electromotive forces is probably one or other modification of the potentiometer principle. Since a special section is devoted to the application of the potentiometer principle to the comparison of resistances, currents, and electro-motive forces, description of this method will be held over.

Another null method available is one employing a condenser.

195. *Condenser Methods of Comparing E.M.F.'s.*—If Q

represents the quantity of electricity required to charge a condenser of capacity, K, to a potential difference, V, then—

$$Q = KV$$

$$\text{and } V = \frac{Q}{K}$$

We may therefore, using the same condenser, compare two E.M.F.'s by comparing the quantities of electricity with which they charge the condenser, the plates of which are initially at zero potential. To measure the quantity of electricity, a ballistic galvanometer is employed, the amplitude of the first swing of the needle being to a first approximation proportional to the quantity of electricity which has passed through the galvanometer. (The student is here recommended to refer to the section dealing with the ballistic galvanometer for full details of its theory and action.)

The arrangement of apparatus for the comparison of the E.M.F.'s of two cells by this method is shown in the accompanying diagram (see Fig. 91). The ballistic galvanometer

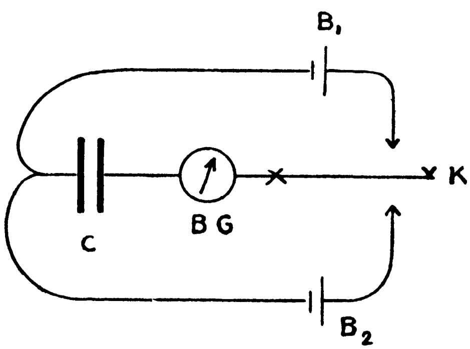


FIG. 91.

BG and the condenser C are connected in series with the tongue of the two-way key K. One of the cells, B_1, is connected to the condenser and the upper contact of K, the other cell, B_2, from the condenser to the lower contact of K. The condenser C is provided with a short-circuiting plug, by means of which the plates may be connected together. The condenser must be short-circuited (K being midway between the

upper and lower contact), and the plug removed before every reading.

When K is raised, the condenser is charged by the battery B_1, and the throw δ_1 is observed on the ballistic galvanometer. After the condenser has been discharged by means of the short-circuit plug, and the galvanometer needle has been brought to rest, K is depressed and a throw, δ_2, is obtained, the condenser being charged from B_2. If V_1 and V_2 are the E.M.F.'s of B_1 and B_2 respectively, then—

$$\frac{\delta_1}{\delta_2} = \frac{KV_1}{KV_2} = \frac{V_1}{V_2}$$

If one of the cells, B_1, is a standard cell, the E.M.F. of B_2 may be calculated from the above relation and the known value of V_1.

The capacity of the condenser employed may be about $\frac{1}{3}$ microfarad.

Temperature Coefficient of a Battery.

196. The total effect of change of temperature on the electro-motive force of a cell is made up of several separate temperature effects, and it may either be determined as a whole, or synthesized out of its component parts. It forms an interesting experiment to determine it both ways and compare the results.

A good example of the synthetic process has been worked out by Professor Carhart, in connection with the Daniell and Clark cells,[1] which, since it is of considerable interest, we propose to describe.

In an ordinary cell having two electrodes and two electrolytes, there are three quantities which may be separately affected by change of temperature, and which will affect the total E.M.F. of the cell; these are (1) the E.M.F. between the positive electrode and the electrolyte round it, (2) that between the negative electrode and its electrolyte, and (3) the E.M.F. between the two electrolytes, these forming, as it were, three

[1] Carhart, *Electrician*, vol. xxvii. p. 167; also Bouty, *Jour. de. Phys.*, 1880, p. 229.

thermal couples. The effect of change of temperature may be determined separately in each case, the algebraic sum representing the total effect.

197. We shall first describe the method of measuring the effect of change of temperature between the electrodes and the electrolytes. For this a special form of cell must be employed, consisting of two tubes communicating with each other near their upper ends by means of a long tube of narrow bore (see Fig. 92). Both tubes and the connecting tube are filled with

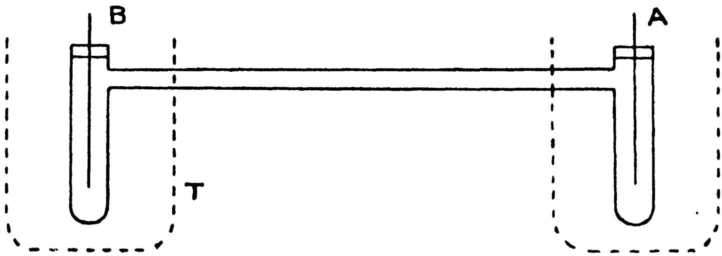


FIG. 92.

one of the electrolytes; the electrodes A and B are made of the same metal that is placed in that electrolyte in the cell. The two tubes containing the electrodes are put each into a thermostat, T, so that the temperature may be regulated and kept constant at any desired value.

First the temperatures of both are kept the same, and then there should be no difference of potential between A and B if tested on a potentiometer or other instrument for measuring E.M.F.'s; then, keeping one tube at constant temperature, the other is raised to different temperatures, and the E.M.F. between A and B noted for the various differences of temperature. If the connecting tube is of sufficiently fine bore, and the apparatus is tilted so that the heated tube is at a slightly higher level than the cold one, there will be no fear of liquid convection currents being set up inside the tubes.

A curve should be plotted from the results, showing the connection between the E.M.F. and the difference of temperature between A and B; from this curve the mean E.M.F. per 1° C.

difference of temperature may be calculated. Record must also be kept of which electrode is at the higher potential.

The above is then repeated, using electrodes of the other metal employed in the cell, and for electrolyte the liquid in which it is immersed.

198. Lastly, there is the determination of the thermo-electric effect at the junction of the two electrolytes. This is more difficult to carry out experimentally than the other, Fig. 93 showing a

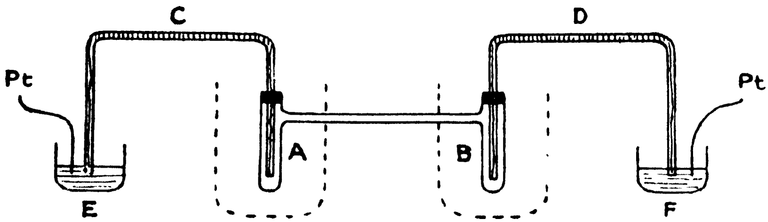


FIG. 93.

form of apparatus which may be employed. Into the tubes A and B, through tightly fitting corks, pass the tubes C and D, the other ends of which terminate in the vessels E and F. The tubes A and B are filled with one electrolyte, whilst EC and DF are filled with the other. The junctions between the electrolytes are near the ends of the tubes C and D, where they dip into A and B. The tubes A and B are placed as before into thermostats. Contact is made by means of platinum wires dipping into E and F, the temperatures at E and F being kept constant during the experiment. The tube A is then heated up, B being kept at constant temperature, and the mean E.M.F. per 1° C. rise of temperature calculated from a curve plotted as in the last case. The total thermal effect in the cell will be the algebraic sum of these separate effects.

199. In the case of the Daniell cell, Carhart found that the E.M.F. generated at the zinc-zinc-sulphate junction (the zinc sulphate being saturated at 0° C.) was 0·00079 volt per 1° C., the cold zinc rod being at the higher potential, whilst that of copper in copper sulphate (density 1·11) was 0·00073 volt per 1° C., the cold plate again being positive to the heated

one; the E.M.F. generated at the junction of the two liquids was found to be negligible, being less than 0·00003 volt per 1° C. The total temperature coefficient should therefore be—

$$0{\cdot}00079 - 0{\cdot}00073 = 0{\cdot}00006 \text{ volt per } 1^\circ \text{ C.}$$

A Daniell cell was then made up with the same solutions, and its E.M.F. measured at different temperatures. This may be done by placing it in a thermostat, and steadying the temperature for some time before each reading. This was found to give a variation of 0·000073 volt per 1° C., and dividing this by 1·09, the E.M.F. of the cell, we get the true temperature coefficient—

$$\frac{0{\cdot}000073}{1{\cdot}09} = 0{\cdot}000063$$

which agrees closely with the value as determined by the first method. The student is recommended to make the same investigation for the other standard cells.

Testing a Primary Battery.

200. In order to make a complete test of a primary battery so as to report upon its merits, the following data must be obtained:—

(1) The E.M.F. of the cell on open circuit;
(2) The temperature coefficient of the cell;
(3) The internal resistance;
(4) The behaviour of the cell when sending a current;
(5) The life and cost of working the cell.

In order to make the above measurements, any of the foregoing methods may be employed; but probably the simplest will be the potentiometer method, for the details of which see par. 219.

The methods of measuring E.M.F., temperature coefficient, and internal resistance, have already been dealt with, and need not be repeated. In (4), however, we have the most important test of all.

201. This test should be made in two parts, first taking a relatively small current from the cell, and another test taking a large current from it. In this, as in all the other tests, the

results obtained from experiments on one cell cannot be taken as conclusive; a number of cells must be tested individually, and from all the results thus obtained the general behaviour of the battery may be deduced.

In the first test, using small currents, after the E.M.F. and temperature coefficient have been determined, the cell is connected up to a resistance, which may have any value between 20 and 100 ohms—about 50 ohms will be found convenient. Time readings of the P.D. of the cell are then taken at intervals of five minutes at first, and afterwards at longer intervals; also the circuit of the cell is broken, from time to time, just long enough to allow of a measurement of its E.M.F. being made. This test may be continued for twelve hours; the circuit is then broken, and time-readings of the recovery of the E.M.F. are taken.

Curves showing the variations of E.M.F., P.D., and internal resistance with time of discharge may then be plotted from the data obtained, also a recovery of E.M.F. curve.

From these curves may be deduced the mean percentage fall of P.D. per minute and the mean rate of recovery. The approximate value of the current flowing should be stated on the curves. Several determinations of the above are made, using fresh cells each time. Another batch of cells are then tested for the high rate of discharge. In this case, the external resistance may have any value between 1 and 5 ohms, and readings similar to those made above are taken. The discharge may be kept on for 15 min. at a time, and recovery-readings taken after each discharge until the cell is completely exhausted; also, some cells may be discharged right on to exhaustion. The time-intervals between the readings must be smaller in this experiment than in the slow-discharge experiment, for obvious reasons.

From the curves plotted from the above, the percentage fall of P.D. and recovery of E.M.F. per minute may be calculated, and also the number of ampère hours supplied by the cell may be estimated from a curve showing the variation of the current with the time.

202. In the case of cells required to give out intermittent

currents in practice, such for instance as cells required to ring bells, etc., a test should be made in which as nearly as possible the actual working conditions are reproduced. The circuit may easily be arranged so as to be made and broken at fixed intervals, by driving the key by clockwork, such for instance as a series of radial spokes attached to the main spindle of the minute hand of an American clock, which may, by dipping into mercury cups, be arranged to make and break the circuit at any desired interval, readings of E.M.F. and P.D. being taken from time to time, and the life of the cell calculated.

203. The question of cost of working the cell is not very important, as, except for running bells, etc., primary cells are not much used commercially. But an estimate of the cost may be got by (in the case of cells requiring fresh electrolyte, etc.) supplying the cell with fresh chemicals until the plates are worn out; then, adding the price of the chemicals to the price of the cell, and dividing by the number of ampère hours supplied, we can get the price per ampère hour. This, of course, will only give a rough estimate, and is only intended to be done in the case of isolated cells; when large numbers of cells are employed, other things must be taken into consideration. But even with the best conditions obtaining, Ayrton has shown that the cost per Board of Trade unit, with the best-known cells, varies from 1*s.* to 1*s.* 6*d.*, whilst from engines and dynamos the cost price is about 1*d.*

ELECTROMETERS.

204. Since electrostatically charged conductors attract or repel one another according as they are oppositely or similarly charged, and since, other things being the same, the force between the charged bodies depends on their difference of potential, the value of this difference of potential may be calculated, provided the value of the force is known.

It is on this principle that the absolute electrometers of Lord Kelvin have been constructed.

It must, however, be distinctly borne in mind that any such measurement of potential difference will be expressed in electrostatic units, whilst the practical unit of potential difference—the

volt—is expressed in electro-magnetic units; so that if we desire to convert the potential difference as given by the absolute electrometer into volts, we must know what relation the one system of units bears to the other. This ratio is not known with great accuracy, but is very approximately—

$$\frac{\text{C.G.S. unit of potential in the electrostatic system of units}}{\text{C.G.S. unit of potential in the electro-magnetic system of units}}$$
$$= \frac{3 \times 10^{10}}{1}$$

The volt, it must be remembered, is 10^8 electro-magnetic units.

205. *Absolute Electrometer.*—The value of the attractive force between two charged conducting plates can be shown[1] to be expressed by the following equation—

$$f = \frac{AV^2}{8\pi d^2}$$

where f is the force in dynes, A the area of one of the attracting plates in square centimetres, d the distance between the plates in centimetres, and V the potential difference between them in electrostatic units. This relation assumes that the distribution of the electric intensity is uniform over the plates. This latter condition, however, does not obtain in the case of a flat plate of finite area, the distribution at the edges not being the same as that over the surface. To overcome this difficulty, Lord Kelvin adopts an ingenious device known as a "guard-ring." This consists of a ring of metal, G, which surrounds one of the plates, B, the plates being circular (see Fig. 94),

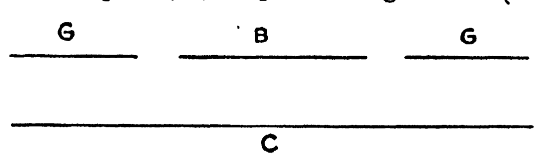


FIG. 94.

and leaving a very small gap between them all round the circumference of B. If G and B are raised to the same potential, the inductive action of G on B prevents the distribution of the electric intensity at the edge of B from differing

[1] See J. J. Thomson's "Elements of the Mathematical Theory of Electricity and Magnetism," p. 97.

from that over its surface. The other plate, C, is much larger than B, so that although the distribution at the edges may not be uniform, that over the part that attracts B may be assumed quite uniform.

One effect of the air gap between G and B is to increase the effective area of B by an amount equal to half the area of the

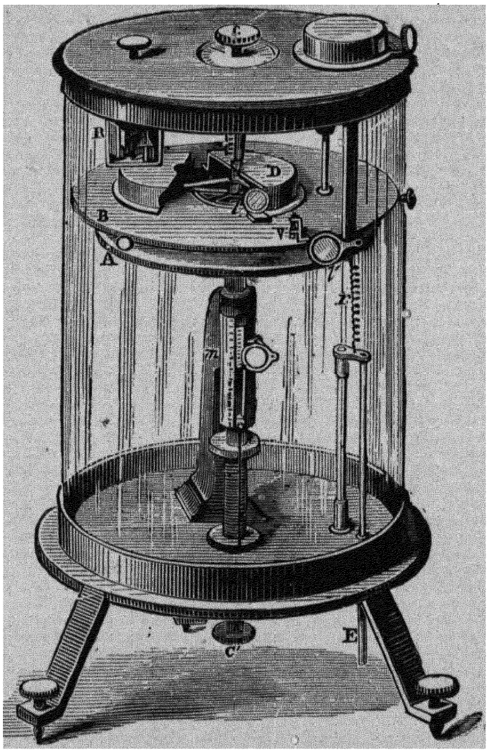


FIG. 95.

annular gap between G and B, the effective area of B being the quantity A referred to in the above equation.

In one form of Lord Kelvin's absolute electrometer (see Fig. 95), the attracted plate is held in position by means of a delicate spring, so that it rests a little above the level of the guard-ring, and can be charged by means of a dry pile or Leyden jar. The attracting plate is movable vertically by

means of a screw of accurately known pitch, the head of the screw being supplied with a graduated disc, so that fractions of a revolution may be read off accurately.

In order to use the instrument, the plate C is earthed, (Fig. 94), and G and B are charged. The plate C, still earthed is then gradually raised by means of the screw until the plate B is pulled into the plane of the guard-ring by the force of the attraction between them, the exact position of B being indicated by sights attached to the instrument. The position occupied by C is noted; it is then insulated from earth and connected to the body whose potential (V) is required, and again adjusted until, by means of the sights, the plate B is seen to occupy a position in the plane of the guard-ring. The position of C is again noted.

Let d = distance between the first and second position of C;

V_B = the potential of B, and A its effective area;

then, firstly—

$$f \propto \frac{A(V_B - 0)^2}{8\pi}$$

and secondly—

$$f \propto \frac{A(V_B - V)^2}{8\pi d^2}$$

from this we get—

$$V = d\sqrt{\frac{8\pi f}{A}}$$

In order to determine f, which is a constant of the apparatus, and depends on the spring supporting B, the plates and guard-ring are discharged, and known weights are placed on B until it sinks to the plane of the guard-ring; if w represents this weight in grammes, then—

$$f = w \times 981 \text{ dynes}$$

From the formula it will be seen that, since the force varies as the square of the potential difference, it will be very small for small potential differences. The instrument can therefore only be used to measure large potential differences.

206. *Quadrant Electrometer.*—For the measurement of small potential differences, the quadrant electrometer is generally

employed. This, in its simplest form, consists of four metal quadrants supported on insulating stands, adjacent quadrants being insulated from one another, but the alternate quadrants are connected together. Over the top of these is suspended a flat paddle-shaped needle of aluminium, which, in its normal condition, occupies a position symmetrically over the gaps between the quadrants (see Fig. 96). When the needle is charged, the quadrants being all connected to earth, there should be no deflection. If the needle does deflect, it shows that it is not symmetrical with regard to the quadrants, and the position of one of the latter, which is adjustable, is altered until equilibrium is obtained. If now alternate quadrants are brought to different potentials, a deflection of the needle will result, it being attracted by one pair of quadrants and repelled by the other; the controlling force acting on the needle being the torsion of the suspending fibre, the earth's magnetic force, or gravity, according to circumstances.

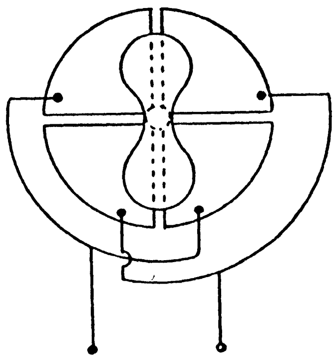


Fig. 96.

207. In the better forms of electrometer the quadrants completely enclose the needle. Fig. 97 shows one of the most modern forms of electrometer, designed by Messrs. Ayrton, Perry & Sumpner.[1] This instrument has been designed especially with a view to sensitiveness and accessibility of the working parts.

The quadrants, which are smaller than in most electrometers, are mounted on long glass rods which are attached to the base-plate of the instrument, one of the quadrants being free to move out or in by means of a screw attached to the slide which carries the glass stem and quadrant. The needle is suspended by a silk fibre attached to the movable piece N; from the lower side of the needle projects a piece of platinum wire which dips into a lead vessel, L, containing strong H_2SO_4, the lead

[1] *Pro. Roy. Soc.*, June, 1891.

vessel being supported by a strong wire from a screw on the vertical rod R, which ends in a glass stem, and is therefore insulated from the base. The rods attached to the terminals T, T pass through the holes in the base of the instrument and are fixed to the ends of two glass stems, contact being made from these points to the quadrants.

The charging is performed through the charging rod *cc*, which is attached to a glass stem, but hinged so that its upper end,

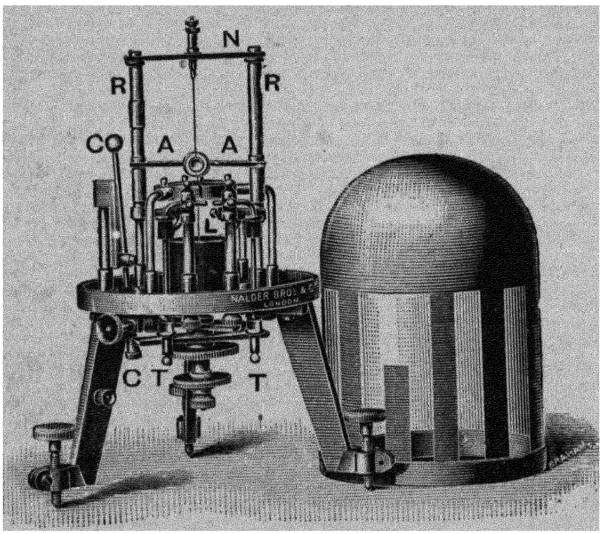


FIG. 97.

on which there is a lead ball, may either be in contact with RR or not, as desired. To charge the needle, the rod *cc* is turned so as to bring the ball into contact with RR; to the other end of the rod there is applied a charged electrophorus, or other source of electrification. After charging, the rod is disconnected (by a tap from the charging body) from RR, thus minimizing the chance of leakage. The mirror attached to the needle is surrounded by a guard-ring, made in two halves, carried by the arms A, A.

The whole of the parts are enclosed in a glass shade, which also serves as a Leyden jar to keep the needle charged.

The controlling force in this instrument is magnetic, a small magnet being attached to the back of the mirror.

208. One of the chief difficulties in connection with an electrometer is the difficulty of getting good insulation for the needle and quadrants. To insure this, the glass-supporting stems must be perfectly clean and well dried, the air inside the case of the instrument being kept dry by the strong H_2SO_4.

Before using an electrometer it must be tested for insulation. To do this the needle should be charged, the quadrants being earthed, then one pair of quadrants is charged and the deflection noted; if this remains steady (the quadrant being insulated after charging) the leakage from this pair of quadrants is negligible. The other pair of quadrants are then tested in the same way, the first pair being earthed.

Should the deflection fall off, it shows that either the needle or quadrants are leaking. If the quadrants are then connected to the poles of a cell, the needle being charged, and the deflection falls, it points to leakage from the needle, since the quadrants are at a constant potential. Where this leakage has been stopped a repetition of the first test will show whether the quadrants are leaking.

209. One other point must be borne in mind when using the quadrant electrometer, namely, that if part of the circuit is brass and part copper, as the quadrants and the leads, or in fact any pair of dissimilar metals, a deflection may be obtained due to the contact difference of potential between the dissimilar metals; this can be determined experimentally.

210. With regard to the law of the instrument, it can be shown[1] that, if V_1 and V_2 are the potentials of the quadrants, V_n the potential of the needle, and θ the angular deflection of the needle, then—

$$\theta = \kappa\left\{(V_1 - V_2)\left(V_n - \frac{V_1 + V_2}{2}\right)\right\}$$

κ being a constant depending on the construction of the instrument. From this it will be seen that, if V_n is very large

[1] "Elements of Mathematical Theory of Electricity and Magnetism" (J. J. Thomson), p. 98.

compared with V_1 and V_2, then the angular deflection θ is approximately proportional to the difference of potentials of the quadrants. This method of using the electrometer with the needle highly charged is called the heterostatic method. On the other hand, if the needle is connected to one pair of quadrants, V_n becomes equal to V_1 or V_2 and—

$$\theta = \kappa (V_1 - V_2)^2$$

the deflection being proportional to the square of the potential difference. The instrument in this form is said to be used idiostatically, and is not so sensitive, but is adapted for measuring greater P.D.'s; it is also capable of measuring alternating E.M.F.'s, since the needle changing its potential with the quadrants causes the deflection to be always on the same side of zero.

211. In some forms of electrometer a special arrangement, known as a replenisher, is employed to charge the needle, the potential of the latter being roughly indicated by means of a small auxiliary gauge, working on the principle of the absolute electrometer; if leakage occurs the gauge falls, and may be brought back to its original position by recharging the needle from the replenisher. Many experimenters, however, object to using it, and prefer to charge the needle some time before the instrument is required for use, so as to give the potential time to settle down to a steady value. The potential of the needle can always be tested for constancy by taking the deflection produced by a standard cell.

The most usual way of using the electrometer for the comparison of E.M.F.'s is to charge the needle to a high potential by means either of an electrophorus or of a dry pile kept at a constant temperature.

The E.M.F. to be measured is then applied, and the deflection obtained recorded; the connections are then reversed and the deflection to the other side taken. The mean deflection is calculated. The same process is repeated, using a standard cell of known E.M.F., and from the ratio of the two mean deflections the ratio of the E.M.F.'s may be obtained.

212. In order to measure the E.M.F. of a Daniell cell an

electrometer was employed the needle of which had been charged to a high potential by means of an electrophorus some hours previously, and had retained its charge as was proved by the instrument giving always the same deflection when connected up to a standard Carhart Clark cell.

The temperature of the Clark cell was 18° C., and the following readings were obtained:—

Deflection to right with standard cell	...	123 scale-divisions.
,, left ,, ,, ,,	...	125 ,, ,,
Mean deflection ...	...	124
Deflection to right with Daniell cell	...	91·5 scale-divisions.
,, left ,, ,, ,,	...	93·6 ,, ,,
Mean deflection ...	...	92·5

$$\text{E.M.F. of standard cell} = 1{\cdot}438\{1 - 0{\cdot}000387(18° - 15°)\}$$
$$= 1{\cdot}436 \text{ volt}$$

$$\text{Hence } \frac{1{\cdot}436}{x} = \frac{124}{92{\cdot}5}$$
$$x = 1{\cdot}071 \text{ volt}$$

The Potentiometer.

213. We have several times referred to this instrument as one suitable for making comparisons of resistances, currents, and electro-motive forces. But instead of describing it in each of the chapters dealing with these measurements, we have considered it better to devote a special section to it and its applications.

In all cases the comparisons, whether of resistance, current, or E.M.F., are reduced to a comparison of E.M.F.'s, resistances being compared in terms of the fall of potential in them when carrying the same current, and currents by comparing the potential differences which they produce at the ends of a standard resistance.

Of the various experimenters who have worked at this instrument, it has perhaps gained more at the hands of Mr.

R. E. Crompton than any one else, and the arrangement which we describe here is a modification of the form of potentiometer constructed by him.[1]

214. The potentiometer consists of a wire which is divided up into fifteen parts, such that all the segments have the same electrical resistance. The last segment at one end consists of bare wire stretched over a graduated scale, and is provided with a sliding tapping-contact similar to that on a wire bridge. The other fourteen segments may be wound into coils, potential wires being brought away from the ends of the segments (see Fig. 98).

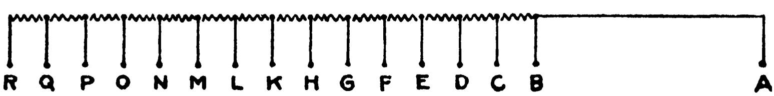


FIG. 98.

In constructing the potentiometer, it is not necessary to make the fifteen lengths separately and then join them in series; it will, in fact, be found much more convenient to cut off one long piece of wire sufficient for the whole instrument and then solder on the potential wires A, B, C, D, etc., so as to divide up the whole wire into fifteen parts of equal resistance. The first fourteen segments may then be coiled up and the fifteenth stretched over its scale. In this way we avoid soldered joints in the wire itself. The wire should be made of some substance possessing high specific resistance and low temperature coefficient, such, for instance, as manganin, the resistance of each segment being somewhere near 2 ohms.

After the coils have been adjusted and the wire stretched, the latter must be carefully calibrated by some of the methods described already (see par. 32), and then each of the fourteen coils accurately compared with the stretched wire, by sending a current through them and comparing the fall of potential down each coil with that down the stretched wire. In this way, taking the length of the stretched wire as a standard, the effective lengths of the coils may be obtained in terms of it,

[1] See *Electrician*, vol. xxxi. p. 32, and vol. xxxvi. p. 158.

and a table of corrections constructed if necessary. A convenient method of arranging the coils is shown in Fig. 99. This can be used in conjunction with a metre bridge, the wire of which, if of equal resistance to the coils, may be used as the stretched wire.

The fourteen coils are wound on bobbins and enclosed in a box, the potential wires being soldered to the contact studs 1, 2, 3, 4, etc. The sliding arm A makes contact between the

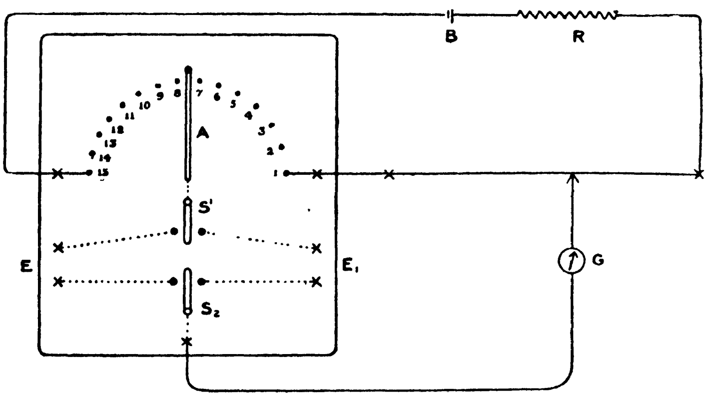


FIG. 99.

studs and the two-way switch S_1, to which one terminal of E and E_1 is connected, the switch S_2 making contact between the other terminals of E and E_1 and the galvanometer G.

In using the potentiometer, a steady current must be maintained in the fourteen coils and stretched wire, from a secondary battery, B, in series with a variable resistance, R. Primary batteries cannot be used satisfactorily for this purpose, on account of their E.M.F. falling off rapidly when supplying a current.

The most recent pattern of Crompton potentiometer is shown in Fig. 99*a*, whilst the internal connections are shown in the diagram Fig. 99*b*.

In Fig. 99*b*, *ab* is the potentiometer wire, and *c* the fourteen equal resistance coils in series with it, *e* and *f* are regulating resistances, A, B, C, D, E, F are terminals

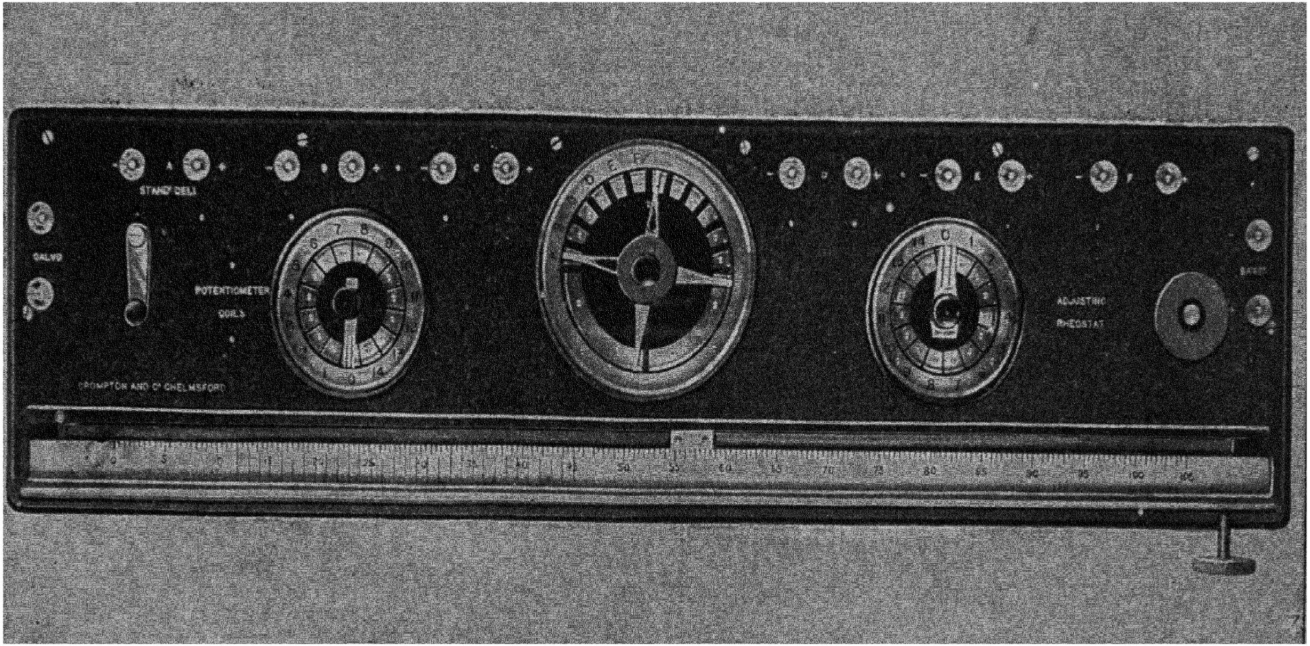

STAND CELL
GALVO
POTENTIOMETER
COILS
ADJUSTING
RHEOSTAT
CROMPTON AND Cº CHELMSFORD

FIG. 99a.

to which the various E.M.F.'s to be compared are brought, and which may be connected when required to the sliding contacts by means of the double pole switch *d*. All the sliding contacts are under glass covers, and the coils and scale wire are inside the case of the instrument.

214*a*. A useful portable form of potentiometer for commercial testing, due to Messrs. Nalder Bros., is shown in Fig. 99*c*.

This instrument is intended for measuring any E.M.F. between 2 and 600 volts by tenths of a volt in terms of the E.M.F. of a standard cell. The diagram of connections is given in Fig. 99*d*, and explains itself. It will be noticed that the standard cell is connected permanently across the ends of a resistance R, and the balance is obtained by varying the resistance between the potential terminals marked + volts and − volts, until

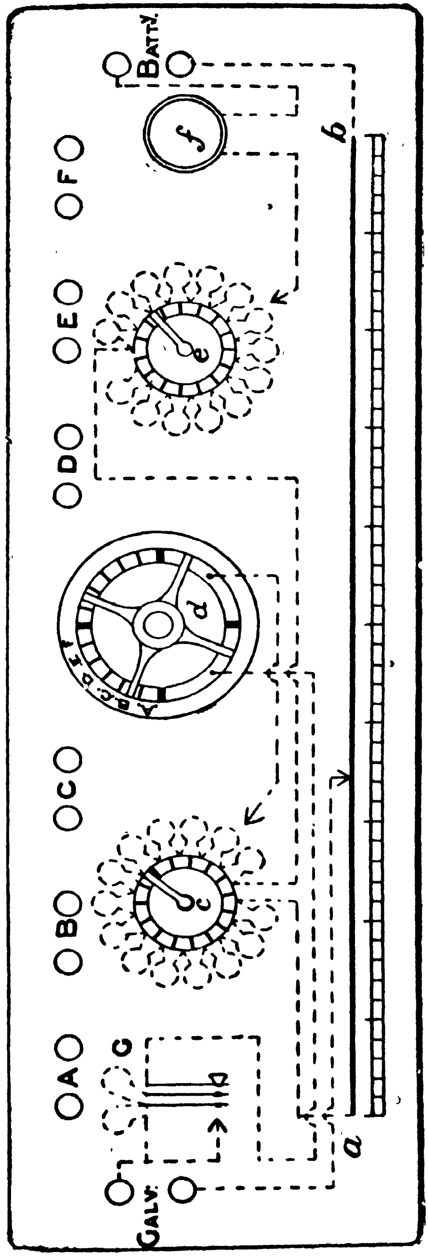


Fig. 99*b*.

the current in R has a potential drop equal to that of the cell E.M.F.

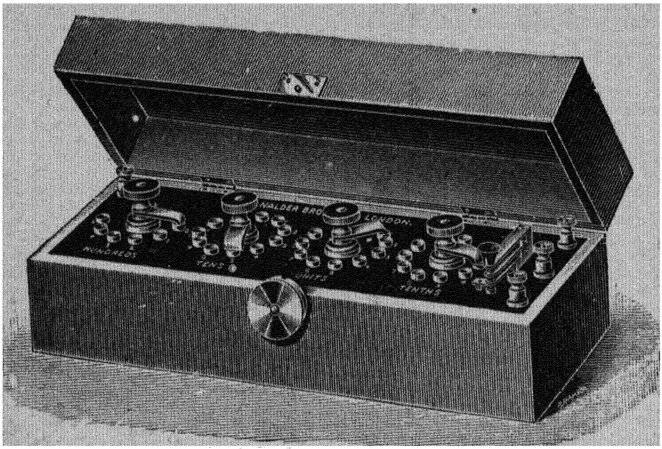


FIG. 99*c*.

215. *Comparison of Resistances.*—In order to compare two resistances by means of the potentiometer, the resistances are connected in series with one another and a current from a

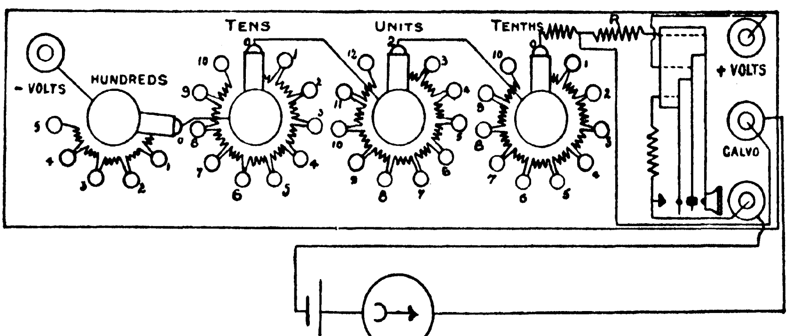


FIG. 99*d*.

secondary battery sent through them. Should the resistances be small, it is necessary to have a variable resistance in series with them, to prevent the current from the battery being too

great. Potential wires are then taken from the ends of the resistances to terminals E and E_1 respectively of the potentiometer. The switches S_1 and S_2 are then adjusted so as to connect the terminals E to the potentiometer wire, and by adjusting the rotating arm A and the tapping-contact connected to G, the fall of potential down the coil may be balanced against the fall of potential down a portion of the potentiometer wire, as shown by the galvanometer G giving no deflection. The positions of A and of the tapping-point are noted; the switches S_1 and S_2 are then altered so as to connect the terminals E_1 to the potentiometer wire, and another pair of balancing-points are found for the other coil. The ratio of the effective lengths of potentiometer wire between the balancing-points in the two cases is the ratio of the two resistances, and if one of the resistances is a standard coil, the resistance of the other coil can be calculated.

In making such a comparison of resistances, care must be taken to insure that the direction of the current in the coils to be compared is such that the potential difference at their ends is in opposition to the potential difference on the wire, and also that the fall of potential down the larger of the two resistances is not greater than the total difference of potential at the ends of the potentiometer wire.

The accuracy of a comparison such as the above depends on the sensitiveness of the galvanometer and on the accuracy of calibration of the potentiometer wire and coils, and may be made very high. The standard resistances employed with the potentiometer should be of the form described in par. 21, which is specially designed for this kind of measurement. The galvanometer used may be of any sensitive type, but one or other modification of the D'Arsonval galvanometer will be found most convenient to work with, on account of its high sensibility and freedom from magnetic control.

All precautions respecting the measurement of the temperature of the coils which have been previously dealt with, apply equally in this case.

216. The following comparison of resistances was made on a potentiometer.

The wire and coils of the potentiometer were calibrated by the fall of potential method, the galvanometer employed having been previously tested and found to have a straight-line law within the limits of the deflections employed in the experiment. The following readings were obtained in the calibration:—

Coil.	Deflection.
1—2	110·0
2—3	110·0
3—4	110·0
4—5	110·0
5—6	110·0
6—7	110·0
7—8	110·0
8—9	110·0
9—10	110·0
10—11	110·0
11—12	110·0
12—13	109·5
13—14	109·0
14—15	109·5
Stretched wire	111·0

A calibration of the stretched wire, which, like the coils, was of manganin, proved it to be perfectly uniform.

The first balance, when the coil connected to the terminals E was tested, was obtained with the arm A at stud 4, and the tapping-contact at 97·50, and therefore the fall of potential was proportional to—

$$(3 \times 110) + \frac{97{\cdot}50}{100} \times 111 = 438{\cdot}22.$$

The second balance of the fall of potential down a standard 1-ohm coil (manganin) against the fall of potential in the wire gave arm A at stud 8, and the tapping-point at 83·55, or a fall of potential proportional to—

$$7 \times 110 + \frac{83{\cdot}55}{100} \times 111 = 862{\cdot}74$$

And the ratio of resistances is—

$$\frac{x}{1} = \frac{438 \cdot 22}{862 \cdot 74}$$

$$\text{or } x = 0 \cdot 508 \text{ ohm.}$$

The temperature of the coil x was $12°$ C.

217. *Measurement of Current.*—In the measurement of current by the potentiometer, the current is sent through a standard resistance, which, if the current is large, must be specially constructed to carry heavy currents without seriously heating. The P.D. at the ends of the coil is balanced against the fall of potential in the potentiometer wire, as in the last case. The E.M.F. of a standard cell is then balanced against the fall of potential in the wire; from these two readings, and the known E.M.F. of the standard cell, the P.D. in volts at the terminals of the known resistance, may be calculated, and, dividing this by the resistance in ohms, we get the current in ampères.

The limit of accuracy in the measurement of current by this method depends on the accuracy to which the E.M.F. of the standard cell is known, *i.e.* to about 1 part in 1400, the E.M.F. of the cell being of course corrected for temperature in the above measurement.

218. In measuring a current on the potentiometer, the calibration table of which is given in par. 216, the current was sent through a standard 1-ohm manganin coil, and the potential difference at its ends was balanced against the fall of potential in the wire and coils, and gave the following reading: Rotating arm A at stud 10, sliding contact at 98·30 on the scale. A Clark cell was then balanced on the potentiometer, and gave sliding arm A at stud 14, tapping-contact at 38·00. The temperature of the cell was $15°$ C.

$$\text{The E.M.F. of the Clark cell was} = 1 \cdot 438(1 - \cdot 000387(t - 15°)$$
$$= 1 \cdot 438 \text{ volt}$$

The first fall of potential was proportional to—

$$9 \times 110 + \frac{98 \cdot 30}{100} \times 111 = 1099 \cdot 11$$

The second fall of potential was proportional to—

$$1649 + \frac{38}{100} \times 111 = 1691 \cdot 18$$

Hence, calling E the potential difference at the ends of the 1-ohm coil, we have—

$$\frac{E}{1 \cdot 438} = \frac{1099 \cdot 11}{1691 \cdot 18} = 0 \cdot 940 \text{ volt}$$

$$\text{Therefore the current} = \frac{E}{R} = \frac{0 \cdot 940}{1} = 0 \cdot 940 \text{ ampère}$$

219. *Comparison of E.M.F.'s.*—The comparison of E.M.F.'s is made in exactly the same manner as that just described above, only two cells are compared instead of a cell and the P.D. at the terminals of a resistance. The instrument, however, may be adjusted so as to read directly in volts, and thus saves time and calculation. To do this the E.M.F. of the standard cell corrected for temperature is calculated; suppose it is 1·4346 volt. The sliding contact A (Fig. 99) is then turned to the extreme left-hand stud 15, and the tapping-point on the wire is adjusted to read 34·6 (the wire being divided into a hundred parts) on the scale. The length of wire between the tapping-points is then 14·346 segments. The variable resistance R is now adjusted until the fall of potential down the 14·346 segments is equal to the E.M.F. of the standard cell. If any other cell is balanced against the fall of potential down the wire now, the number of segments between the balancing-points will be ten times the E.M.F. in volts. Thus supposing, in order to get a balance with another cell, the arm A had to be turned so as to make contact with stud 13, and the tapping-point gave a balance when placed at 57·0, the E.M.F. required would be $\frac{12 \cdot 570}{10} = 1 \cdot 257$ volt. It is assumed here that, after the instrument has been standardized by the standard cell, the current in the wire keeps perfectly constant. In order to test this, readings with the standard cell should be taken from time to time. It is also assumed that the resistance of each of the fourteen coils is equal to that of the stretched wire, and that the stretched wire is perfectly uniform.

The accuracy which the potentiometer is capable of, is very great when comparisons only are to be made; but if a measurement of E.M.F. or current is desired, it is limited to the

accuracy of our knowledge of the E.M.F. of the standard cell.

220. As an exercise on the use of the potentiometer the following investigation was carried out on a Daniell cell in which dilute sulphuric acid was used as the electrolyte surrounding the zinc instead of zinc sulphate solution. The cell was prepared in the following manner. The copper-plate was freshly plated with copper before using. The zinc rod employed was of double distilled zinc, and free from impurity; it was well amalgamated with pure mercury before using. The copper sulphate solution was made by dissolving crystals of $CuSO_4$ in tap-water till the solution was saturated at 15° C. First of all, experiments were made by adding acid to the water in the porous pot surrounding the zinc only; the results were, however, not satisfactory, on account of the alteration of E.M.F. produced by the slow passage of the acid into the copper sulphate solution. After many experiments, the best and most constant results were obtained by adding acid to both the liquids.

The table below gives the E.M.F.'s of the cell when made up with different quantities of acid. In making these measurements, after the liquid had been made up, the cell was short-circuited for two minutes, and then allowed to rest, its E.M.F. being measured on the potentiometer against a standard Clark cell. After two minutes it was found to reach a steady value, the numbers recorded in the table representing the value of the E.M.F. after fifteen minutes' rest.

Clark cell balance.	Daniell cell balance.	Electrolyte surrounding copper plate.	Electrolyte surrounding zinc rod.	E.M.F. of Daniell cell.
				Volts.
150·2	114·8	$CuSO_4$ dissolved in tap-water	tap-water	1·095
150·2	119·7	100 c.c. $CuSO_4$ + 0·5 c.c. H_2SO_4	100 c.c. H_2O + 0·5 c.c. H_2SO_4	1·142
150·2	119·5	" " + 1 " "	" " + 1 " "	1·140
150·2	118·3	" " + 2 " "	" " + 2 " "	1·128
150·2	116·8	" " + 5 " "	" " + 5 " "	1·114
150·2	115·9	" " + 10 " "	" " + 10 " "	1·106
150·2	115·2	" " + 15 " "	" " + 15 " "	1·099
150·2	114·7	" " + 20 " "	" " + 20 " "	1·094

The E.M.F. of the Clark cell corrected for temperature was 1·434 volt. The following curve (Fig. 100) shows the variations in voltage with amount of acid added.

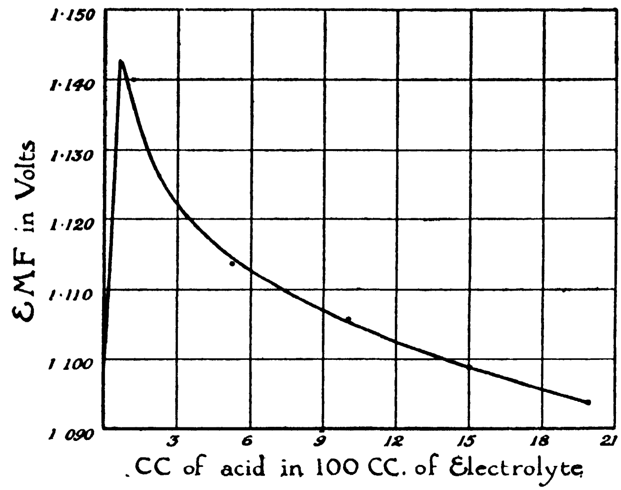


FIG. 100.

The first reading, with no acid in the cell, is very variable and liable to sudden changes if the cell is shaken.

221. References to Scientific Papers.

Title of Paper.	Author.	Reference.
Standard Cells.		
A Standard Voltaic Battery	Clark	*Trans. Roy. Soc.*, 1874; *Pro. Roy. Soc.*, vol. 20, p. 444; *Ibid.*, vol. 21, p. 421.
On the Electro-Motive Force of Standard Cells	Rayleigh and Sedgwick	*Trans. Roy. Soc.*, 1885.
On the Clark Cell as a Standard of Electro-Motive Force	Rayleigh	*Ibid.*, 1886.
,, ,, ,,	Glazebrook and Skinner	*Ibid.*, 1892.
Variation of the E.M.F. of Clark Cells with Temperature	Ayrton and Cooper	*Pro. Roy. Soc.*, vol. 59, p. 368.
On the Clark Cell as a Source of Small Currents	Threlfall and Pollock	*Phil. Mag.*, vol. 28, Nov., 1889.
The Clark Cell when producing a Current	Skinner	*Ibid.*, vol. 38, Sept., 1894; vol. 39, April, 1895.
,, ,, ,,	Threlfall	*Ibid.*, vol. 39, Mar., 1895.
An Improved Standard Clark Cell	Carhart	*Ibid.*, vol. 28, Nov., 1889.
Instructions for preparing Clark Cells	Kahle	*Elect.*, vol. 31, p. 265.
On the Variation of Clark Cells	Swinburne	*Ibid.*, vol. 27, p. 500.
Standard Clark Cells	Glazebrook	*Ibid.*, vol. 27, p. 98.
Clark Cells	Kahle	*Ibid.*, vol. 29, p. 516.
On a Form of Daniell Cell convenient for a Standard of Electro-Motive Force	Lodge	*Phil. Mag.* vol. 5, Jan., 1878.
On the use of Daniell's Cell as a Standard of E.M.F.	Fleming	*Ibid.*, vol. 20, Aug., 1885.
Weston Standard Cell	——	*Elect.*, vol. 30, p. 741.
E.M.F. and Temperature Coefficient of Cadmium Cells	Dearlove	*Ibid.*, vol. 31, p. 645.
The Cadmium Standard Cell	Jaeger and Wachsmuth	*Jour. Elec. Eng.*, vol. 25, p. 726.
E.M.F. of Daniell Cell	Carhart	*Am. Jour. of Science*, vol. 28, 1884.
Cadmium Cells	Jaeger	*Annal. Phys. Chem.*, vol. 65, 1, p. 106.
,, ,,	Kohnstamm and Cohen	*Ibid.*, vol. 65, 2, p. 344.

Title of Paper.	Author.	Reference.
Standard Cadmium Cells	Henderson	*Phil. Mag.*, vol. 48, p. 152.
Silver Voltameter and Standard Cells	Kahle	*Zeitschr. Instrumentenk*, vol. 18, p. 229.
Standard Cells	Henderson	*Elect. Eng.*, vol. 26, p. 121.
" "	Jaeger and St. Lindeck	*Zeitschr. Instrumentenk*, vol 21, p. 33.
Electrometers.		
Quadrant Electrometers	Ayrton, Perry, Sumpner	*Trans. Roy. Soc.*, 1892.
A Guard-Ring Electrometer	Fitzgerald	*Phil. Mag.*, vol. 10, July, 1880.
On the Quadrant Electrometer	Hopkinson	*Ibid.*, vol. 19, Apr., 1885.
An Absolute Spherical Electrometer	Lippmann	*Elect.*, vol. 22, July, 1886.
A Method of determining the Value of Rapid Variations of Potential Difference by the Capillary Electrometer	Burch	*Pro. Roy. Soc.*, vol. 48, p. 89.
The Time Relations of the Excursions of the Capillary Electrometer	——	*Trans. Roy. Soc.*, 1892.
Capillary Electrometer	Berget	*Elect.*, vol. 27, p. 252.
Capillary Electrometer in Theory and Practice	Burch	*Ibid.*, vol. 37, p. 380, etc.
Contact Difference of Potential	Clifton	*Pro. Roy. Soc.*, vol. 26, p. 299.
On Dry Charging Piles	Elster and Geitel	*Phil. Mag.*, vol. 16, Aug., 1883.
The Capillary Electrometer	Burch	*Elect.*, vol. 49, p. 235.
The Addenbrooke Electrometer	Addenbrooke	*Elect. Eng.*, vol. 25, p. 150.
Battery Tests, etc.		
On a Constant Daniell Battery	Sir Wm. Thomson	*Pro. Roy. Soc.*, vol. 19, p. 253.
Effects of Temperature on the E.M.F. and Resistance of Batteries	Preece	*Ibid.*, vol. 35, p. 48; vol. 35, p. 250; vol. 36, p. 464.
Determination of Chemical Affinity in Terms of E.M.F.	Alder Wright	*Phil. Mag.*, vol. 9, Apr., 1880, etc.
On the E.M.F. of Alloys	Trowbridge and Stevens	*Ibid.*, vol. 16, Nov. 1883.

Title of Paper.	Author.	Reference.
On the Application of Standard Current Balances to the Determination of the E.M.F. of Voltaic Cells	Sir Wm. Thomson	*Phil. Mag.*, vol. 24, Dec., 1887; vol. 25, Feb., 1888.
Divergence of E.M.F.'s from Thermo-Chemical Data	Herroun	*Ibid.*, vol. 27, Mar., 1889.
Temperature Coefficient of a Battery	Carhart	*Elect.*, vol. 27, p. 167.
Report of the Hellesen Dry Battery	Krehbiel	*Ibid*, vol. 26, p. 419.
Report of the Lessing Dry Battery	Walmsley	*Ibid.*, vol. 36, p. 589.
The Crompton Potentiometer	——	*Ibid.*, vol. 31, p. 32.
Use and Capabilities of the Crompton Potentiometer	Fisher	*Ibid.*, vol. 36, p. 158, etc.; vol. 37, p. 5, etc.
The Legalized Standard of E.M.F.	Carhart	*Ibid.*, vol. 52, p. 828.
Measurement of Potential of the Electrodes in Stationary Liquids	Sands	*Ibid.*, vol. 54, p. 66.
Methods of Measuring High Electrical Pressures	Kintner	*Ibid.*, vol. 55, p. 702.
The Cadmium Standard Cell	Hulett	*Ibid.*, vol. 57, p. 861.
Mercurous Sulphate and Standard Cells	Hulett	*Ibid.*, vol. 57, p. 708.
The Standard Cell	Wolff and Waters	*Ibid.*, vol. 58, p. 692.
Standard Cells	Guthe and von Ende	*Ibid.*, vol. 59, p. 723.
Cadmium Standard Cell	Hulett	*Ibid.*, vol. 59, p. 841.
Stability of Weston Cells	Carhart	*Ibid.*, vol. 60, p. 848.
Normal Weston Cadmium Cell	Smith	*Ibid.*, vol. 60, p. 403.

IV.

QUANTITY.

222. THE practical unit of quantity of electricity is the coulomb, and is the quantity of electricity conveyed along a conductor by a current of one ampère flowing for one second.

The practical measurement of quantity of electricity may be made by means of a voltameter, which is really a quantity, and not a current measuring apparatus, since the weight of the ions liberated by electrolysis is proportional to the quantity of electricity which has passed through the instrument.

For the measurement of very small quantities of electricity the voltameter is not a convenient instrument to use, a ballistic galvanometer being employed instead.

223. In general construction, the main difference between a ballistic galvanometer and an ordinary reflecting galvanometer is in the shape of the needle, which in the former instrument is made very massive, and shaped so as to offer as small a surface to air friction as possible. When a discharge of electricity takes place through the coils of the galvanometer, the needle receives a sudden impulse, which causes it to deflect, the amplitude of the first swing of the needle being proportional to the quantity of electricity which has passed through the coils, provided that the whole of the discharge took place before the needle started on its swing. It is with the object of insuring this latter condition that the galvanometer needle is made so massive.

THEORY OF THE BALLISTIC GALVANOMETER.

224. The following is the proof of the relation between the constants of the ballistic galvanometer and the quantity of electricity producing a given swing of the needle.

Let M = magnetic moment of the galvanometer needle;
I = moment of inertia of galvanometer needle;
G = galvanometer coil constant;
ω = angular velocity of the needle at any instant;
Q = total quantity of electricity in absolute units;
α = amplitude of the first swing of the needle;
H = horizontal intensity of the earth's magnetic field;
T = periodic time of swing of the needle.

The quantity of electricity $Q = c\tau$ where c is the current strength, and τ the time during which it flows.

$$\text{Moment of the force on the needle due to } c = \text{MG}c$$
$$\text{The total impulse given to the needle} = \text{MG}c\tau = \text{I}\omega$$

$$\therefore \omega = \frac{\text{MG}c\tau}{\text{I}} \qquad (1)$$

If the pole strength of the needle is m, and its length $2l$, the work done in deflecting the needle through an angle α is—

$$2ml\text{H}(1 - \cos \alpha) = \text{MH}(1 - \cos \alpha)$$

But the work done on the needle = kinetic energy in needle

$$= \tfrac{1}{2}\,\text{I}\omega^2$$

$$\therefore \tfrac{1}{2}\,\text{I}\omega^2 = \text{MH}(1 - \cos \alpha)$$

$$\text{and } \omega = \sqrt{\frac{2\text{MH}(1 - \cos \alpha)}{\text{I}}} \qquad (2)$$

Now, combining equations (1) and (2), we get—

$$\frac{\text{MGQ}}{\text{I}} = \sqrt{\frac{2\text{MH}(1 - \cos \alpha)}{\text{I}}}$$

$$\text{and } \text{Q} = \frac{2 \sin \frac{\alpha}{2}}{\text{G}} \sqrt{\frac{\text{HI}}{\text{M}}} \qquad (3)$$

Now, the periodic time $\text{T} = 2\pi\sqrt{\dfrac{\text{I}}{\text{MH}}}$, and equation (3) may be written—

$$\text{Q} = \frac{\text{HT}}{2\pi\text{G}}\, 2 \sin \frac{\alpha}{2}$$

Multiplying this result by 10 gives the number of coulombs, since there are 10 coulombs in 1 C.G.S. unit of quantity.

225. *Correction for Damping.*—In the above calculation we have assumed that the only retarding force acting on the needle is that due to the controlling-field H. There are, however, other causes which affect the amplitude of the swing of the needle. Of these the most important are (1) the induced currents set up in the galvanometer coil, due to the motion of the needle inside it, in accordance with Lenz's law; (2) the resistance to the motion of the needle due to the friction of the air; (3) the temporary alteration in M due to the field produced by the current in the galvanometer coil; and (4) the torsion of the suspending fibre.

The effect of (3) may be made small by sending discharges through the galvanometer, which will only produce small swings; and (4) may be allowed for in the same manner as described in par. 138.

The effects of (1) and (2), which tend to diminish the amplitude of the swings, are usually termed the "damping" effects, and may be determined experimentally.

If the needle of a ballistic galvanometer is set swinging, the amplitude of the swings gradually diminishes, on account of this damping action, till finally the needle is brought to rest.

If the damping is not too great, then it is found that the amplitudes of successive swings diminish in a geometrical series, calling $a_1, a_2, a_3, \ldots a_n$ successive swings, then—

$$\frac{a_1}{a_2} = \left(\frac{a_1}{a_3}\right)^{\frac{1}{2}} = \left(\frac{a_1}{a_4}\right)^{\frac{1}{3}} = \left(\frac{a_1}{a_5}\right)^{\frac{1}{4}} = \left(\frac{a_1}{a_n}\right)^{\frac{1}{n-1}} = c$$

or, calling a_m = amplitude of the mth swing
and a_n = amplitude of the nth swing

$$\text{then } \left(\frac{a_m}{a_n}\right)^{\frac{1}{n-m}} = c$$

$$\text{hence } \log_e c = \frac{1}{n-m}(\log_e a_m - \log_e a_n)$$

$\log_\epsilon c$ is the log of the constant ratio, and is known as the logarithmic decrement (λ);

$$\text{or } \lambda = \frac{1}{n-m}(\log_\epsilon a_m - \log_\epsilon a_n)$$

226. In order to correct any swing for damping we can now proceed as follows. Let a represent the observed swing of the ballistic galvanometer needle. If there had been no damping, the swing would have been greater, say a', so that the effect of the damping has been, during a half-period of a complete swing of the needle, to reduce the swing from a value a' to a value a; therefore, since—

$$\lambda = \frac{1}{n-m}(\log_\epsilon a_m - \log_\epsilon a_n)$$

$$\lambda = \frac{1}{\frac{1}{2}}(\log_\epsilon a' - \log_\epsilon a)$$

$$\text{and } \frac{\lambda}{2} = \log_\epsilon a' - \log_\epsilon a$$

$$\text{or } \log_\epsilon a' = \frac{\lambda}{2} + \log_\epsilon a$$

from which, by the theory of logarithms—

$$a' = \epsilon^{\frac{\lambda}{2} + \log_\epsilon a}$$

$$= \epsilon^{\frac{\lambda}{2}} \times \epsilon^{\log_\epsilon a}$$

but $\epsilon^{\frac{\lambda}{2}}$ when expanded gives $1 + \frac{\lambda}{2}$ + etc., and since λ is small it is not necessary to go further up the series than the second term; also—

$$\epsilon^{\log_\epsilon a} = a$$

$$\text{hence } a' = a\left(1 + \frac{\lambda}{2}\right)$$

So that, in order to correct the observed swing of a ballistic galvanometer needle for damping, we must multiply by

$\left(1 + \frac{\lambda}{2}\right)$. We may therefore write the full formula for the ballistic galvanometer as—

$$Q = \frac{HT}{2\pi G} 2 \sin \frac{a}{2}\left(1 + \frac{\lambda}{2}\right)$$

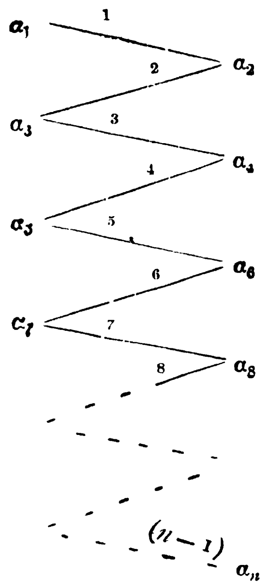
This simple correction for damping may be applied in all cases when λ does not exceed 0·5, without introducing any sensible error.

MEASUREMENT of λ.

227. From what has just been said, it will be seen that we may determine λ from the observation of a series of swings of the needle. It must, however, be borne in mind that λ is not a constant for all conditions under which the galvanometer may be used; since it depends partly on the induced currents set up in the coil in accordance with Lenz's law, and these depend on the resistance of the circuit in which they are induced, the damping must obviously vary with the resistance of the galvanometer circuit. The measurement of λ should therefore be made for a number of cases, the galvanometer circuit resistance being specified in each case, starting with the galvanometer on open circuit and ending with it short circuited. A curve may then be plotted showing the relation between λ and the resistance of the galvanometer circuit. From such a curve the value of λ for any particular experiment may be deduced.

To measure λ the galvanometer needle should be set swinging, either by means of a magnet, which is afterwards removed from its vicinity, or else by passing a current momentarily through the galvanometer coils. The successive swings to right and left of the scale zero are then noted by two observers. Let these be $a_1, a_2, a_3, a_4, \ldots a_n$.

These should be tabulated in two parallel columns, and for convenience a zigzag line connecting them, thus—



If the zero lies between a_1 and a_2, then the arithmetical sum of $a_1 + a_2$ gives the amplitude of the first swing, or, in general, calling readings to the left hand of zero minus, and to the right hand plus—

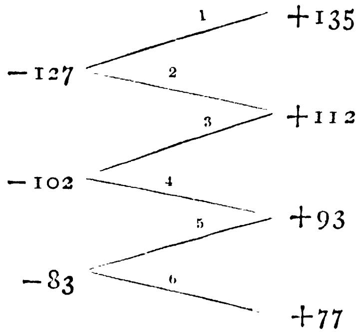
The algebraic diff. of		$a_1 - a_2$ is the amplitude of the			1st swing	
,,	,,	$a_2 - a_3$	,,	,,	2nd	,,
,,	,,	$a_3 - a_4$	,,	,,	3rd	,,
,,	,,	$a_{n-1} - a_n$	,,	,,	$(n-1)$th	,,

The logarithmic decrement may be calculated from the above, taking various combinations of swings, and using the relation—

$$\lambda = \frac{1}{n-m}(\log_e a_m - \log_e a_n)$$

228. In order to determine the logarithmic decrement of a ballistic galvanometer, the needle was adjusted so that the spot of light was at the zero on the scale. The needle was then started swinging by bringing a magnet near the galvano-

meter and then removing it. The following readings were taken with the galvanometer on open circuit :—



Amplitude of 1st swing	=	135 + 127	= 262
" 2nd "	=	127 + 112	= 239
" 3rd "	=	112 + 102	= 214
" 4th "	=	102 + 93	= 195
" 5th "	=	93 + 83	= 176
" 6th "	=	83 + 77	= 160

Taking swings 1 and 3, $\lambda = \frac{1}{3-1}(\log_e 262 - \log_e 214) = 0{\cdot}103$

" 3 " 5, $\lambda = \frac{1}{5-3}(\log_e 214 - \log_e 176) = 0{\cdot}098$

" 4 " 6, $\lambda = \frac{1}{6-4}(\log_e 195 - \log_e 160) = 0{\cdot}103$

" 2 " 6, $\lambda = \frac{1}{6-2}(\log_e 239 - \log_e 100) = 0{\cdot}101$

Mean value of logarithmic decrement $\lambda = 0{\cdot}101$

Calibration of a Ballistic Galvanometer

229. The calibration of a ballistic galvanometer, in order to obtain the relation between the quantity of electricity discharged through the coils and the throw produced on the needle, may be carried out in three ways—

(1) From the relation $Q = \frac{HT}{2\pi G} 2 \sin \frac{\alpha}{2}\left(1 + \frac{\lambda}{2}\right)$

(2) By means of an earth inductor or standard current inductor.

(3) By means of a standard cell and condenser.

230. *First Method.*—In calibrating the ballistic galvanometer by means of the relation $Q = \frac{HT}{2\pi G} 2 \sin \frac{\alpha}{2}$ we have to first find the value of $\frac{H}{G}$. This may be got by sending a steady current of known value, C (in C.G.S. units), through the galvanometer, and observing the steady deflection, δ, produced by it; the conditions of control, etc., remaining the same as when the galvanometer is used ballistically. Assuming the deflection δ of the galvanometer to follow a tangent law for steady currents, we have—

$$C = \frac{H}{G} \tan \delta$$

and therefore—

$$\frac{H}{G} = \frac{C}{\tan \delta}$$

So that—

$$Q = \frac{T}{2\pi} \times \frac{C}{\tan \delta} \times 2 \sin \frac{\alpha}{2}\left(1 + \frac{\lambda}{2}\right) \text{ (absolute units)}$$

$$\text{or } Q \text{ (coulombs)} = \frac{10TC}{\pi \tan \delta} \sin \frac{\alpha}{2}\left(1 + \frac{\lambda}{2}\right)$$

and the constant K of the galvanometer is—

$$K = \frac{10TC}{\pi \tan \delta}$$

The measurement of C, the steady current producing the deflection δ, which in general will be very small, may be effected either by means of a zinc iodide voltameter in series with it, or by connecting the galvanometer in series with a large resistance and battery, and measuring the fall of potential between the galvanometer terminals on a potentiometer, against the

E.M.F. of a standard cell, then if the galvanometer resistance be known, the current passing through it may be calculated.

In order to determine T, the periodic time of swing of the galvanometer needle, the latter should be set swinging through a small angle, and the time required for the spot of light on the scale to make fifty transits in the same direction across the zero, should be taken on a stop-watch. The time between two successive transits in the same direction may then be calculated, and is the periodic time of swing T.

In reading the steady deflection δ of the galvanometer needle, should the instrument be a mirror-galvanometer, it must be borne in mind that the angular deviation of the spot of light will be twice that of the needle. The same remark applies to the swing α. In connection with this point the student should refer back to par. 137, where a similar case is dealt with.

231. In order to determine the constant of a 6000-ohm reflecting ballistic galvanometer, it was set up with its mirror a distance of 1000 mm. from the scale on which the divisions were measured and found to be half-millimetres. A battery of constant E.M.F. was connected to the ends of a resistance of 10,000 ohms, and the galvanometer terminals were connected across 20 ohms of the 10,000-ohm resistance. The deflection of the spot of light was 303 scale-divisions. The potential difference at the ends of a 5000-ohm coil, which made up part of the 10,000-ohm resistance, was measured on a potentiometer against a standard Clark cell, and found to be exactly 1 volt.

The galvanometer needle was then set swinging, and by means of a stop-watch it was found to make fifty swings in 260 seconds. Then—

$$K = \frac{10TC}{\pi \tan \delta}$$

$$T = \frac{260}{50} = 5{\cdot}20 \text{ seconds}$$

If the fall of potential was 1 volt per 5000 ohms, the fall of potential between the galvanometer terminals will be—

$$\frac{20}{5000} = 0{\cdot}004 \text{ volt}$$

and the current $C = \frac{0{\cdot}004}{6000} = 0{\cdot}00000066$ ampère

$= 0{\cdot}000000066$ absolute unit

The distance between the scale and the mirror of the galvanometer = 1000 mm. = 2000 scale-divisions, hence $\frac{303}{2000} = 0{\cdot}151$ = tangent of angular deflection of spot of light;

Hence the angular deviation of the spot of light = 8·6°
and angular deviation of galvanometer needle $\delta = 4{\cdot}3°$
and $\tan \delta = {\cdot}0752$

$$\text{therefore } K = \frac{10TC}{\pi \tan \delta}$$

$$= \frac{10 \times 5{\cdot}20 \times 0{\cdot}000000066}{3{\cdot}142 \times 0{\cdot}0752}$$

$$= 0{\cdot}0000145$$

Inductor Methods of Calibration.

232. (*a*) *Earth Inductor.*—When a coil of wire is rotated in a magnetic field so as to cut lines of force, an electro-motive force is induced in it, the value of which in volts is—

$$E = \frac{1}{10^8} \frac{dN}{dt}$$

$\frac{dN}{dt}$ denoting the rate of cutting the lines of force. If the circuit of the coil is completed through a ballistic galvanometer a current will be produced, and if R is the total resistance of the circuit in ohms (galvanometer and coil), the current, according to the above equation, must be—

$$\frac{E}{R} = C = \frac{dN}{10^8 R dt}$$

From this we have—

$$Cdt = \frac{1}{10^8} \frac{dN}{R}$$

integrating this expression, we get—

$$Q = \frac{1}{10^8} \frac{N}{R}$$

where Q is the quantity of electricity discharged through the galvanometer in coulombs.

If the earth inductor consists of a coil of n turns, each of area A sq. cm., and placed with its plane parallel to the direction of the horizontal component of the earth's magnetic field **H**, then, when the coil is rotated through 180°, the total number of lines of force cut will be—

$$N = 2n\mathbf{H}A$$

and therefore—

$$Q = \frac{2n\mathbf{H}A}{10^8 R}$$

We can now proceed to calibrate the galvanometer for quantity of electricity. Thus, if on turning the earth inductor through 180°, we obtain on the galvanometer needle a swing α_1, we have—

$$Q = \frac{2n\mathbf{H}A}{10^8 R} = \frac{HT}{2\pi G} 2 \sin \frac{\alpha_1}{2}\left(1 + \frac{\lambda}{2}\right)$$

and therefore the constant of the galvanometer (K) will be—

$$K = \frac{HT}{\pi G} = \frac{2n\mathbf{H}A}{10^8 R \sin \frac{\alpha_1}{2}\left(1 + \frac{\lambda}{2}\right)}$$

So that in order to calculate the quantity of electricity in coulombs which produces any swing, α, of the galvanometer needle, we may write—

$$Q = K \sin \frac{\alpha}{2}\left(1 + \frac{\lambda}{2}\right)$$

It must again be noticed here that α refers to the angular swing of the needle, and not that of the spot of light.

In making a calibration of this kind, a separate experiment must be performed in order to determine the value of **H**, the horizontal intensity of the earth's magnetic field, at the earth inductor coil (this may be done by some of the methods described in par. 268); also the relative value of the controlling field H, acting on the ballistic galvanometer needle, must be specified by determining its periodic time of swing.

233. Earth inductors, of a more or less elaborate kind, mounted so as to admit of rotation about two axes, and provided with graduated dials and stops such as that shown in Fig. 101, may be used. A much simpler form, however, with

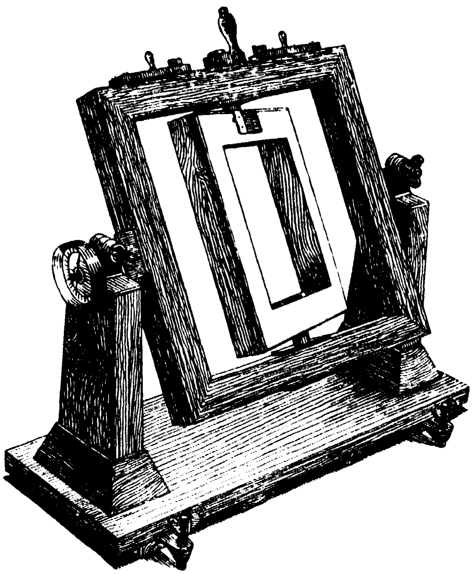


FIG. 101.

which very satisfactory results may be obtained, consists of a large circular coil of many turns wound on a wooden ring, the ends of the coil being brought to two terminals on the inner side of the ring, and the whole entirely unmounted.

The total area is calculated by very carefully measuring the circumference of each layer of wire, and adding together the areas of each turn so calculated. To use this coil, it is placed on a flat table with its plane parallel to the table, and turned over through 180° by hand, thus cutting the vertical component of the earth's field, V. The value of V may be found from a measurement of **H** and the angle of dip δ, since—

$$V = \mathbf{H} \tan \delta$$

A convenient value for the total area of such a coil is about 200,000 sq. cm.

In using the earth inductor, care must be taken that the whole rotation of the coil has taken place before the ballistic needle commences to deflect.

234. The following data were obtained in an experiment to determine the constant of a ballistic galvanometer :—

Distance of mirror from scale = 2000 scale-divisions
Swing of spot of light = 36 „ „
Effective area of earth inductor nA = 192,300 sq. cm.
Resistance of galvanometer = 6000 ohms

The earth inductor was laid flat on a table and turned through 180°, so as to cut the vertical component of the earth's magnetic field. The strength of the horizontal component **H** had been carefully measured previously, and was found to be = 0·184 C.G.S. unit. The angle of dip was also measured, and found to be 70·3°.

$$\text{Hence the vertical component } V = \mathbf{H} \tan 70{\cdot}3$$
$$= 0{\cdot}184 \times 2{\cdot}793$$
$$= 0{\cdot}514 \text{ C.G.S. unit}$$

The logarithmic decrement of the galvanometer was determined, and was $\lambda = 0{\cdot}30$.

The angular deviation of the spot of light was the angle whose $\tan = \frac{36}{2000} = 0{\cdot}018$ *i.e.* 1·05°.

$$\text{Hence } \alpha = \frac{1{\cdot}05}{2} = 0{\cdot}525^\circ$$

$$\text{and } \sin\frac{\alpha}{2} = \sin 0{\cdot}262^\circ = 0{\cdot}0045$$

$$\text{Therefore the constant } K = \frac{2nVA}{10^8 R \sin\frac{\alpha}{2}\left(1 + \frac{\lambda}{2}\right)}$$

$$= \frac{2 \times 192300 \times 0{\cdot}514}{10^8 \times 6000 \times 0{\cdot}0045\,(1 + 0{\cdot}15)}$$
$$= 0{\cdot}0000633.$$

235. (*b*) *Current Inductor.*—Instead of using a coil moving in the earth's field, we may produce a field of known strength

inside a coil by means of a long solenoid through which a measured current is flowing. The form of apparatus employed usually consists of a solenoid of length ($2l$), which is at least ten times its radius (r), mounted horizontally on a stand (see Fig. 102). Round the centre portion of the solenoid a small

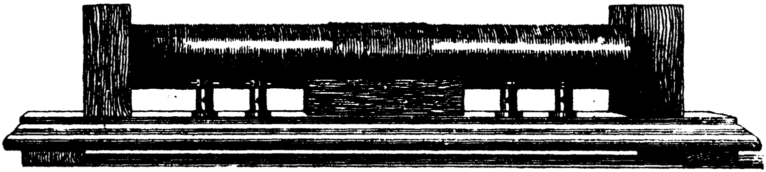


FIG. 102.

test-coil is wound, consisting of many turns of fine wire, the number of turns, n_1, being known.

The test-coil is connected to the ballistic galvanometer. If now a current of C ampères is sent through the solenoid, the intensity of the field produced inside it at its centre is[1]—

$$H = \frac{4\pi n C}{10}\left(1 - \frac{1}{2}\frac{r^2}{l^2}\right)$$

n being the number of turns per centimetre on the solenoid. If A is the mean area of the solenoid, the total number of lines of force (N) produced is—

$$N = \frac{4\pi n CA}{10}\left(1 - \frac{1}{2}\frac{r^2}{l^2}\right)$$

and this magnetic flux cuts the test-coil n_1 times. Hence the effective number of lines, N_1, cut by the test-coil during the starting or stopping of the current in the solenoid is—

$$N_1 = \frac{4\pi n n_1 CA}{10}\left(1 - \frac{1}{2}\frac{r^2}{l^2}\right)$$

If R is the total resistance in ohms of the ballistic galvanometer and test-coil, the number of coulombs of electricity (Q) discharged through the circuit will be—

$$Q = \frac{N_1}{10^8 R} = \frac{4\pi n n_1 CA}{10^9 R}\left(1 - \frac{1}{2}\frac{r^2}{l^2}\right)$$

[1] *Phil. Mag.*, vol. xxii., Oct. 1886, p. 369.

Hence, if when a current of C_1 ampères is started in the solenoid we get a throw, α_1, on the ballistic galvanometer, we have—

$$Q = \frac{4\pi n n_1 C_1 A}{10^9 R}\left(1 - \frac{1}{2}\frac{r^2}{l^2}\right) = \frac{HT}{2\pi G}\, 2 \sin\frac{\alpha_1}{2}\left(1 + \frac{\lambda}{2}\right)$$

and the constant of the galvanometer K will then be—

$$K = \frac{HT}{\pi G} = \frac{4\pi n n_1 C_1 A}{10^9 R \sin\frac{\alpha_1}{2}\left(1 + \frac{\lambda}{2}\right)}\left(1 - \frac{1}{2}\frac{r^2}{l^2}\right)$$

so that the quantity of electricity in coulombs required to produce any swing, α, of the ballistic needle will be—

$$Q = K \sin\frac{\alpha}{2}\left(1 + \frac{\lambda}{2}\right)$$

236. The method of calibration by means of the current inductor has certain advantages over the earth inductor method, since it is independent of the earth's magnetic field, if the axis of the coil is placed at right angles to the magnetic meridian. It is also very useful for calibrating ballistic galvanometers of the D'Arsonval type where the ordinary ballistic formula does not necessarily apply; in this case it is usual to observe the ballistic throws produced by starting or stopping currents of different values in the coil, and plotting an absolute calibration curve with the scale-readings for abscissæ against the corresponding quantities of electricity in coulombs which produced them.

The time of swing of the galvanometer coil, the distance of the mirror from the scale, and the length of a scale-division in centimetres should be carefully specified.

The current sent through the large coil of the solenoid may be measured on a tangent galvanometer, or on some form of ammeter which has been calibrated absolutely by comparison with a standard current measuring instrument.

In using the apparatus, it will be found more satisfactory to take the ballistic swing on "breaking" a known current in the solenoid, rather than on "make."

237. The following are the dimensions of the coil shown in Fig. 102 :—

Large Coil.—640 turns of No. 20 wire in two layers—

Internal diameter = 5·00 cm.
External " = 5·53 "
Total length = 41·85 "
Resistance = 2·82 ohms.

Small Coil.—130 turns No. 36 wire one layer—

Resistance = 2·35 ohms.

238. In the calibration of a ballistic galvanometer by the current inductor, the dimensions of which have just been given, the galvanometer needle was 2000 scale-divisions from the scale zero, and on breaking a current of 5 ampères in the large coil, a ballistic swing of 50 scale-divisions was obtained on the galvanometer. The logarithmic decrement of the galvanometer was determined, and was $\lambda = 0·30$.

The number of turns per cm. on large coil $n = \frac{640}{41·85} = 15·30$

The number of turns on test-coil $n_1 = 130$

Mean area of coil $A = 21·74$ sq. cm.

$$\text{Value of } \left(1 - \tfrac{1}{2}\frac{r^2}{l^2}\right) = 0·992$$

Resistance of galvanometer = 6000 ohms

Tangent of angular deflection of spot of light $= \frac{50}{2000}$

$= 0·025$

Angular deflection of spot of light $= 1·43°$

Angular deflection of needle $\alpha = 0·71°$

$$\text{Value } \sin\frac{\alpha}{2} = 0·0061$$

$$\text{Hence } K = \frac{4\pi n n_1 C_1 A}{10^9 R \sin\frac{\alpha}{2}\left(1 + \frac{\lambda}{2}\right)}\left(1 - \tfrac{1}{2}\frac{r^2}{l^2}\right)$$

$$= \frac{4 \times 3·142 \times 15·30 \times 130 \times 5 \times 21·74 \times 0·992}{10^9 \times 6000 \times 0·0061 \times 1·15}$$

$= 0·0000647$

Calibration of Ballistic Galvanometer by Means of a Standard Cell and Condenser.

239. If a condenser having a capacity of K farads is charged to a potential of V volts, the quantity of electricity stored in it in coulombs is—

$$Q = KV$$

Since the capacity of standard condensers is usually measured in microfarads, it will have to be divided by 10^6 to bring it to farads before using in the above formula.

In order to calibrate the ballistic galvanometer by this method, it is connected up with a battery, condenser, and suitable key, so that the condenser may be charged or discharged through the galvanometer. The following diagram (Fig. 103) shows

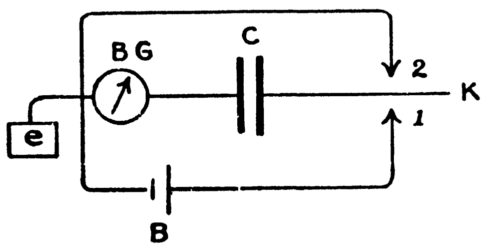


FIG. 103.

one method of arranging the circuit. The condenser C is placed in series with the ballistic galvanometer BG and the discharge key K; the other terminal of the galvanometer being connected to the upper contact (2) of K, to the lower contact (1) of K through the standard cell B, and to earth.

When K is depressed on contact (1), the condenser is charged, and the amplitude of the first swing of the needle noted. By raising K to the upper contact (2) the condenser is discharged. This should be repeated several times, and the mean of the swings taken; also, if necessary, with several cells in series. The E.M.F. of the cell is corrected for temperature, and the value of Q calculated as shown above. K may be measured by a method to be described in the next chapter, or if a standard condenser is used it will be known. The cell B and the key K must be carefully insulated from earth. If the galvanometer

to be calibrated is one in which the sine of half the angle of swing of the needle is proportional to the quantity of electricity discharged through it, then—

$$Q = K_1V_1 = \frac{HT}{2\pi G} 2 \sin \frac{\alpha_1}{2}\left(1 + \frac{\lambda}{2}\right)$$

where the capacity, potential, and swing of the needle have the values K_1, V_1, and α_1 respectively, and the constant of the galvanometer K will therefore be—

$$K = \frac{HT}{\pi G} = \frac{K_1V_1}{\sin \frac{\alpha_1}{2}\left(1 + \frac{\lambda}{2}\right)}$$

and the quantity of electricity corresponding to any throw of the needle will be—

$$Q = K \sin \frac{\alpha}{2}\left(1 + \frac{\lambda}{2}\right)$$

The condenser method of calibration may also be employed in obtaining the absolute calibration curve of a D'Arsonval galvanometer for quantity of electricity, the condenser in this case being charged to different potential differences, either by employing several standard cells in series or by standardizing, by means of a Clark cell, the fall of potential down a long uniform wire, and charging the condenser across various known lengths of the wire. A curve may then be plotted with scale-readings for abscissæ and corresponding quantities of electricity in coulombs for ordinates, the distance of the mirror from the scale, length of scale-division and relative value of the controlling force being specified.

240. In a calibration of a ballistic galvanometer by means of a standard condenser and known E.M.F., a battery of two Leclanche cells in series was connected to a resistance of 10,000 ohms; whilst a condenser of 10 microfarads capacity was charged through the ballistic galvanometer across 300 ohms of the 10,000-ohm resistance. A ballistic swing of 230 scale-divisions was obtained, the scale being 2000 scale-divisions distant from the mirror. The difference of potential across a coil of 1000 ohms resistance, which formed part of the

10,000-ohm resistance, was then carefully measured on a potentiometer against a standard Clark cell, and was found to be 0·280 volt. The difference of potential employed to charge the condenser was therefore—

$$\frac{300 \times 0{\cdot}28}{1000} = 0{\cdot}084 \text{ volt}$$

The angular swing of the spot of light $= 6{\cdot}55^\circ$
The angular swing of the galvanometer needle $\alpha = 3{\cdot}27$

$$\text{Hence } \sin\frac{\alpha}{2} = 0{\cdot}028$$

The logarithmic decrement was found to be $\lambda = 0{\cdot}40$

$$\text{Hence } K = \frac{K_1 V_1}{\sin\frac{\alpha_1}{2}\left(1 + \frac{\lambda}{2}\right)}$$

$$= \frac{10 \times 0{\cdot}084}{10^6 \times 0{\cdot}028 \times 1{\cdot}2}$$

$$= 0{\cdot}0000250$$

Remarks on the Use of the Ballistic Galvanometer.

241. In using a ballistic galvanometer, where great accuracy is not required, and the swings of the needle are small, the sines of the angles of swing are nearly proportional to the angles themselves, and therefore, in a comparison of two quantities of electricity, the ratio of the amplitudes may be taken instead of the ratio of the sines of the half-angles.

The student will probably experience some difficulty at first in reading the exact value of the swing of the needle, this becoming more difficult the quicker the periodic time of swing of the needle. A few trial swings should be taken first, and thus the part of the scale reached by the spot of light will be located; the observer may then at once turn his attention to this spot after having completed the circuit. A sliding pointer moving over the scale may be used with advantage to locate the exact position of the swing. It is advisable, however, for measurements of this kind, that there should be at least two observers.

242. As stated before, the needle of the galvanometer must be perfectly stationary before the discharge is sent through the coils. To enable this to be attained quickly, a small auxiliary coil, connected to a battery through a reversing key, and known as a damping coil, may be set up near the galvanometer, so that when a current is flowing in it, its magnetic field will act on the galvanometer needle. By properly timing the duration and direction of the current in this coil, the needle may be rapidly brought to rest. It has been stated that almost any type of galvanometer may be used ballistically; for accurate work, however, a special design is necessary, since the damping in

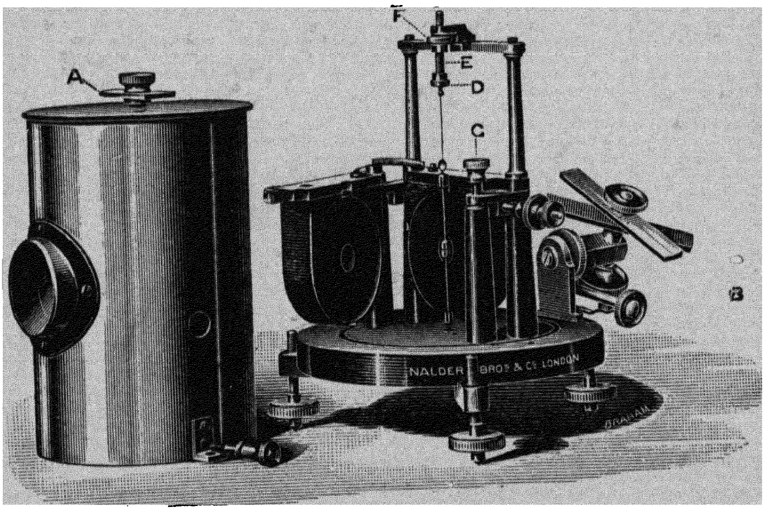


FIG. 104.

most galvanometers is pretty large, and when this is so, the correction is difficult, and cannot be made by taking the logarithmic decrement, but the galvanometer must be calibrated directly in coulombs per degree of swing, throughout the scale, by some of the inductor methods which are independent of the damping. A special form of ballistic galvanometer, designed by Messrs. Nalder Bros., is shown in Fig. 104; in it the magnets, which are bell-shaped to diminish air-friction, are arranged astatically, two being at the centre of the coil and two outside, one above and one below the coil.

The coils, which are hinged to give easy access to the needle, are mounted in ebonite cases, and the whole highly insulated; there is as little solid metal as possible in the vicinity of the needles, as eddy currents might be set up, which would increase the damping action. The mirror is also reduced in size as much as possible, on account of air-friction.

V.

CAPACITY.

243. THE capacity of a conductor is defined as the quantity of electricity required to raise its potential from zero to unity. The electro-magnetic practical unit of capacity is the farad, which is that capacity requiring one coulomb of electricity to raise the potential from zero to 1 volt.

In practice this unit is much too large, and the millionth part of it, called the microfarad, is used instead.

1 farad = 10^{-9} C.G.S. electro-magnetic units of capacity.
1 farad = 10^{6} microfarads.
1 microfarad = 10^{-15} C.G.S. electro-magnetic units of capacity.

MEASUREMENT OF THE CAPACITY OF A CONDENSER.

244. In order to measure the capacity of a condenser we take advantage of the relation $K = \frac{Q}{V}$, where K is the capacity in farads, Q the quantity of electricity in coulombs, and V the potential in volts. The quantity of electricity, Q, in the condenser may be found by discharging it through a ballistic galvanometer, and calculating from the ballistic galvanometer formula; whilst the potential difference, V, of the plates may be found in terms of a current and resistance.

The following diagrams (Figs. 105 and 106) show the connections of the apparatus.

The condenser C, whose capacity is required to be measured, is connected in series with a ballistic galvanometer, BG, and high-resistance key, K, across the ends of a large resistance R_1,

which is at least 10,000 ohms (see Fig. 105). A battery, B, of low internal resistance and constant E.M.F. is also connected to the ends of R.

When the circuit of the condenser is completed by pressing the key K, the condenser is charged to a potential, V, which

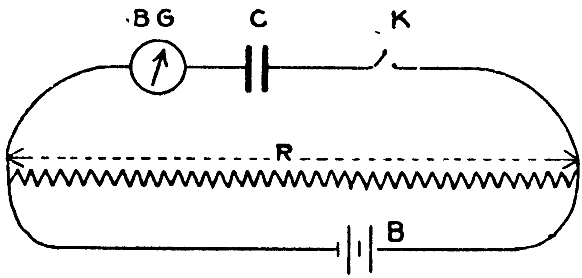


FIG. 105.

represents the fall of potential between the ends of R, and the amplitude of the first swing of the galvanometer needle (a) is noted. This should be repeated several times, the condenser being discharged, after each throw, by means of its short-circuiting plug, *after the circuit at* K *has been broken.* Using the same symbols as in par. 224, we have—

$$Q = \frac{10HT}{2\pi G} 2 \sin\frac{a}{2}\left(1 + \frac{\lambda}{2}\right)$$

Q being in coulombs.

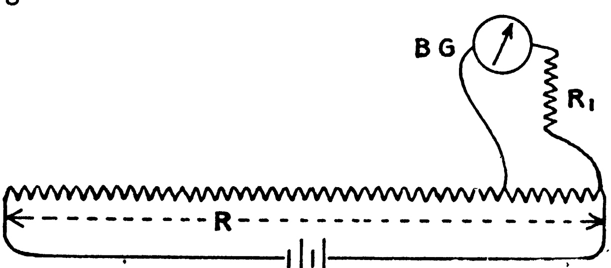


FIG. 106.

In order to get the value of V, the charging potential, the connections are now altered as below (see Fig. 106).

The condenser is removed and a resistance box, R_1, is

placed in series with the galvanometer, and the terminals are brought to two points on R, so as to include a known fraction, r, of the resistance R, in order to get a steady deflection, δ, on the ballistic galvanometer. The value of δ should not be very different from that of α. This may be obtained by altering the resistance R_1 or the resistance r, or, if necessary, both of them. Then, if R_g = galvanometer resistance, e = potential difference at the ends of the resistance r, and assuming that for steady currents the galvanometer follows a tangent law, we have—

$$c = \frac{e}{R_g + R_1} = 10\frac{H}{G}\tan\delta$$

$$\therefore e = \frac{10(R_g + R_1)H}{G}\tan\delta$$

$$\text{But } \frac{V}{e} = \frac{R}{r}$$

$$\text{therefore } V = \frac{eR}{r}$$

$$= \frac{10(R_g + R_1)HR}{rG}\tan\delta$$

$$\text{and } K = \frac{Q}{V} = \frac{\dfrac{10HT}{2\pi G}\,2\sin\dfrac{\alpha}{2}\left(1 + \dfrac{\lambda}{2}\right)}{\dfrac{10(R_g + R_1)HR}{rG}\tan\delta}$$

$$= \frac{Tr\sin\dfrac{\alpha}{2}\left(1 + \dfrac{\lambda}{2}\right).}{\pi R(R_g + R_1)\tan\delta}$$

245. In the second part of the experiment the resistance r must be small compared with the resistance of the galvanometer and R_1, otherwise the fall of potential down R will not be the same as in the first part of the experiment. Also the controlling force acting on the ballistic galvanometer is supposed to be the same in the two experiments. Both α and δ represent actual angular deflections of the galvanometer needle, and must be calculated from the scale-readings of the spot of light as shown in par. 137. The logarithmic decrement should be determined when the apparatus is connected up as in Fig. 105. The

result will give the capacity of the condenser in farads Multiplying by 10^6 we get it in microfarads.

246. The following are the details of a measurement of capacity of a condenser by the above method. Three constant cells were connected to the ends of a resistance of 10,000 ohms, and the condenser was charged from two points, the resistance between which was 4700 ohms, and a ballistic throw of 151 scale-divisions was obtained on a scale 100 cm. distant from the mirror; the scale-divisions were in millimetres. The condenser was then removed and the galvanometer terminals connected across two points on the 10,000 ohms, the resistance between which was 20 ohms, and a steady deflection of 303 scale-divisions was obtained. The galvanometer resistance was measured and found to be 6000 ohms. The periodic time of swing and logarithmic decrement of the galvanometer needle were taken and were $T = 5{\cdot}20$ seconds and $\lambda = 0{\cdot}300$.

The angle of swing of the needle α is half the angle whose tangent is $\frac{15{\cdot}1}{100} = \frac{1}{2} \times 8{\cdot}6^\circ = 4{\cdot}3^\circ$; and the angle of steady deflection is half the angle whose tangent is $\frac{30{\cdot}3}{100}$ *i.e.* $\frac{1}{2} \times 16{\cdot}8^\circ = 8{\cdot}4^\circ$

$$\text{Hence } K = \frac{Tr \sin\frac{\alpha}{2}\left(1 + \frac{\lambda}{2}\right)}{\pi R R_g \tan\delta}$$

$$= \frac{5{\cdot}20 \times 20 \times 0{\cdot}0377(1 + 0{\cdot}15)}{3{\cdot}1416 \times 4700 \times 6000 \times 0{\cdot}1477}$$

$$= 0{\cdot}344 \times 10^{-6} \text{ farads}$$

$$= 0{\cdot}344 \text{ microfarads}$$

The temperature was $11{\cdot}8^\circ$ C.

METHODS OF COMPARING CAPACITIES.

247. If it is desired to compare two capacities with one another, one of them being a standard, there are several methods which may be adopted. Since the capacity is the

ratio of the quantity of electricity in a condenser to the potential difference at its terminals, we may either charge both condensers to the same potential and compare the quantities of electricity in them, or we may give them a charge of known quantity and measure the potential differences.

248. *Comparison of Condensers by Means of a Ballistic Galvanometer.*—In this method both are charged to the same potential and the quantities compared by means of a ballistic galvanometer.

The two condensers, A and B (see Fig. 107), have each one terminal connected to earth and to the upper and lower contacts of a Kemp discharge key K, a battery C being inserted between the lower contact of K and the common terminal. The other terminals of the condensers are connected

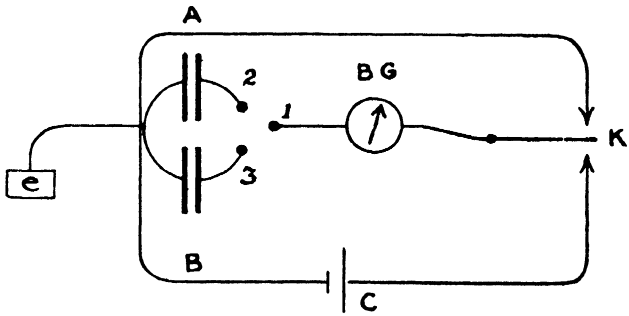


FIG. 107.

to the terminals (2) and (3) of a high-resistance two-way plug key respectively; the terminal (1) of the two-way key being connected through the ballistic galvanometer BG, to the movable tongue of the key K. The terminals (1) and (2) of the two-way key are connected together, and the key K depressed, thus charging condenser A from the cell C; the ballistic swing a_1 is noted. The key K is then raised to the upper contact, discharging the condenser. The terminals (1) and (2) are now disconnected, and (1) and (3) are connected together and the process repeated, charging condenser B this time, and obtaining a ballistic swing, a_2. If K_A and K_B are the capacities of A and B respectively, and V is the potential of

the charging battery, which is assumed to have remained constant during the experiment, then—

$$Q_1 \propto a_1 \propto K_A V$$
$$Q_2 \propto a_2 \propto K_B V$$
$$\text{therefore } \frac{K_A}{K_B} = \frac{a_1}{a_2}$$

If K_B is a standard condenser, then $K_A = \dfrac{K_B a_1}{a_2}$

249. Comparing a condenser with a standard $\frac{1}{3}$ microfarad condenser by the above method, a ballistic swing of 183 scale-divisions was obtained with the standard, and 273 scale-divisions with the condenser whose capacity was required.

$$\text{Hence } \frac{0{\cdot}333}{x} = \frac{183}{273}$$
$$x = 0{\cdot}498 \text{ microfarads}$$

The charging battery employed was a Hellesen dry cell.

250. *Comparison of Condensers by the Electrometer Method.*—In this method, one of the condensers is charged and its potential measured, the charge is then divided between the two condensers, and from the alteration in potential the ratio of their capacities may be calculated.

The following diagram (Fig. 108) shows the arrangement of the apparatus.

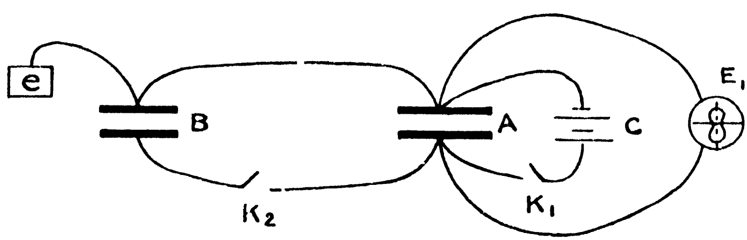


FIG. 108.

The condensers A and B are connected in parallel with each other and with the battery C and electrometer E_1, highly insulating keys, K_1 and K_2, being placed in the battery circuit and between the two condensers respectively. It will also be advisable to keep the battery well insulated and to earth one pair of plates, as at **e**.

In making the measurement, the keys K_1 and K_2 are opened and the electrometer needle adjusted to zero. The key K_1 is then closed, thus charging condenser A with a certain quantity of electricity, Q, and raising its potential to V_1, which is measured by the deflection of the electrometer needle, δ_1. The battery circuit is now broken at the key K_1, and key K_2 is closed; this allows the charge Q to mix between the two condensers, and the potential will fall to V_2, as shown by the deflection δ_2 of the electrometer needle. Now, calling K_A and K_B the capacities of A and B respectively, we have—

$$(1)\ Q = K_A V_1 \propto K_A \delta_1$$
$$(2)\ Q = (K_A + K_B)V_2 \propto (K_A + K_B)\delta_2$$
$$\text{whence } \frac{K_A}{K_B} = \frac{\delta_2}{\delta_1 - \delta_2}$$

In comparing two condensers in this way, a preliminary test must be made to see that the apparatus is insulated properly. This may be assumed to be the case if the deflections δ_1 and δ_2 remain constant for at least two minutes. The electrometer deflections are assumed proportional to the potential, and therefore the instrument must be used heterostatically, with the needle potential largely in excess of that of the quadrants. The battery C should consist of a sufficient number of cells to give a large deflection on the electrometer for δ_1.

251. In a comparison of two condensers, one of which was a standard condenser of capacity 0·333 microfarads, when the standard alone was charged the electrometer deflection was 240 scale-divisions; on dividing the charge between the two condensers, the deflection fell to 121 scale-divisions.

$$\text{Hence } \frac{K_A}{K_B} = \frac{0{\cdot}333}{K_B} = \frac{121}{240 - 121}$$
$$\text{and } K_B = 0{\cdot}327 \text{ microfarads}$$

The battery employed consisted of six Lessing dry cells in series, and the electrometer needle was charged to a high potential by means of a Zamboni dry pile.

252. *Comparison of Capacities by the Method of Mixtures.*—One of the most satisfactory methods of comparing the

capacities of two condensers, and one which may be applied even when there is a considerable difference in their values, is the method of mixtures, the principle of the method being the measurement of the potentials required to give equal charges to the two condensers. The condensers are charged to different potential differences, the charges being afterwards mixed by placing the condensers in series, and the resultant charge tested for; when this latter is zero the condensers must have contained equal quantities of electricity, and therefore the capacities will be in the inverse ratio of the charging potentials.

One method of arranging the apparatus is shown in Fig. 109.

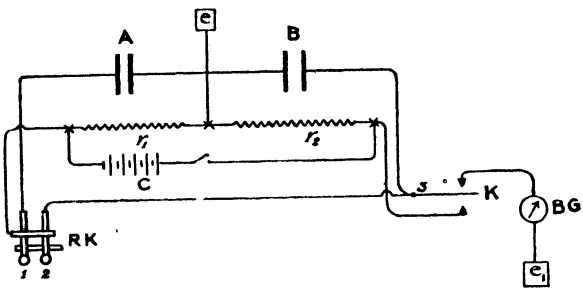


FIG. 109.

The battery C, consisting of several cells of constant E.M.F., is connected in series with the two resistance boxes r_1 and r_2. The condensers A and B, by an arrangement of keys—a reversing key, RK, supplied with cams by means of which the keys can be insulated midway between the upper and lower contacts, and a Kemp discharge key, K—may be charged across the ends of r_1 and r_2 respectively, connected in series so as to allow the charges to mix, and finally connected so that the resultant charge (if any) may discharge through the ballistic galvanometer BG to earth, the junction of r_1 and r_2 being connected to one plate of each of the condensers and to earth. Calling (1), (2), and (3) the contacts of the two keys, the following operations are gone through :—

(1) The contacts (1), (2), and (3) are all insulated; *i.e.* they are held midway between the upper and lower contacts.

(2) Key (1) is allowed to rise to the upper contact, thus charging condenser A across the ends of r_1.

(3) Key (3) is depressed so as to touch the lower contact and charge condenser B across the ends of r_2.

(4) Key (3) is insulated and keys (1) and (2) are depressed, to make contact with the lower contact, which is common to both; by this means the positively charged plate of A is connected to the negatively charged plate of B, and *vice versâ*, and the two charges mix.

(5) Key (3) is now raised to the upper contact, and the resultant charge (if any) in the condensers escapes to earth through the ballistic galvanometer.

Should there be any swing on the ballistic needle after the last operation, it signifies that one of the charges must have been greater than the other. From the direction of the swing the direction of the discharge, and consequently the condenser having the larger charge, can be found, and the potential difference to which it is charged must be diminished; this may be effected either by diminishing the resistance, between the ends of which it is connected, or by increasing the resistance across which the other condenser is charged. The above operations are repeated, and r_1 and r_2 altered until there is no resultant charge left, and the ballistic galvanometer shows no deflection after operation (5). Then, since the charging potential differences are proportional to the resistances r_1 and r_2, we have—

$$\frac{K_A}{K_B} = \frac{r_2}{r_1}$$

The value of $r_1 + r_2$ must always be large, never being less than 10,000 ohms. The keys may be arranged close together, so that all the above connections may be made in a second or two, thus, as far as possible, eliminating the effects of dielectric absorption. The keys must be highly insulated, as also must all the connecting wires and the condensers.

The ballistic galvanometer must be arranged so as to be sufficiently sensitive to detect small discharges. The accuracy of the comparison may be tested by finding the range through which r_1 or r_2 may be altered without the galvanometer showing

a deflection due to the resultant charge. This should be done, and the percentage accuracy of the comparison stated.

253. In a comparison between the standard one-third microfarad condenser and a large condenser, the method of mixtures was employed. For the resistances r_1 and r_2, two large resistance boxes wound with platinoid wire were used. The ballistic galvanometer employed was very sensitive, but in order to make the test still more sensitive, instead of having the galvanometer needle at rest before sending the resultant charge through it, the following procedure was adopted. The needle was set swinging through a small arc, and the amplitudes of two successive swings observed; the resultant charge was then sent through the galvanometer when the needle was in the middle of its third swing, and the effect on the amplitude of that swing noted. If the discharge was in such a direction as to deflect the needle in the same direction as that in which it was moving, the amplitude was greater that it otherwise would have been as might be calculated from the two observed swings, allowing for damping; if the discharge was in the opposite direction, the amplitude would be smaller than the theoretical; and if the discharge was zero, *i.e.* if the charges in the two condensers were exactly equal, there would be no effect on the swing at all.

The condenser A was the standard 0·333 microfarad, and in order to get zero effect on the ballistic galvanometer needle, the resistances were $r_1 = 15{,}015$ ohms, and $r_2 = 500$ ohms. A change of about 5 ohms could be detected. The coils were correct at 12·5° C., and the temperature during the test was 15·5° C. Taking 0·021 per cent. per 1° C. as the temperature variation of resistance of platinoid, the resistances corrected for temperature are $r_1 = 15024·4$, and $r_2 = 500·315$.

$$\text{Hence } \frac{0·333}{B} = \frac{500·315}{15024·4}$$

$$B = 9·99 \text{ microfarads at } 15·5^\circ \text{ C.}$$

Specific Inductive Capacity.

254. The ratio of the capacity of a condenser with some medium other than air as dielectric to the capacity of the same

condenser with air as the dielectric, is termed the specific inductive capacity (S.I.C.) of that medium.

In order to determine the value of the S.I.C. of various substances, we must therefore have some form of condenser between the plates of which various dielectrics may be introduced. The method here described is that due to Dr. John Hopkinson.[1]

The apparatus required is a guard-ring condenser, an adjustable condenser, and a quadrant electrometer.

The guard-ring condenser consists of a brass disc about 15 cm. in diameter (see Fig. 110), surrounded by a guard-ring,

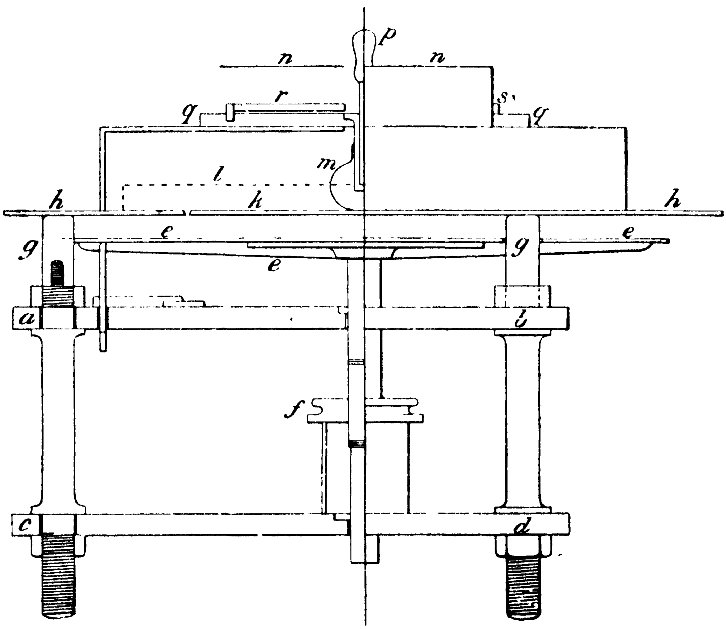


FIG. 110.

h, also of brass, about 7·5 cm. wide, and separated from it by a space of 0·1 cm., the ring *h* being supported from the base of the instrument by means of ebonite rods.

The other plate of the condenser, *e*, is parallel to *h*, and

[1] *Phil. Trans.*, 1878.

mounted on a rod which has a screw of $\frac{1}{25}''$ pitch accurately cut on it, and passes through a nut, *f*, the circumference of which is graduated into 100 parts. The plate *e* is free to move up and down in a vertical plane, but is prevented from rotating.

The disc is supported from two rods of ebonite, the ends of which rest on the guard-ring, and is thus insulated from it. A metal shield attached to *qq*, covers and protects the upper surface of the plate *hh*. When in use, a vessel containing pumice and strong sulphuric acid is placed on the upper surface of *h*, inside the metal shield, in order to keep the insulation perfect. The rods supporting *gg* must also be kept well insulated.

255. The adjustable condenser used is of the form due to Lord Kelvin,[1] and consists essentially of two hollow brass cylinders about 5 cm. diameter, one 26·5 cm. long, the other 35·3 cm. long, supported on ebonite insulators with their axes in the same line and their ends separated by a small air gap. Inside these, and insulated from them, is a third brass cylinder, placed coaxial with the other two, 36·6 cm. long and 2·3 cm. diameter, arranged so that it can slide backwards and forwards, and so project by different amounts inside the interior of the larger tubes. On the outside of all is a metal cover, which can be earthed to protect the tubes within from outside induction. In using this instrument, the outside cover *d*, the cylinder *b*, and the inner cylinder *c* are all earthed, whilst the cylinder *a* is charged. By sliding *c* in or out of *a*, the capacity of the condenser may be increased or decreased. If necessary, the capacity of this sliding condenser may be calculated from its dimensions; in the following measurements, however, the knowledge of its capacity is not required.

The electrometer employed may be any form of sensitive electrometer.

256. In making a measurement of the S.I.C. of a substance, the following diagrams show the connections. GR represents the guard-ring condenser, SC the sliding condenser, E the electrometer, and + and – the poles of the charging battery.

[1] *Phil. Trans.*, 1871, p. 573.

Fig. 111 shows the connections in the first part of the experiment. The plates of the sliding condenser are connected to + and earth respectively, the disc and guard-ring of GR to −, whilst the movable plate of GR is earthed, as are also the

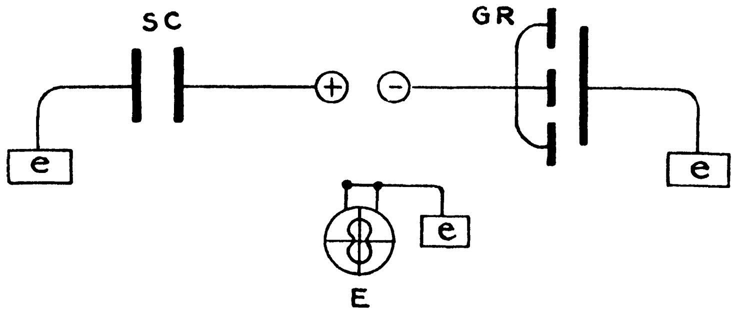


FIG. 111.

quadrants of the electrometer E. In this way the condensers will both receive a charge of opposite sign. The connections are now altered to those of Fig. 112. The battery is entirely disconnected from the guard-ring and disc, and from the

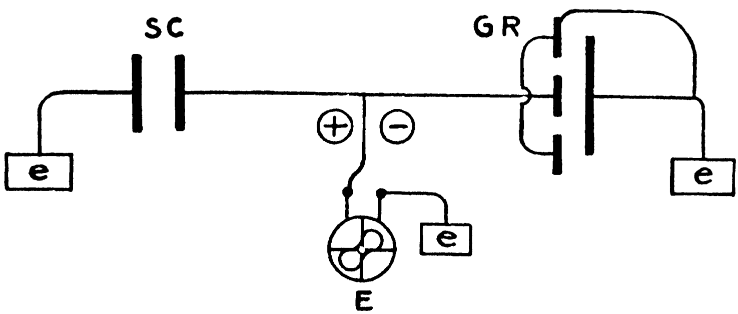


FIG 112.

charged plate of SC, and the guard-ring is earthed. This allows the charges in the condensers to mix.

On connecting now, as in Fig. 113, one quadrant of the electrometer, being connected to the condensers, will be charged to the same potential. If the two condensers are of equal

capacity, this charge will be zero, and there will be no electrometer deflection, because the + and – charges would mix and neutralize.

If the electrometer does deflect, it proves that one of the condensers has a greater capacity than the other, the sliding condenser SC is then adjusted until no deflection is obtained.

257. It is important that the connection in Fig. 113 should follow that of Fig. 112 within a fraction of a second, and it may be found advisable to earth both poles of the battery during the second and third part of the experiment. The connections in

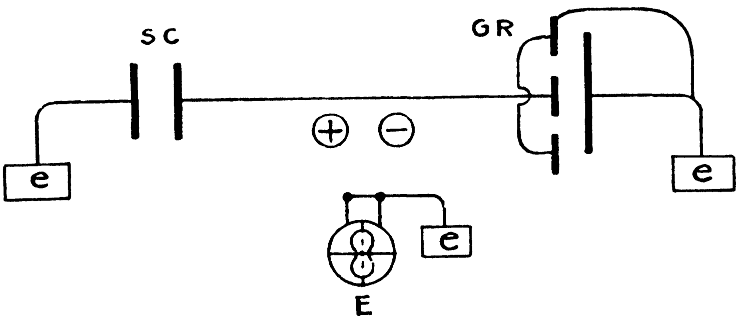


Fig. 113.

these experiments may be made by means of separate keys. It is, however, desirable to devise a special form of key, so that the various changes may be made rapidly. One such key, devised by Dr. Hopkinson, and described by him,[1] is shown below (Fig 114). It consists of an ebonite plate, *qq*, which is screwed to the shielding cover of the condenser; a steel spring, *r*, connected to earth; a similar spring, *s*, connected to one pole of the battery; *tv* are segments of brass connected to the brass cover; *wu* are similar segments insulated from the cover and guard-ring, and connected to the sliding condenser and electrometer respectively; *p* is an ebonite handle and brass pin which turns in an insulated brass socket connected with the disc *k*; the ebonite piece *x* moves the springs *r* and *s* from *t* and *v* to *u* and *w* respectively; the spring *yy* may connect *tv* with *k*, or *w*

[1] *Phil. Trans.*, 1878, p. 19.

with k, and immediately after, both with the electrometer ; the other battery pole is connected to hh.

258. Having got the sliding condenser and guard-ring condenser adjusted until they have equal capacities, a plate of dielectric, carefully cut with its opposite faces parallel, is then placed on the movable plate of the guard-ring condenser, its diameter being greater than that of the disc opposite to it. On repeating the process of comparing the condensers, it will be found that they are no longer equal, since the capacity of GR has been increased by the plate of dielectric. The movable plate of GR is now lowered by means of the screw until a balance of the capacities is again obtained, and the distance s, through which it has been lowered, is noted. Then, if A is the effective area of the guard-ring condenser, d the distance between the plates in the first experiment, and t the thickness of the slab of dielectric of S.I.C. = K, the capacity in the first case is—

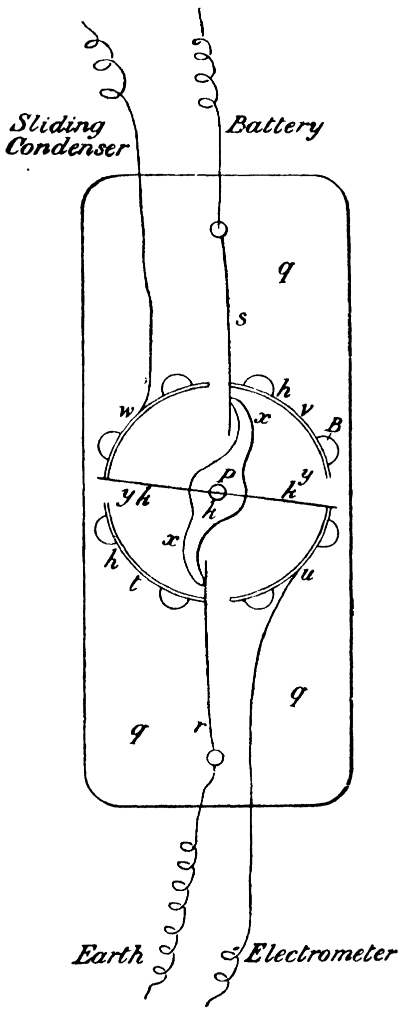


FIG. 114.

$$= \frac{A}{4\pi d} \quad [1]$$

[1] See J. J. Thomson, "Mathematical Theory of Electricity and Magnetism," pp. 91 and 129.

and in the second case—

$$= \frac{A}{4\pi\left(d + s - t + \frac{t}{K}\right)}$$

and since these capacities are equal, we have—

$$s = t\left(1 - \frac{1}{K}\right)$$

$$\text{and } K = \frac{t}{t - s}$$

The thickness t of the dielectric slab may also be measured by means of the guard-ring condenser as follows. The dielectric slab is removed, and the movable plate is screwed up until it comes into contact with the disc S, the exact position of contact being judged by inserting slips of tissue paper between them and adjusting the screw until they just become loose. The plate is then screwed down, the dielectric slab placed on it, and again screwed up until the upper surface of the slab makes contact with the disc, the tissue paper being again used to test the exact position; from the difference in the readings of the screw in the two positions the thickness of the slab may be calculated.

The battery employed in the above experiment consists of from 50 to 100 cells, and the electrometer should be sufficiently sensitive to give a large deflection with one cell.

259. The specific gravity of the slab of dielectric should be determined, and also, if possible, its index of refraction for light of known wave-length, since, according to the electromagnetic theory of light—

$$\mu^2 = K$$

where μ = index of refraction for waves of infinite wave-length. A sufficiently close approximation may, however, be obtained by using the sodium D line in the spectrum in the measurement of μ. Since K increases slightly with temperature, a measurement of the temperature of the slab should be recorded.

260. There are many other methods of measuring the S.I.C.

of dielectrics, references being given at the end of the chapter, one of considerable importance being the five-plate method of Gordon. By this method Gordon obtained results which differed widely from those of Hopkinson. This difference has been discussed by Hopkinson,[1] who showed that it was due to the distance between the plates not being small compared with their diameter.

261. The following data have been taken from Hopkinson's results :—

Material.	Density.	t (revs. of screw.)	s (revs. of screw.)	Battery.	K
Light flint glass	3·2	15·01	12·83	48 cells	6·89
,, ,,	3·2	15·01	12·78	72 ,,	6·76

262. *Measurement of the Specific Inductive Capacity of the Dielectric of a Cable.*—A wire of circular cross-section covered uniformly with insulating material may be looked upon as a cylindrical condenser, and if the dimensions of the cable are known, and its total capacity measured, the specific inductive capacity of the dielectric may be calculated. The measurement of the capacity of the cable is made by some of the methods described in par. 247, the cable being immersed in a tank of water as in the measurement of dielectric resistance, when the metal tank forms one of the plates of the condenser and the wire core the other.

In order to calculate the specific inductive capacity, we pro proceed as follows. Let a represent the radius of the wire core, and b the radius of the outside of the dielectric surrounding the wire; then, if we consider any ring of dielectric of radius r, and of infinitely small thickness dr, whose length in the direction of the length of the cable is l (centimetres), its capacity is practically that of two parallel plates of length l and breadth $2\pi r$, which are separated by a distance dr, and is—

$$c = \frac{2\pi r l}{4\pi dr} = \frac{rl}{2dr} \text{ (electrostatic units)}$$

[1] *Phil. Trans.*, 1881, p. 366.

But since the measurement of the capacity of the cable will be made in microfarads, this had better be expressed in microfarads, and will be—

$$c = \frac{rl}{2dr} \times \frac{10^{15}}{9 \times 10^{20}} \text{ microfarad}$$

the ratio of the electrostatic unit of capacity to the electromagnetic unit being proportional to v^2, where $v = 3 \times 10^{10}$ cm. per second; the microfarad being 10^{-15} C.G.S. electro-magnetic units of capacity.

If the specific inductive capacity of the dielectric of the cable be denoted by K, the above expression becomes—

$$c = \frac{Krl}{2dr} \times \frac{10^{15}}{9 \times 10^{20}} \text{ microfarads}$$

Now, this gives the capacity of a single ring of dielectric, but the insulation of the cable may be looked upon as consisting of a number of such rings in series, and if C denotes the total capacity in microfarads of the whole cable, by the laws of condensers in series, we get—

$$\frac{1}{C} = \frac{2}{Kl} \times \frac{9 \times 10^{20}}{10^{15}} \times \int_a^b \frac{dr}{r}$$

$$= \frac{2}{Kl} \times \frac{9 \times 10^{20}}{10^{15}} \times \log_\epsilon \frac{b}{a}$$

from which—

$$K = \frac{2C \log_\epsilon \frac{b}{a}}{l} \times 9 \times 10^5$$

In making a measurement of the specific inductive capacity in this way, the temperature of the water in the tank containing the cable should be taken.

263. In order to measure the specific inductive capacity of G.P. insulating covering, experiments were made with three wires of different diameter, all covered with the same material.

The length and the ratio of the external to the internal radius were first measured, and were—

$$\text{No. 1. Length} = 0{\cdot}25 \text{ mile, } \frac{b}{a} = 2{\cdot}50$$

$$\text{No. 2. Length} = 0{\cdot}25 \text{ mile, } \frac{b}{a} = 2{\cdot}62$$

$$\text{No. 3. Length} = 0{\cdot}25 \text{ mile, } \frac{b}{a} = 3{\cdot}00$$

The capacities were next measured by comparison with a standard one-third microfarad condenser, the method of mixtures being employed. The following capacities were found:—

No. 1 = 0·0986 microfarads at 23·8° C.
No. 2 = 0·0938 „ „
No. 3 = 0·0800 „ „

From these data we get, expressing the length in centimetres—

$$\text{No. 1.}\quad K = \frac{2 \times 0{\cdot}0986 \times \log_\epsilon 2{\cdot}50 \times 9 \times 10^5}{40233} = 4{\cdot}03$$

$$\text{No. 2.}\quad K = \frac{2 \times 0{\cdot}0938 \times \log_\epsilon 2{\cdot}62 \times 9 \times 10^5}{40233} = 4{\cdot}03$$

$$\text{No. 3.}\quad K = \frac{2 \times 0{\cdot}0800 \times \log_\epsilon 3{\cdot}00 \times 9 \times 10^5}{40233} = 4{\cdot}02$$

264. *Measurement of the Specific Inductive Capacity of Liquids.*—The method of Hopkinson for the measurement of the S.I.C. of solids may also be applied to the measurement in the case of liquids.

The liquid is contained in a double metal cylinder, C (see Fig. 115), into which, but insulated from it, hangs the metal cylinder P, the position of P relatively to C being fixed by means of ebonite stops.

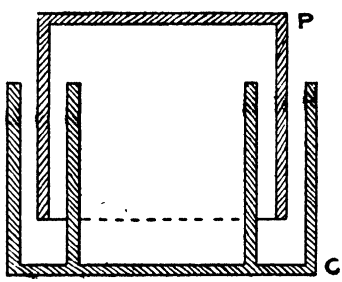


Fig. 115.

The cylinder P forms one plate, and C the other, of the condenser. This condenser is balanced against the sliding condenser, first without and afterwards with the dielectric liquid

between the plates, the sliding condenser being in this case adjusted until a balance is obtained, the capacities being calculated from the positions of the sliding tube in the cylindrical condenser, which, for two concentric cylinders of length l and external and internal radii b and a respectively, air being the medium between them, is—

$$C = \frac{l}{2 \log_e \frac{b}{a}}$$

265. *Silow's Method.*—A much simpler method is a modification of one due to Silow.[1]

A cylindrical quadrant electrometer is made by pasting four tinfoil strips, each 10 cm. × 10 cm., symmetrically round the inner sides of a glass jar 10 cm. deep and 15 cm. diameter. In the interior, suspended by a wire, w, from a torsion head, hangs a platinum needle, consisting of two curved plates of platinum P, P (see Fig. 116) connected together by a platinum wire, the whole needle being well within the vessel. The opposite tinfoil quadrants of the glass jar are connected together as in an electrometer; one pair is connected to one pole of a battery the other pole of which is earthed, whilst the other pair and the needle are both earthed.

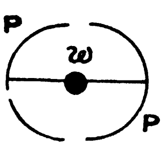


FIG. 116.

The needle is first brought to zero with the quadrants both earthed, one pair is then connected to the battery, and the needle deflects; by turning the torsion head at the top of the wire w, the needle may be brought back to zero, and the angle α_1 through which the wire has been turned noted. The vessel is then filled up with the liquid and the above repeated; a torsion α_2 will now have to be applied to restore the needle to its original position, the E.M.F. of the battery being assumed constant.

The ratio $\frac{\alpha_2}{\alpha_1}$ is the ratio of the capacities, and therefore equal to the S.I.C. of the liquid.

[1] *Pogg. Ann.*, 155, p. 389.

266. References to Scientific Papers.

Title of Paper.	Author.	Reference.
An Instrument for reproducing an Invariable Quantity of Electricity	Deprez	*Phil. Mag.*, vol. 21, June, 1886.
Measurements of S.I.C. of Dielectrics	Gibson and Barclay	*Trans. Roy. Soc.*, 1871.
S.I.C. of Glass	Hopkinson	*Ibid.*, 1878, 1881; *Pro. Roy. Soc.*, vol. 26, p. 298.
Dielectric Properties of Different Glasses	,,	*Trans. Roy. Soc.*, 1878; *Pro. Roy. Soc.*, vol. 25, p. 496.
Measurement of Electrostatic Capacity of Glass	,,	*Pro. Roy. Soc.*, vol. 31, p. 148.
Dielectric Capacity of Liquids	,,	*Ibid.*, vol. 31, p. 347.
Note on S.I.C.	,,	*Ibid.*, vol. 41, p. 453.
On S.I.C.	,,	*Ibid.*, vol. 43, p. 88.
Measurement of S.I.C.	Gordon	*Ibid.*, vol. 27, p. 270; vol. 28, p. 155.
S.I.C. of Dielectrics where acted on by Very Rapidly Alternating Currents	J. J. Thomson	*Ibid.*, vol. 46, p. 292.
Effect of Temperature on the S.I.C. of a Dielectric	Cassie	*Ibid.*, vol. 46, p. 357.
A New Form of Air Leyden with Application to Measurements of Small Electrostatic Capacity	Kelvin	*Ibid.*, vol. 52, p. 6.
Dielectric Hysteresis	Porter	*Ibid.*, vol. 57, p. 469.
On a Condenser of Variable Capacity	Boys	*Phil. Mag.*, vol. 7, Feb., 1879.
The Refractive Index and S.I.C. of a Transparent Media	Hopkinson	*Ibid.*, vol. 13, Apr., 1882.
On a Method of Measuring the Electrostatic Capacity of a Condenser	Glazebrook	*Ibid.*, vol. 18, Aug., 1884.
Researches on the Dielectric Constants of some Gases	Klemencic	*Ibid.*, vol. 19, May, 1885.
Specific Inductive Capacity of Electrolytes	Rosa	*Ibid.*, vol. 31, Mar., 1891; vol. 34, Oct., 1892.
On a Method of determining the S.I.C. of a Dielectric	Trouton and Lilly	*Ibid.*, vol. 33, June, 1892.
Determination of the S.I.C. of Conducting Liquids	Slschegtiaeff	*Ibid.*, vol. 34, Oct., 1892.
On Dielectrics	Appleyard	*Ibid.*, vol. 38, Oct., 1894.

Title of Paper.	Author.	Reference.
Measurement of the S.I.C. of Water and Alcohol	Fessenden	*Phil. Mag.*, vol. 38, Dec., 1894.
On a Method of Comparing the Values of S.I.C. under Slow and Rapidly Changing Fields	Northrup	*Ibid.*, vol. 39, Feb., 1895.
Modification of the B.A. Method of determining the Capacities of Condensers	Womack	*Ibid.*, vol. 39, Feb., 1895.
Effect of Temperature on the S.I.C. of Dielectrics	Appleyard	*Ibid.*, vol. 42, Aug., 1896.
Experiments with Condensers	Leblanc	*Elect.*, vol. 27, p. 306.
Absorption in Mica Condensers	Bouty	*Jour. Elec. Eng.*, vol. 19, p. 755.
S.I.C. of Mica	,,	*Ibid.*, vol. 20, p. 499
Capacity of Coils	Cauro	*Ibid.*, vol. 24, p. 378
S.I.C. of Ice	Ayrton and Perry	*Phil. Mag.*, vol. 5, Jan., 1878.
Platymeter	Sir Wm. Thomson	*B.A. Report*, 1855.
Determination of the Capacity of a Condenser	Jenkin	*Ibid.*, 1867.
Comparison of Capacities by Method of Mixtures	Sir Wm. Thomson	*Jour. Elect. Eng.*, vol. 1, p. 394.
Dielectric Constants of Ice and Alcohol at Very Low Temperatures	Fleming and Dewar	*Elect.*, vol. 38, p. 748.
Dielectric Constants of Liquid Oxygen and Liquid Air	,, ,,	*Ibid.*, vol. 38, p. 285
Apparent Resistance of a Ballistic Galvanometer of the Moving Coil Type	Robertson	*Ibid.* vol. 46, p. 901.
S.I.C. of Oils	Clark	*Phys. Rev.*, vol. 6, p. 120.
Theory of Moving Coil and other Ballistic Galvanometers	Wilson	*Elect.*, vol. 57, p. 860.
Electric Inductive Capacities of Dry Paper and Solid Cellulose	Campbell	*Ibid.*, vol. 57, p. 784.
Secohmmeter for Measurement of Combined Resistance and Capacities	Milner	*Ibid.*, vol. 58, p. 60.
Loss of Energy in the Dielectric of Condensers and Cables	Monasch	*Ibid.*, vol. 59, p 416.
Measurement of S.I.C.	Boltzmann	*Wien. Ber.*, vols. 66, 67 : 1872.
S.I.C. of Mica	Klemencic	*Beiblätter*, vol. 12 : 1888.
S.I.C. of Liquids	Silow	*Pogg. Ann.*, vol. 156 : 1875.

Title of Paper.	Author.	References.
S.I.C. of Liquids	Quincke	*Wied. Ann.*, vol. 19, 1883; vol. 33, 1888.
" "	Negreano	*Comptes Rendus*, vol. 104: 1887.
S.I.C. of Gases	Boltzmann	*Pogg. Ann.*, vol. 15: 1755.
" "	Ayrton and Perry	*Trans. Asiatic Soc. Japan*, 1877.
Dielectrics	V. Hoor	*Electrotechn. Zeitschr.*, vol. 22, p. 716.

VI.

MAGNETISM.

DETERMINATION OF H.

268. IN very many of the experiments performed in a physical laboratory, especially those involving the deflection of a magnetic needle, which is either wholly or partly controlled by the magnetic field of the earth, it becomes necessary to determine the value of this controlling field. Also, since the direction of this field varies from place to place, and since ordinarily it is only the component of the total force which acts parallel to the surface of the earth that we have to do with, we do not require to know the total force, but only **H**, the horizontal component of it. The relation connecting the value of **H** with the total force is—

$$\mathbf{H} = \mathrm{T} \cos \delta$$

where T is the total force in C.G.S. units, and δ is the angle between H and T, usually known as the angle of dip.

269. *Gauss's Method of determining* **H**.—Several methods have been devised for the measurement of **H** in absolute units, but that most frequently employed is due to Gauss. This method has been thoroughly investigated, and improved by Professor T. Gray, to whose researches the student is referred.[1]

A small magnetic needle is suspended so as to swing freely in the earth's field, and a permanent magnet, *the length between the poles of which* is $2l$, is placed with its centre at a distance, d, from the centre of the needle, so that the line joining the centres is at right angles to the magnetic meridian, and also

[1] *Phil. Mag.*, vol. vi., Nov., 1878; vol. xx., Dec., 1885.

the magnetic axis of the magnet is at right angles to the magnetic meridian (see Fig. 117); or so that the line joining the centres lies in the magnetic meridian, and the magnetic axis of the magnet at right angles to the meridian (see Fig. 118).

In each case the effect of the magnet is to deflect the needle out of the plane of the magnetic meridian. Then, in the first

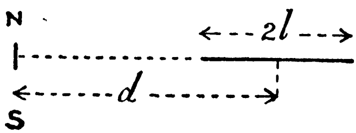


Fig. 117.

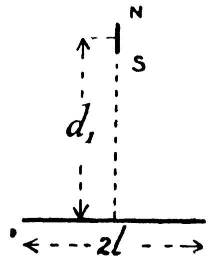


Fig. 118.

case, usually called the A position of Gauss, we have, calling M the magnetic moment of the deflecting magnet, and θ the angular deflection of the needle—

$$\frac{\mathbf{M}}{\mathbf{H}} = \frac{(d^2 - l^2)^2}{2d} \tan \theta$$

and in the second case, the B position of Gauss, if d_1 is the distance apart of the centres, and θ_1 the angular deflection of the needle—

$$\frac{\mathbf{M}}{\mathbf{H}} = (d_1^2 + l^2)^{\frac{3}{2}} \tan \theta_1$$

If we now remove the needle, and suspend the deflecting magnet in its place, so that it is free to oscillate about a vertical axis, then, calling I the moment of inertia of the magnet, and T its periodic time of swing, we have—

$$\kappa \mathbf{MH} = \frac{4\pi^2 \mathrm{I}}{\mathrm{T}^2}$$

κ being a constant depending on the inductive action of the earth's field on the magnetic moment of the magnet. Where

great accuracy is not required, κ may be taken as equal to unity. By combining this last equation with either of the previous ones, we can get an expression for either M or **H**.

270. *Corrections.*—If a very accurate determination of **H** is to be made, there are certain corrections that must be taken into account in the above calculation. Of these the most important are—

(1) The correction for the distribution of magnetism in the bar.

(2) The correction for the inductive action of the earth's field.

271. (1) The correction for the distribution of magnetism in the bar magnet resolves itself into the measurement of $2l$, the distance between the poles of the magnet, as distinguished from l_0, the actual length of the magnet. The difference between these two quantities is very small, especially in the case of short magnets, and for ordinary measurements they may be taken as identical. Kohlrausch has shown that the distance of the poles from the ends of a bar magnet is about one-twelfth the length of the bar. The exact length, $2l$, may be determined from observations of the deflections produced by the magnet on the needle in the A and B positions, as follows:—

$$\text{In the A position } \frac{M}{\mathbf{H}} = \frac{(d^2 - l^2)^2}{2d} \tan \theta$$

$$\text{In the B position } \frac{M}{\mathbf{H}} = (d_1^2 + l^2)^{\frac{3}{2}} \tan \theta_1$$

$$\text{Hence } \frac{(d^2 - l^2)^2}{2d(d_1^2 + l^2)^{\frac{3}{2}}} = \frac{\tan \theta}{\tan \theta_1}$$

and expanding, we get, neglecting small terms—

$$l^2 = \frac{\theta_1 d^3 - 2\theta d_1^3}{2\theta_1 d + 3\theta d_1}$$

the value l thus calculated is used in the calculation for **H**.

272. (2) The correction for the inductive effect of the earth's field on the magnetic moment of the deflecting magnet is due to the fact that during the first part of the experiment the magnet is placed with its axis at right angles to the field of the

earth, whilst during the determination of its periodic time of swing it lies in the magnetic meridian, and is therefore liable to a slight increase in magnetization by induction from the earth's field. In order to find the relation between the magnetic moment of the magnet in the two positions, the following method may be adopted. The magnet is placed inside a long solenoid, with its centre at the centre of the solenoid, and at a distance, d, from the centre of the needle, so that it is in the A Gauss position with respect to it. In series with the solenoid is a compensating coil, which is adjusted with respect to the needle so as to neutralize the magnetic effect due to the solenoid alone; this adjustment is made before the magnet is placed in position, and with a much stronger current than is to be used afterwards. The magnet is now placed inside the solenoid, and the deflection, ϕ, of the needle noted; a current of C ampères is sent through the solenoid sufficient to produce a magnetic field at the centre of the coil approximately equal to the field **H** to be measured, and the deflection ϕ_1 of the needle noted.

Now, since—

$$\text{M} \propto \tan \phi$$
$$\text{and } \kappa\text{M} \propto \tan \phi_1$$
$$\kappa = \frac{\tan \phi_1}{\tan \phi}$$

The solenoid should be considerably longer than the needle, and may consist of a coil of wire one layer deep, wound on a long glass tube of a diameter slightly larger than the magnet. Then, if n represents the total number of turns on the coil, and l its length in centimetres, the strength of the field at the centre is—

$$\frac{4\pi n\text{C}}{10l}$$

Putting in the values for n, l, and the strength of field **H** required, we get—

$$\text{C} = \frac{10\mathbf{H}l}{4\pi n}$$

The current may be measured on a tangent galvanometer

whose constant has been determined by copper deposition, and which is sufficiently far off from the other apparatus not to exercise any magnetic effect upon it, or else measured by its fall of potential down a known resistance.

273. *The Magnetometer.*—The magnetometer needle should consist of two steel magnets about 1 cm. long and 0·08 cm. diameter, cemented to the back of a light galvanometer mirror 0·8 cm. diameter, so that they are parallel to one another and have like poles pointing in the same direction. This mirror is suspended by a silk or spider line inside a glass tube, so that the magnets lie in the horizontal plane.

The glass tube may be mounted on a stand provided with levelling screws, great care being taken to remove all iron or other magnetic substance from the neighbourhood, and for this reason the apparatus must be entirely constructed of non-magnetic material; any brass work should be tested, as it sometimes contains iron as an impurity.

An ordinary galvanometer lamp and scale are placed at a carefully measured distance from the mirror, the scale being parallel to the plane of the mirror, and adjusted so that when the needle is in the magnetic meridian the spot of light is at the zero. To test the scale for accuracy of adjustment, the deflection produced by the magnet when its centre is at a given distance from the needle should be the same as when it is turned end for end.

Two lines at right angles to one another should now be marked out to indicate the positions to be occupied by the centre of the magnet in the A and B positions.

If the galvanometer scale is not graduated in centimetres its length must be measured, and the distance from the mirror to it expressed in scale-divisions, instead of in centimetres; also, in reading off the angle θ from the deflection of the spot of light, the remarks in par. 137 apply here.

If necessary, the coefficient of torsion τ of the suspending fibre of the mirror must be measured by the method described previously (par. 138), and the angle θ increased in the ratio $\theta(1 + \tau)$.

274. *Deflecting Magnets.*—It is advisable that readings should

be taken with more than one deflecting magnet, in order to eliminate errors in the determination of $2l$ and of I.

These magnets should have a length at least forty times their diameter, and may be made as follows. From a long, uniform steel rod of about 0·2 cm. diameter, which has been softened, lengths from 8 to 10 cm. are cut off, and the ends carefully filed flat; these lengths are then glass hardened by heating to a bright red heat and at once plunging into cold water; after drying, they are magnetized by placing them inside a solenoid through which a strong current is flowing and at the centre of which there is an intense magnetic field, or else by placing them between the poles of a powerful electro-magnet. After having been magnetized in this way, it is advisable to artificially age the magnets, as they are in a very unstable condition, and liable to lose a large amount of their magnetism on getting a slight knock; they are therefore plunged once or twice into a vessel of boiling water, being allowed to cool between each immersion. In this way they lose some of their magnetism, but are in a much more permanent condition than before.

275. *Method.*—The needle having been carefully adjusted, one of the magnets is taken and placed with its centre at a distance, d, from the centre of the needle in the A position of Gauss, and the deflection noted; it is then turned end for end, its centre being still kept at the same point, and the deflection to the opposite side of the zero on the scale noted. This is now repeated with the magnet at the opposite side of the needle, and at such a distance from it that the deflection produced is the same, the exact distance of the centre of the magnet from the centre of the needle being half the distance between the positions of the centre of the magnet on either side of the needle.

The above should be repeated at two or three different distances for each of the magnets. Care must be taken to keep the magnets apart and to note the particular magnet which gave each set of readings.

Similar sets of readings are taken for the B position of Gauss. The readings may be tabulated thus—

Magnet.	Gauss position.	d	Deflections.				Mean deflection	Value of θ calculated from mean deflection.
			Right.	Left.	Right.	Left.		

From readings in the A and B positions the effective lengths of the various magnets may be calculated.

276. After having got all the deflection experiments performed, the determinations of the periodic times of swing of the various magnets must be made. To do this the magnetometer is removed and the magnets one after another are suspended in its place, inside a glass shade, or box with glass sides, the suspension consisting of a long silk fibre with two loops at one end to hold the magnet in (see Fig. 119). It is not advisable to use a brass clip at the end of the suspension to hold the magnet, as it introduces an additional amount of inertia which would have to be determined by a separate experiment and allowed for.

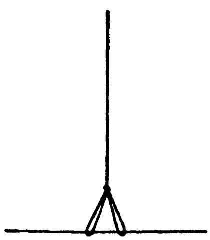


FIG. 119.

On the bottom of the box, underneath the suspended magnet, there should be placed a strip of mirror glass with a line scratched on it, so that when the magnet lies in the magnetic meridian it is exactly over this line. The magnet is set oscillating about the fibre as a vertical axis by bringing a piece of soft iron near and then removing it, care being taken not to set up a pendulum swing; and when the amplitude of oscillation has fallen to about 3° on either side of the zero line, a stop-watch with centre seconds hand reading to 0·2 sec. is started just as the magnet crosses the line. After ten such transits in the same direction the watch is stopped and the time of one complete double vibration calculated (T). The accuracy with which this measurement can be made increases

with the number of swings observed, so that, if possible, more than ten complete swings should be timed.

The periodic time of swing of each of the magnets is taken in this way. The amplitude of the swing should always be kept small, otherwise a correction will have to be made for the arc of oscillation, this, however, being less than 0·02 % for an arc of 3° on either side of zero.

277. Having obtained the time of swing of the magnets, we next proceed to determine their moments of inertia (I). This, for a cylindrical rod of length l_0 (actual length of magnet), radius r, and weight w, is—

$$\mathrm{I} = \left(\frac{l_0^2}{12} + \frac{r^2}{4}\right) w$$

The length and diameter of each bar should be measured in centimetres by a pair of bar-callipers, the diameter being taken at various points along the bar, and the mean taken in the calculation of the radius. The weight in grammes of each bar is taken on a delicate balance.

278. The magnetometer needle should now be replaced in position and the induction constant κ determined for each magnet, in the manner described above.

The values—

$$\frac{\mathrm{M}}{\mathbf{H}} = \frac{(d^2 - l^2)^2}{2d} \tan\theta$$

$$\text{and } \frac{\mathrm{M}}{\mathbf{H}} = (d^2 + l^2)^{\frac{3}{2}} \tan\theta_1$$

are calculated for each set of observations on each magnet and the mean for each of the Gauss positions obtained; also the value of

$$\mathrm{M}\mathbf{H} = \frac{4\pi^2\mathrm{I}}{\kappa\mathrm{T}^2}$$

being known for each magnet, the value of **H** as determined by each of the magnets may now be calculated for the two Gauss positions by combining the two equations. Thus we have—

$$\mathbf{H}^2 = \frac{8\pi^2\mathrm{I}d}{\kappa\mathrm{T}^2(d^2 - l^2)^2 \tan\theta} \text{ for the A position}$$

$$\text{and } \mathbf{H}^2 = \frac{4\pi^2 I}{\kappa T^2 (d^2 + l^2)^{\frac{3}{2}} \tan \theta_1} \text{ for the B position}$$

279. The values of $\frac{M}{w}$, or the magnetic moment per gramme weight of the different magnets, should also be calculated, in order to give some idea of the quality and magnetic condition of the magnets employed. The following table, taken from Gray's paper, is of interest in this respect :—

$\frac{M}{w}$	l_0	$2l$	Diameter.
	centimetres.	centimetres.	centimetre.
44·9	8·03	6·91	0·25
58·5	8·05	7·10	0·25
35·2	4·00	3·12	0·25
55·8	14·93	13·22	0·25
71·0	10·01	9·14	0·20

280. The effect of change of temperature on the magnetic moments of the magnets is very slight, being only about 0·005 % per 1° C., and since the alteration in temperature of the room during the determination is never likely to be more than a few degrees, it may be neglected.

In order to show the magnitude of the error due to the inductive action of a field on a magnet, the following numbers are taken from a table by Gray for glass-hard steel magnets, showing the percentage alteration of magnetic moment due to a field about five times the strength of the earth's field :—

l	$\frac{l_0}{\text{diameter}}$	Per cent. variation of moment in unit field.	$\frac{M}{w}$
centimetres.			
3	10	0·85	27
4	16	0·70	32
6	20	0·69	35
7	31	0·54	39
8	32	0·54	54
10	34	0·51	40
10	44	0·48	43
10	50	0·51	67
10	105	0·44	66

281. The following determination of **H** was made by the foregoing method. The deflecting magnets, of which there were three, were glass hard, and had the following dimensions :—

(A) Length, 8·05 cm.; diameter, 0·25 cm.
(B) Length, 15·03 cm.; diameter, 0·25 cm.
(C) Length, 10 cm.; diameter, 0·2 cm.

The mirror of the magnetometer needle was 108·7 cm. distant from the scale, which was graduated in millimetres. The deflection magnets were adjusted until the readings on each side of zero were equal, and their distance from the mirror of the magnetometer was taken as half the distance between the positions on either side of the needle where they produced the same deflection. From the readings in the A and B positions of Gauss, the effective lengths of the magnets were calculated.

Number of experiment.	Magnet.	Gauss position.	d	Scale reading.	θ (of needle).	tan θ	$2l$
			cm.				
1	A	A	32·06	154·7	4·05°	0·0707	7·11
2	A	B	28·75	97·6	2·31°	0·0406	7·11
3	B	A	51·90	77·6	2·04°	0·0356	12·82
4	B	B	38·85	72·9	2·26°	0·0395	12·82
5	C	A	35·00	99·1	2·75°	0·0481	9·14
6	C	B	30·00	83·0	2·18°	0·0381	9·14

In order to determine the effect of the inductive action of the earth's field on the magnets, the magnetizing coil, with its compensator, were fitted up in the A Gauss position, and the deflection noted, when a field of approximately 0·15 C.G.S. units was produced in the interior of the coil. This gave the following readings :—

Magnet.	Deflection.	Deflection with current on.
A	154·7	154·9
B	77·6	77·7
C	99·1	99·2

The effect of the earth's field on the magnet as calculated from these readings was taken as 0·08 per cent.

The following table gives the results of the experiments to determine the time of swing of the magnets, and the final calculated value of **H**.

Magnet.	I.	T.	H	M.
		seconds.	C.G.S. units.	
A	16·58	5·177	0·1522	160·4
B	106·50	8·074	0·1525	422·8
C	19·32	5·549	0·1526	162·2

The mean value of **H** may be taken as 0·1524.

Vibration Method of determining **H.**

282. The following method of determining the value of the horizontal component of the earth's magnetic force, due to the author, is very much simpler to carry out than the method of Gauss.

The method consists in comparing, by the method of vibration of a magnetic needle, the strengths of two magnetic fields, one due to the earth's magnetism alone, and the other made up partly of the earth's field and partly of an artificial field of known strength produced in the interior of a long solenoid. Knowing the strength of field due to the solenoid the value of the earth's field may be calculated.

283. The apparatus for carrying out this experiment consists of a solenoid containing a known number of turns and of a length at least ten times its radius, similar to that described in par. 237. Inside this solenoid there fits a cardboard tube, rather longer than half the length of the solenoid, near the end of which a mark is made, so that when the mark is flush with one end of the solenoid the other end of the cardboard tube is exactly at the centre of the solenoid.

At the end of the tube which fits into the solenoid there is suspended, by means of a fine silk fibre or spider-line, a small magnetic needle so that it hangs exactly in the axis of the coil. A concave galvanometer mirror is attached to the needle so

that the plane of the mirror is at right angles to the axis of the needle. When the tube has been adjusted in position, glass plates are cemented over the open ends of the coil, to prevent air-currents from affecting the needle. The solenoid is mounted on a horizontal stand, so that its axis lies in the magnetic meridian, and a galvanometer lamp and scale are arranged so as to indicate the motions of the mirror attached to the needle. The solenoid is connected through a reversing key to a constant battery, resistance-box, and tangent, or other galvanometer whose constant has been obtained by a copper deposition experiment. The needle is set oscillating about the fibre as a vertical axis before the current is sent through the coil, and the periodic time of swing taken by means of a stop-watch, the oscillations of the needle being taken from the movement of the spot of light across the galvanometer scale. In no case should the angle of oscillation exceed three degrees on either side of zero, and should be kept as small as possible. Let T_1 represent the periodic time of swing in this experiment.

A current of known strength is now sent through the solenoid, sufficient to produce a field at the centre of strength rather less than that due to the earth, and in the same direction as the earth's field. The periodic time of swing T_2 is again observed.

The current, still of the same value, is reversed in the solenoid so as to produce a field in opposition to the earth's field, and the time of swing T_3 of the needle is taken.

284. Now, calling **H** the horizontal component of the earth's magnetic field, H_1 the strength of field produced by the current in the solenoid, M the magnetic moment of the needle, and κ a constant depending on the moment of inertia of the needle, and equal to $4\pi^2 I$, we have—

$$(1) \qquad \mathrm{M}\mathbf{H} = \frac{\kappa}{T_1^2}$$

$$(2) \qquad \mathrm{M}(\mathbf{H} + \mathrm{H}_1) = \frac{\kappa}{T_2^2}$$

$$(3) \qquad \mathrm{M}(\mathbf{H} - \mathrm{H}_1) = \frac{\kappa}{T_3^2}$$

From (1) and (2) or (1) and (3) we can calculate the value of H, since—

$$\frac{(1)}{(2)} = \frac{M\mathbf{H}}{M(\mathbf{H} + H_1)} = \frac{T_2^2}{T_1^2}$$

$$\text{and } \mathbf{H} = H_1 \frac{T_2^2}{T_1^2 - T_2^2}$$

$$\text{or } \frac{(1)}{(3)} = \frac{M\mathbf{H}}{M(\mathbf{H} - H_1)} = \frac{T_3^2}{T_1^2}$$

$$\text{and } \mathbf{H} = H_1 \frac{T_3^2}{T_3^2 - T_1^2}$$

The value of H_1, the field strength at the centre of the solenoid due to a current of c ampères, is—

$$H_1 = \frac{4\pi nc}{10}\left(1 - \frac{1}{2}\frac{r^2}{l_2}\right)$$

n being the number of turns per centimetre on the solenoid, r its radius in centimetres, and $2l$ its length.

285. *Corrections.*—In the above calculation it is assumed that the magnetic moment of the needle remains constant in the different fields; this, however, is hardly true, on account of the inductive action on the needle of the field in which it is placed. This error, however, may be eliminated by taking the readings with the direction of the current in the solenoid reversed as above, provided we assume that the increase in magnetic moment (m) of the needle, when placed in a field of the same direction as the earth's, is the same as the decrease when placed in a reverse field of equal strength. This has been proved to be the case for strongly magnetized steel bars in fields of strength up to 0·2 C.G.S. unit.[1] Re-writing the above equations, we get—

$$(1) \qquad M\mathbf{H} = \frac{\kappa}{T_1^2}$$

$$(2) \qquad (M + m)(\mathbf{H} + H_1) = \frac{\kappa}{T_2^2}$$

[1] See Sack on the "Specific Induction Constants of Magnets," *Phil. Mag.*, vol. xxii., Oct., 1886.

$$(3) \qquad (\mathrm{M} - m)(\mathbf{H} - \mathrm{H}_1) = \frac{\kappa}{\mathrm{T}_3^2}$$

From (2) and (3) we get—

$$2\frac{\mathrm{M}}{\kappa} = \frac{1}{\mathrm{T}_2^2(\mathbf{H} + \mathrm{H}_1)} + \frac{1}{\mathrm{T}_3^2(\mathbf{H} - \mathrm{H}_1)}$$

and from (3)—

$$\frac{2}{\mathrm{T}_1^2\mathbf{H}} = \frac{\mathrm{T}_2^2(\mathbf{H} + \mathrm{H}_1) + \mathrm{T}_3^2(\mathbf{H} - \mathrm{H}_1)}{\mathrm{T}_2^2\mathrm{T}_3^2(\mathbf{H}^2 - \mathrm{H}_1^2)}$$

from which $\mathbf{H} =$

$$\frac{\mathrm{T}_1^2\mathrm{H}_1(\mathrm{T}_3^2 - \mathrm{T}_2^2) \pm \sqrt{\mathrm{T}_1^4\mathrm{H}_1^2(\mathrm{T}_2^2 - \mathrm{T}_3^2)^2 - 8\mathrm{T}_2^2\mathrm{T}_3^2\mathrm{H}_1^2(\mathrm{T}_1^2\mathrm{T}_2^2 + \mathrm{T}_1^2\mathrm{T}_3^2 - 2\mathrm{T}_2^2\mathrm{T}_3^2)}}{2(\mathrm{T}_1^2\mathrm{T}_2^2 + \mathrm{T}_1^2\mathrm{T}_3^2 - 2\mathrm{T}_2^2\mathrm{T}_3^2)}$$

286. In a measurement of **H** by the vibration method, the coil employed consisted of a solenoid 41·7 cm. long, containing 640 turns of wire, the mean radius of the coil being 2·63 cm. The oscillations of the magnet suspended at the centre of the solenoid were observed through a telescope with spider-lines in the eye-piece, instead of by means of a mirror, lamp, and scale. The solenoid was connected in series with a standard resistance-coil of 300 ohms, a battery, and reversing-key. In order to measure the current in the coil, the potential difference at the ends of the 300-ohm coil was measured against a standard cell by the potentiometer, and during the experiment it was 1·434 volt. The following readings were obtained:—

Field in solenoid.	Total number of swings observed.	Time in seconds for total swings.	Time of one double vibration.	
$\mathbf{H}$	40	114·6	2·865	
$\mathbf{H}$	40	116·0	2·900	2·880 mean
$\mathbf{H}$	40	115·0	2·875	
$\mathbf{H} + \mathrm{H}_1$	100	232·6	2·326	
$\mathbf{H} + \mathrm{H}_1$	100	232·0	2·320	2·325 mean
$\mathbf{H} + \mathrm{H}_1$	100	233·0	2·330	
$\mathbf{H} - \mathrm{H}_1$	30	129·6	4·320	
$\mathbf{H} - \mathrm{H}_1$	30	129·6	4·320	4·310 mean
$\mathbf{H} - \mathrm{H}_1$	30	128·6	4·290	

The current $C = \frac{1 \cdot 434}{300}$ ampères

$$\text{Hence } \mathbf{H} = H_1 \frac{T_2^2}{T_1^2 - T_2^2} = H_1 \frac{T_3^2}{T_3^2 - T_1^2}$$

$$= \frac{H_1}{2}\left(\frac{T_2^2}{T_1^2 - T_2^2} + \frac{T_3^2}{T_3^2 - T_1^2}\right)$$

Therefore—

$$\mathbf{H} = \frac{4 \times 3 \cdot 14 \times 640 \times 1 \cdot 434}{41 \cdot 7 \times 300 \times 2 \times 10}\left(1 - \frac{(2 \cdot 63)^2}{2 \times (41 \cdot 7)^2}\right)\left(\frac{(2 \cdot 325)^2}{(2 \cdot 88)^2 - (2 \cdot 325)^2}\right.$$

$$\left. + \frac{(4 \cdot 310)^2}{(4 \cdot 310)^2 - (2 \cdot 880)^2}\right) = 0 \cdot 162 \text{ C.G.S. unit}$$

and, calculating from the larger formula, we get—

$$\mathbf{H} = 0 \cdot 161 \text{ C.G.S. unit}$$

Determination of the Angle of Dip.

287. The measurement of the angle which the direction of the total force of the earth's magnetic field makes with the horizontal is usually made with an apparatus known as a dip circle, in which a magnetic needle is supported so as to move freely in a vertical plane. One form of this apparatus is shown in Fig. 120.

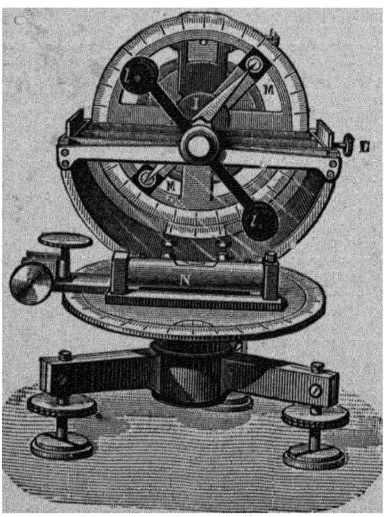


Fig. 120.

The needle, which is made of steel, is supported on a perfectly cylindrical axis between two agate knife-edges in front of a graduated circle; the angle of inclination being read off on the graduated circle by means of the lenses *l*, *l*, the vernier attached to the scale being adjusted until the needle points cover their reflections in the concave mirrors M, M, which move with the vernier.

288. *Adjustments.*—The first adjustment is to get the plane of the graduated circle accurately at right angles to the plane of the table. This is effected by means of a plumb-line, which is hung in front of the circle, and the vernier adjusted until the line covers its reflection in the two mirrors. The vernier reading indicates the position when the line joining the centres of the mirrors is exactly vertical. The spirit-level attached to the base will assist in the levelling process. The next adjustment is to get the plane of motion of the needle in the plane of the magnetic meridian. Here advantage is taken of the fact that when the plane of the needle's motion is at right angles to that of the magnetic meridian, the needle sets exactly vertical. The needle is therefore magnetized and placed in position, and the dip circle turned about its vertical axis until the readings at the ends of the needle correspond with the vertical axis as determined above, and the whole circle is then rotated through 90°, thus placing the plane of motion of the needle in the plane of the magnetic meridian.

Another cause of error is the eccentricity of the axis of rotation of the needle with respect to the centre of the circle, since, as the inclination of the needle alters, it tends to roll along the agate knife-edges. To prevent this, the axis can be caught in V-shaped notches, which raise it off the knife-edges and then replace it exactly at the centre of the circle, the screw E raising and lowering the notches.

In taking the readings, both ends of the needle are observed; the circle is then rotated through 180° about its vertical axis, and the readings repeated. The needle is reversed on its bearings, and above readings taken again. The needle is then removed and its polarity reversed by placing it in a long solenoid through which a current is flowing, and a second set of readings similar to the above taken again. The mean of several such sets of readings gives the angle of dip.

289. *Errors.*—The principal errors to be guarded against in the above determination are—

(1) Defective centreing of the needle with respect to the dial.

(2) Want of coincidence between the magnetic and geometrical axes of the needle.

(3) Eccentricity of the axis of suspension of the needle with respect to its centre of gravity.

(4) Want of level of the agate knife-edges.

Of these errors the first is eliminated by taking readings at both ends of the needle, the second by reversing the position of the needle in its pivots, the third by reversing the polarity of the needle, and the fourth by rotating the dial 180° about a vertical axis. Thus, the mean of all the readings gives the mean value of the angle of dip.

290. *Determination of the Angle of Dip by Induced Currents.* —The angle of inclination may also be determined by experiments with the earth inductor, a form such as that shown in Fig. 101 being used, which is capable of being rotated about two axes at right angles to one another. Either of the following methods may be employed.

(1) The earth inductor is set up on a horizontal table and carefully levelled, so that the axis of rotation of the coil is perpendicular to the plane of the table, this adjustment being made by means of a plumb-line and spirit-level. The coil is connected to the terminals of a ballistic galvanometer placed at some distance off, and on suddenly rotating the coil through 180° a swing, β_1, will be observed on the galvanometer. The coil, in rotating through 180°, has twice cut the horizontal component of the earth's magnetic field. Calling this **H**, and A the total effective area (the sum of the number of turns on the coil multiplied by the area of each) of the coil, we get, for the quantity of electricity, Q_1, discharged through the galvanometer—

$$Q_1 = \frac{2\mathbf{H}A}{R} \propto \sin\frac{\beta_1}{2}\left(1 + \frac{\lambda}{2}\right)$$

R being the resistance of the coil and galvanometer.

The axis of rotation of the earth inductor is now turned through 90°, by the help of the graduated circle attached to it, and the coil again suddenly rotated through 180°, this time cutting the vertical component (V) of the earth's field twice, and discharging a quantity of electricity, Q_2, through the galvanometer, which produces a swing β_2. Then—

$$Q_2 = \frac{2VA}{R} \propto \sin \frac{\beta_2}{2}\left(1 + \frac{\lambda}{2}\right)$$

$$\text{hence } \frac{H}{V} = \frac{\sin \frac{\beta_1}{2}}{\sin \frac{\beta_2}{2}}$$

$$\text{But } \frac{H}{V} = \frac{\cos \delta}{\sin \delta} = \frac{1}{\tan \delta}$$

δ = angle of dip;

$$\text{therefore } \frac{1}{\tan \delta} = \frac{\sin \frac{\beta_1}{2}}{\sin \frac{\beta_2}{2}} = \frac{\beta_1}{\beta_2}$$

when the swing of the needle is small, from which, by the help of a table of tangents, the value of δ may be found.

In making this experiment, care must be taken that the whole rotation of the coil has taken place before the ballistic needle commences to move.

291. (2) The second method of using the earth inductor to determine the angle of inclination is to arrange it so that its axis of rotation lies in the direction of the lines of force of the earth, *i.e.* so that the angle of inclination of the axis of the coil to the horizontal is the angle of dip. When this is the case, then, when connected to a ballistic galvanometer, and rotated through 180°, no swing of the galvanometer needle will be observed, since the coil in rotating does not cut any lines of force of the earth. This method may be made very sensitive by starting the ballistic galvanometer needle swinging slightly, and then timing the rotations of the coil to be synchronous with the swings of the needle, when any increase in the amplitude of the swing will denote a small quantity of electricity produced in the coil.

292. In a measurement of the angle of dip by means of the earth inductor method, the following ballistic throws were observed:—

Component cut by earth inductor.	Ballistic swing.	Mean deflection.	$\tan\delta = \frac{\beta_1}{\beta_2}$.
Vertical	206·5	206·0	2·6042
Vertical	207·0		
Vertical	206·5		
Vertical	205·0		
Horizontal ...	79·0	79·1	
Horizontal ...	80·0		
Horizontal ...	77·7		
Horizontal ...	80·0		

$$\tan\delta = 2\cdot6042, \text{ corresponds to } \delta = 69^\circ$$

This value of δ was verified by placing the axis of rotation of the earth inductor inclined to the horizontal at an angle of 69° in the direction of the magnetic meridian, and on rotating the coil no deflection of the galvanometer needle was observed.

Measurement of the Magnetic Qualities of Iron and Steel.

293. When a current of electricity flows through a solenoid, a magnetic field is produced inside it, the strength of which, expressed in terms of lines of force per square centimetre, is usually denoted by the letter H. If now, keeping the current still the same, a bar of unmagnetized soft iron is introduced into the interior of the solenoid, the number of lines of force is very greatly increased, and the iron bar becomes strongly magnetized. The number of lines of force per square centimetre in the iron bar is denoted by the letter B, and is generally termed the magnetic induction in the iron, or the flux density. If A represents the cross-sectional area of the bar, then BA = N the total flux of lines through it. The ratio of B to H in the above case, or the flux density with iron inside the solenoid to the flux density with air, the current being the same, is termed the magnetic permeability of the iron, and is denoted by the letter μ. So we have—

$$\mu = \frac{B}{H}$$

$$\text{or } \mu H = B$$

The permeability of the iron is sometimes termed its specific conductivity for lines of force, and its value is a measure of the magnetic quality of the iron. Another term sometimes employed in connection with the magnetization of iron is the "intensity of magnetization" (I), which is defined as the magnetic moment per unit volume, and the ratio between I and H is called the magnetic susceptibility (K) of the bar, the relations connecting these quantities being—

$$I = \frac{M}{V}$$

M = magnetic moment, V = volume;

$$K = \frac{I}{H}$$

$$\begin{aligned} B &= H + 4\pi I \\ &= H(1 + 4\pi K) \\ \text{And since } B &= \mu H \\ \mu &= 1 + 4\pi K \\ \text{and } K &= \frac{\mu - 1}{4\pi} \end{aligned}$$

Measurement of Permeability.

294. In order to measure the permeability of a substance, we have to measure both B and H. The methods generally employed for this may be classed under three heads—

(1) Magnetometric methods.
(2) Inductive or ballistic methods.
(3) Traction methods.

In (1) the intensity of magnetization of the bar is deduced from the effect which it exerts on a magnetic needle placed in its vicinity. The magnetic induction is measured in (2) by the quantity of electricity which it induces in a circuit surrounding the specimen, when its value is suddenly altered; whilst in (3) the magnetic induction can be calculated from the force required to overcome the magnectic attraction between two bars.

(1) *The Magnetometric Method of Measuring Permeability.*

295. In this method the specimen to be experimented upon is placed so that it can act on a magnetic needle under the influence of the earth's or other controlling force of known value. The position occupied by the bar with respect to the needle differs with different experimenters; thus, for instance, one or other of the Gauss positions may be chosen. We will, however, follow Ewing, and take what he terms the one-pole method, in which one pole is so much nearer the needle than the other that it practically produces the entire effect; a small correction, however, is made for the effect of the other pole. The relative positions of the specimen and the needle N are shown in Fig. 121. AB represents the distance between the poles of the magnet, which is slightly less than the length of the bar: the method of finding these points experimentally will be described later. Let x represent the distance NA, y the distance NB, and a the cross-sectional area of the rod in square centimetres—

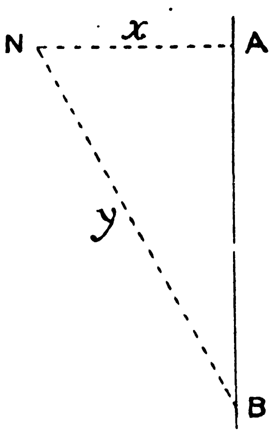


FIG. 121.

Then the force exerted by A at the needle $= \dfrac{\text{I}a}{x^2}$

and " " B " " $= -\dfrac{\text{I}a}{y^2}$

But the horizontal component of the force due to B at N ... ... $= -\dfrac{\text{I}a}{y^2} \times \dfrac{x}{y}$

Hence the total horizontal force (f) acting at N is ... ... $f = \dfrac{\text{I}a}{x^2}\left(1 - \dfrac{x^3}{y^3}\right)$

This force exerts a turning moment on the needle N, which is balanced by the moment of the controlling force. Hence, calling θ the angular deflection of the needle, and F the value of the controlling force, we have—

$$f = F \tan \theta$$

$$\text{or } \frac{Ia}{x^2}\left(1 - \frac{x^3}{y^3}\right) = F \tan \theta$$

$$\text{and } I = \frac{x^2 \, F \tan \theta}{a\left(1 - \frac{x^3}{y^3}\right)}$$

In order to calculate B from this we use the relation—

$$B = H + 4\pi I$$

and the value of the magnetizing force H, at the centre of a long solenoid such as that used in this experiment, is—

$$H = \frac{4\pi SC}{10}$$

where S is the number of turns per centimetre length of the solenoid, and C the current in ampères.

296. *Apparatus.*—In carrying out a measurement of permeability by means of the magnetometric method, the length of the specimen must be great compared with its diameter, in order to reduce the demagnetizing effect of the poles at the ends of the bar to a minimum. Thus, for instance, if a short bar of soft iron is strongly magnetized, and the magnetizing force removed, the specimen will be found to have lost all its magnetism, owing to the large demagnetizing action of the ends; if, however, the bar has a length about four hundred times its diameter, this demagnetizing action is very much diminished. In order to experiment on specimens whose length is four hundred times their diameter, we have practically to work with wires, and this may be looked on as one of the disadvantages of this method of measuring permeability, since there is reason to believe that wires may not behave in the same way as large masses of metal.

The wire to be experimented upon is placed inside a long solenoid, so that the coils project a little way beyond the ends of the specimen, in order that the magnetizing force acting on the wire may be sensibly uniform all along its length. The solenoid may be wound on a thin tube of some non-magnetic substance, which fits closely over the specimen. The arrangement of the apparatus is shown in Fig. 122. S represents

the magnetizing solenoid with the iron wire inside, clamped in a vertical position near the magnetometer needle N; in series with S is the compensating coil C, which neutralizes the magnetic effect of the solenoid on the needle N, a reversing key K, variable liquid resistance R, secondary battery B, and current measuring galvanometer G. A second solenoid S′, in

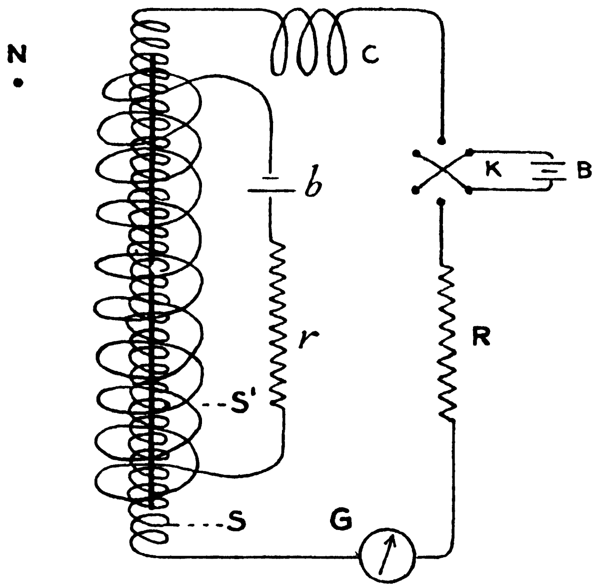


FIG. 122.

series with a single secondary cell *b*, and large resistance *r*, is wound over the solenoid S in order to neutralize the inductive action of the vertical component of the earth's magnetic force on the specimen.

The magnetometer needle N may be the same as that employed in the measurement of **H** (see par. 273), the deflections being read by means of a lamp and scale, it being borne in mind that the angle θ in the formula above is the angular deflection of the needle, and not that of the spot of light. In order to determine the value of the controlling force F acting on the needle, which may be due to the earth alone, or to the earth and a permanent magnet, the method of Gauss

employed in the determination of **H** may be used; or, if **H** has been determined for any particular part of the laboratory, the periodic time of swing of the magnetometer needle should be taken at that place, and then again, when under the controlling force to be used during the experiment. If **H** represents the strength of the earth's field, and T_1 the periodic time of swing in it, while T_2 represents the periodic time of swing under the controlling force F, then—

$$\frac{F}{\mathbf{H}} = \frac{T_1^2}{T_2^2}$$

$$\text{and } F = \mathbf{H}\frac{T_1^2}{T_2^2}$$

The reversing key K employed must be of a form which will allow very rapid reversals to be made, and may conveniently take the form of a commutator mounted so as to rotate on an axle, contact being made by springs or brushes.

The variable resistance R must be so arranged that its value can be altered without breaking circuit, and for this reason a liquid resistance, such as that described in par. 22, is very suitable; or else a carbon resistance may be used (see Fig. 14).

The current measuring instrument G may be a sensitive ammeter which has been carefully calibrated throughout its scale, or a sensitive low resistance galvanometer whose absolute calibration curve has been obtained. In any case the instrument must be so placed that its indications are not affected by the other pieces of apparatus; also all the wires leading to the various pieces of apparatus should be twisted together, to neutralize any magnetic effect they might have.

297. *Adjustments.*—The first adjustment is to place the iron wire so that its upper pole is in a line with the needle of the magnetometer; it is therefore placed into the solenoid, and a current sent through the coils; the solenoid, with the wire inside, is then raised or lowered until the maximum deflection is obtained on the magnetometer; it is then clamped firmly in this position. The iron rod is now removed, a strong current sent through the solenoid, and the compensating coil C adjusted relatively to the needle until it neutralizes the

magnetic effect of the latter on the needle, this being the case when the magnetometer needle returns to zero; this adjustment will be perfect for all currents, but should be made for the strongest current likely to be employed.

298. As it is desirable to start with the bar in a perfectly unmagnetized condition, any residual magnetism must be removed. To do this the wire is adjusted inside the solenoid till it produces its maximum effect at the needle, and starting with the strongest current in the solenoid, the reversing key is worked rapidly, so as to magnetize the bar, first in one direction and then in the opposite direction, at the same time the resistance R is gradually increased until the current is diminished to zero. This treatment should completely wipe out any residual effects of previous magnetizations. It may be found, however, that the bar is always left feebly magnetized in one direction, this being due to the inductive effect of the earth's magnetic field acting on the bar. To neutralize this, a current is sent through the solenoid S′ so as to produce a magnetic field in opposition to that of the earth's, the current being regulated until, after the demagnetizing process described above has been carried out, the bar is left perfectly unmagnetized, as evidenced by the magnetometer needle setting at zero.

299. *Method of Measurement.*—A very weak current is now sent through the solenoid S, and readings taken on the magnetometer and galvanometer G. The current is gradually increased by slowly diminishing the resistance R, and a set of simultaneous readings on the two instruments taken until the current has reached its maximum value. It is then gradually diminished to zero, reversed, and carried to a maximum in the opposite direction, and finally reduced again to zero. The readings may be tabulated thus—

Galvanometer reading.	Current in ampères.	H	Magnetometer deflection.	I	B	μ

Great care must be taken during the above operations not to shake the specimen, since, especially in the case of soft iron, a slight jar is sufficient to seriously affect its magnetic condition.

From the above table a curve should be plotted, having values of B for ordinates, and H for abscissæ; also a curve with μ for ordinates and B for abscissæ.

300. *Correction for the End Effects.*—In the above it has been assumed that the value of H, as calculated from the dimensions of the solenoid and the current flowing in it, represents the actual magnetizing force acting on the specimen; this assumption, however, is not strictly true, on account of the demagnetizing action of the poles at the ends of the bar, which tend to set up lines of force in the reverse direction to those in the solenoid, and therefore diminish the magnetizing force inside the specimen. If we call H′ the effective magnetizing force acting on the specimen, then it has been shown by Ewing[1] that in the case of ellipsoids—

$$H' = H - NI$$

The following table gives values of N for various sizes of specimen:—

Ratio $\frac{\text{length}}{\text{diameter}}$	N
50	0·01817
100	0·00540
200	0.00157
300	0·00075
400	0·00045
500	0·00030

The corrections may be made for each value of H employed, and a separate column of values of H′ added, or the correction may be made graphically on the curve connecting B and H.

301. The following example of the method is taken from Ewing's "Experimental Researches in Magnetism."[2] The specimen consisted of annealed iron wire 0·077 cm. diameter and

[1] See *Electrician*, vol. xxiv. pp. 313, 341.
[2] *Phil. Trans.*, 1885 p. 539.

30·5 cm. long. The curve plotted from these readings is shown in Fig. 123.

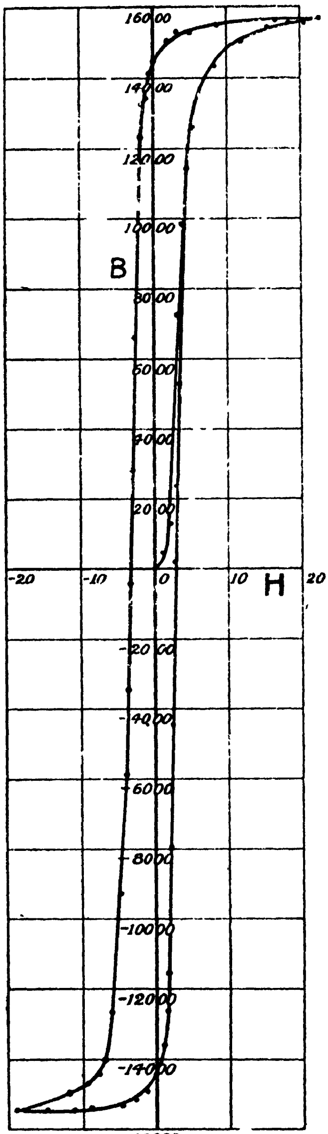


FIG. 123.

Galvanometer reading.	H	Magnetometer reading.	B	Galvanometer reading.	H	Magnetometer reading.	B
0	0·00	0	0	141	−4·99	−278	−11490
9	0·32	1	41	162	−5·73	−306	−12650
24	0·85	4	165	211	−7·47	−338	−14400
39	1·38	10	413	261	−9·23	−352	−14700
59	2·18	28	1460	312	−11·05	−373	−14920
79	2·86	89	3680	411	−18·90	−376	−15550
99	3·50	175	7230	311	−11·01	−375·5	−15530
119	4·21	239	9880	211	−7·47	−375	−15500
139	4·92	279	11540	111	−3·93	−372	−15380
159	5·63	304	12570	61	−2·16	−368	−15210
189	6·69	327	13520	23	−0·81	−361	−14930
239	8·46	348	14390	11·5	−0·41	−358	−14800
289	10·23	359	14840	0	0·00	−352	−14550
342	12·11	365	15090	9	0·32	−348	−14390
441	15·61	373	15420	19	0·67	−339	−14010
574	20·32	378	15630	29	1·02	−327	−13520
629	22·27	380	15710	39	1·38	−306	−12650
464	16·42	379	15670	49	1·73	−274	−11330
239	8·46	375	15560	59	2·09	−195	−8060
139	4·92	372	15380	69	2·44	−103	−4260
89	3·15	369	15270	79	2·80	4	165
39	1·38	363	15010	86	3·04	58	2400
0	−0·00	350	14470	99	3·50	130	5370
11·5	−0·41	342	14140	114	4·03	199	8230
23	−0·81	329	13360	140	4·96	279	11530
31	−1·10	318	13150	190	6·72	332	13720
41	−1·45	295	12200	240	8·50	351	14510
51	−1·80	253	10460	291	10·30	362	14970
62	−2·20	166	6860	339	12·00	368	15210
71	−2·51	70	2890	340	12·04	374	15460
81	−2·87	−12	−496	579	12·50	381	15750
91	−3·22	−83	−3430	630	22·30	383	15830
101	−3·50	−142	−5870	0	0·00	353	14590
121	−4·28	−226	−9340				

Values of μ.

B	μ	B	μ
41	128	12570	2230
165	194	13520	2020
413	299	14390	1700
1460	670	14840	1450
3680	1310	15090	1250
7230	2070	15420	990
9880	2350	15630	770
11540	2350	15710	705

(2) *Inductive or Ballistic Methods of Measuring Permeability.*

302. In the inductive methods of measuring μ, advantage is taken of the fact that if the magnetic flux through any closed conducting circuit is suddenly altered, a current is induced in the circuit during the change, the total quantity of electricity induced being proportional to the change in the magnetic flux. By placing a ballistic galvanometer in series with the conducting circuit, we can measure the quantity of electricity induced, and so get a measure of the change in the number of the lines of force. Also, since we can calculate the magnetizing force H, required to produce this induction change, in terms of the number of spirals in the magnetizing solenoid and the current flowing in them, we have all the necessary data from which to obtain μ.

The specimen to be experimented upon by this method may take the form of either a straight bar or a closed ring. If the former is chosen, a correction must be applied for the demagnetizing action of the ends similar to that employed in the magnetometric method just described; the advantage of the ring form is that, not having any poles, there is no such demagnetizing effect. The ring has usually a circular cross-section, but this is not necessary; in fact, in the case of rings of small diameter, it is better that the section should rather approach that of a rectangle, since, on account of the difference between the external and internal circumferences of the ring, the magnetizing force will be stronger on the inner side where the spirals of the magnetizing coil lie closest together. For this reason, the diameter of the ring should always be large compared with its thickness, and may in the case of a ring of circular cross-section be from forty to fifty times its cross-sectional diameter.

The ring to be experimented upon should have a perfectly uniform cross-section all round, and after its sectional area and mean circumference have been carefully measured, must be uniformly overwound with the magnetizing coil. A small secondary or induction coil of fine wire is wound over a part of the magnetizing coil. The numbers of turns on the two coils must be known.

The apparatus is then arranged as shown in Fig. 124. The primary or magnetizing coil P is connected through a reversing key, K, to a secondary battery, B, current-measuring galvanometer, G, and variable liquid or carbon resistance, R. The secondary or induction coil S is connected in series with the ballistic galvanometer BG and the earth inductor E. Care must be taken in setting up the apparatus that the various

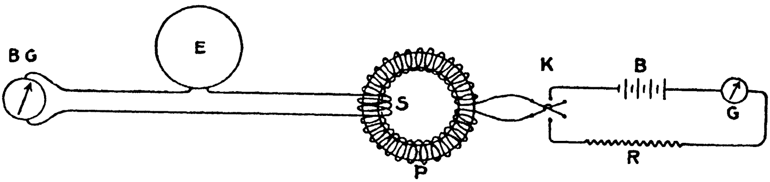


FIG. 124.

instruments have no direct effect on one another. The ballistic galvanometer should be supplied with a damping coil, in order to be able to quickly bring the needle to rest.

303. The method of experiment is what is termed the step-by-step method, the current in the magnetizing spiral being increased suddenly by small amounts, by diminishing the resistance R, and the ballistic throws observed corresponding to the increments in the lines of force in the ring, the total number of lines of force in the ring at any instant being proportional to the algebraic sum of the ballistic throws from the commencement up to that point. When the current in the magnetizing coils has been increased to its maximum value, it is then reduced by steps to zero, reversed, carried to a negative maximum and again reduced to zero, thus carrying the specimen through a complete cycle of magnetization.

The ballistic throws may be calibrated directly, in terms of values of B, by means of the earth inductor. If the earth inductor has a total effective area, A, and is arranged so as to cut the horizontal component of the earth's lines of force, then, when it is rotated through 180°, let it produce a throw, θ, on the ballistic galvanometer, then—

$$\theta \propto 2\text{HA}$$

and $\frac{2\text{HA}}{\theta}$ will represent the number of lines cut per scale-division

throw of the ballistic needle. Hence, if θ_1 be any throw due to an increment of current in the magnetizing coils, and a be the total effective area of the induction coil S—

$$\frac{2H\Lambda\theta_1}{a\theta}$$

will represent the induction B.

It is necessary here to point out a small source of error in the above calculation. The value of B required is the induction in lines per square centimetre *in the iron*. What has been calculated above is the induction in lines per square centimetre in the induction coil *s*. Now, the area of the induction coil, if wound outside the primary coil, will be slightly larger than the area of the iron core. Generally speaking, this correction is so small as to be negligible, but if a correction is to be applied, then from the total flux of lines through the test-coil must be subtracted the number of lines flowing through the air-space between the test-coil and the iron core; or, calling this area α, and the magnetizing force due to the current in the primary H, Hα lines of force must be subtracted from the total flux of lines, and the remainder, when divided by a, will give the true value of B. The magnetizing force H is calculated exactly as in the last case, or—

$$H = \frac{4\pi n C}{10}$$

where n is the number of turns per centimetre on the ring, and C is the current in ampères. The same method of measuring the current C may be employed as before.

304. One of the disadvantages of the ring method of measuring μ is, that if an error is made in reading the ballistic throw at any point, it is carried through all the subsequent readings, and errors of this kind are therefore cumulative; also, the method will give no indication of slow creeping up of the induction, as has been observed by Ewing using the magnetometric method, so that the value of B may really be somewhat larger than will be calculated from the sum of the ballistic throws.

305. The following example of the ballistic method of measuring μ is due to Ewing:—[1]

[1] *Phil. Trans.*, 1885, p. 530.

The ring was welded out of a piece of soft annealed iron wire.

Diameter of wire forming the ring = 0·248 cm.
Area of section of iron = 0·0483 sq. cm.
Mean radius of ring = 5·0 cm.
Mean circumference of ring = 31·4 cm.
Number of turns in magnetizing coil = 474
Number of turns in secondary coil = 167
Area of earth inductor = 1216 sq. cm.
Number of turns in earth inductor = 10
Earth's force cut by the earth inductor = 0·34 C.G.S. units
Ballistic throw on turning over earth inductor = 42·9 scale div.

From the above it follows that the change in B per scale-division ballistic throw = 23·89.

H	Ballistic throw.	Sum of throws.	B	μ
0·13	1·1	1·1	26	—
0·26	1·1	2·2	53	—
0·30	0·5	2·7	65	—
0·40	0·8	3·5	84	—
0·53	1·0	4·5	107	—
0·71	2·1	6·6	158	—
0·93	2·9	9·5	227	—
1·31	3·9	13·4	320	245
1·69	9·2	22·6	540	320
1·89	6·9	29·5	705	370
2·78	77·5	107·0	2560	920
3·36	78·7	185·7	4440	1320
4·01	82·0	267·7	6400	1600
4·95	91·5	359·2	8580	1740
5·86	57·0	416·2	9940	1700
7·20	57·0	473·2	11300	1570
8·10	23·5	496·7	11870	1460
9·14	24·0	520·7	12440	1360
7·83	−4·4	516·3	12330	—
6·21	−6·7	509·6	12170	—
4·75	−7·1	502·5	12000	—
2·70	−14·0	488·5	11670	—
0·00	−33·2	455·3	10880	—

Fig. 125 shows the curve plotted from these readings.

306. An interesting modification of the ring method just described, due to Messrs. Evershed and Vignoles,[1] gets rid of

[1] *Electrician*, vol. xxvii. pp. 49, 77.

the cumulative errors which may occur, and at the same time subjects the specimen to treatment more closely approximating

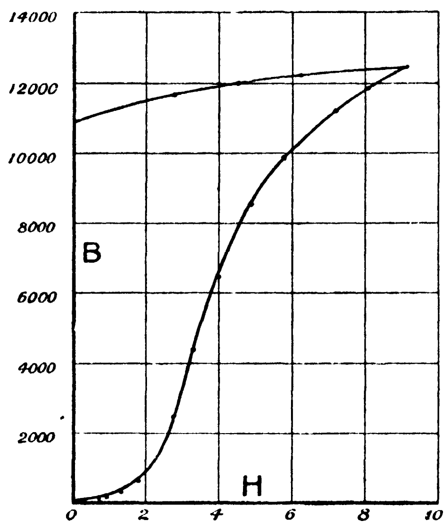


FIG. 125.

to what would occur in actual practice; also, the method admits of any reading being repeated at any time, and as often as desired, under exactly similar conditions, which is not possible with the other methods.

In Evershed and Vignoles' method the ring of iron is overwound in opposite directions with two magnetizing spirals, one having twice the number of turns that are on the other; the connections are shown in Fig. 126, where, for the sake of clearness, the coils are shown as wound on separate parts of the ring, but in reality both coils completely surround the ring, as does also the induction coil T. The magnetizing coils S_1 and S_2 are each connected to reversing keys K_1 and K_2, variable resistances R_1 and R_2, ammeters A_1 and A_2, and batteries B_1 and B_2, the test-coil T being connected by a pair of long leads to the ballistic galvanometer BG.

The method of working is as follows. A current is always kept flowing in coil S_1 of sufficient strength to produce the

maximum amount of magnetization desired in one direction; if now, the current in S_1 being kept on, a current of the same strength is sent through the coil S_2, since the number of spirals on S_2 is twice that on S_1, and the direction of winding is opposite to that on S_1, the iron will be magnetized to the same extent but in the opposite direction, so that by keeping a given current flowing in S_1 the iron may be carried through a com-

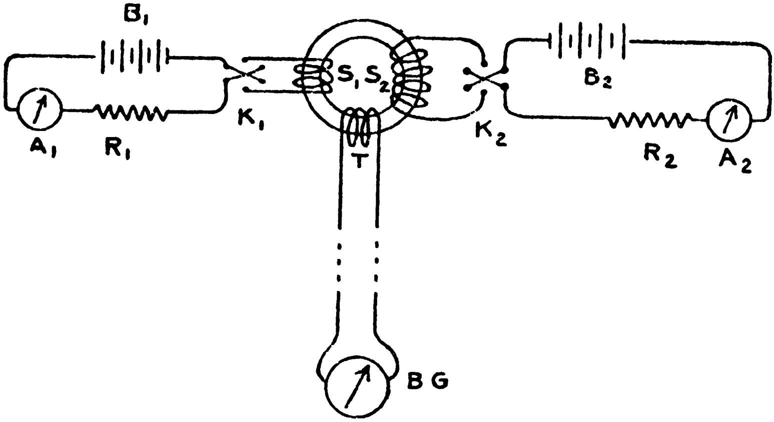


FIG. 126.

plete cycle of magnetization by simply starting and stopping an equal current in S_2. This is one of the principal features of the method, the specimen being carried through a complete cycle of magnetization between each pair of readings.

In order to obtain data for the BH curve, currents of various strengths are started in S_2 and the corresponding ballistic throw noted, the cyclic process being carried through between each pair of readings. The falling curve of magnetization is obtained in the same way by reversing the currents in both coils.

It will also be seen that any reading on the curve can at once be reproduced. The rings employed in the measurement were about 13 cm. diameter and 1 sq. cm. cross-sectional area. The ballistic galvanometer was calibrated by means of a current inductor.

307. The following examples of the method are taken from their paper:—

WROUGHT IRON ANNEALED.

Rising curve.		Falling curve.	
H	B	H	B
−15·75	−12500	22·0	12900
−14·5	−12350	21·5	12900
−11·4	−11800	18·4	12700
−9·3	−11400	16·95	12600
−7·9	−11000	14·9	12300
−5·8	−10400	13·1	12100
−4·4	−9800	12·8	12000
0·35	−7200	11·4	11800
1·0	−6600	8·75	11200
1·4	−5900	7·35	10800
1·9	−5000	5·2	10000
2·5	−3400	3·7	9500
2·6	−2900	0·7	7900
3·3	−250	0·2	7600
4·0	1800	−0·7	7000
4·7	3400	−3·7	−250
6·3	5800	−4·0	−1400
8·2	7900	−5·1	−4800
10·3	9100	−7·2	−7200
14·0	10900	−10·3	−9400
19·3	12400	−11·5	−9800

For curve, see Fig. 127.

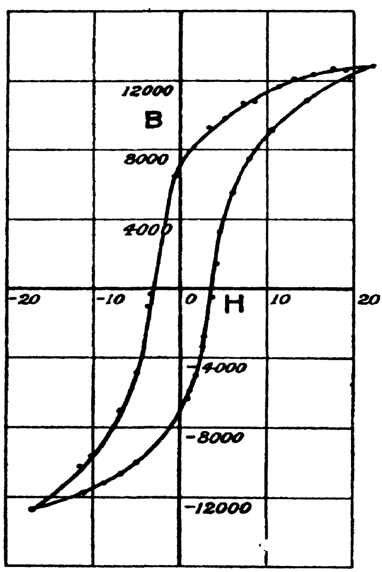


FIG. 127.

308. Magnet Bar Iron.

Average length, 35 cm. ; sectional area, 0·877 sq. cm.

Rising curve.		Falling curve.	
H	B	H	B
−39·5	−15550	48·3	15800
−29·6	−15100	38·5	15400
−19·75	−14500	28·6	15050
−9·9	−13400	19·75	14400
−0·4	−7750	9·9	13200
1·0	−3700	−0·6	−6900
3·0	4400	−2·0	−2000
9·9	11800	−4·9	−8900
20·7	14000	−9·9	−12150
29·6	15000	−29·6	−14950
39·5	15600	−39·5	−15550
48·3	15800	—	—

For curve, see Fig. 128.

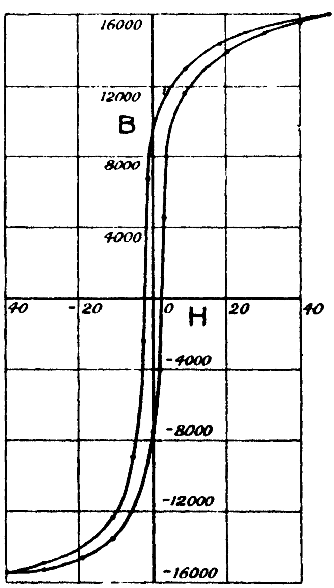


Fig. 128.

309. CAST IRON.

Average length, 32·6 cm.; sectional area, 0·88 cm.

Rising curve.		Falling curve.	
H	B	H	B
−32·3	−5400	43·7	5700
−24·5	−5100	33·2	5300
−15·7	−4600	24·9	4750
−6·6	−3970	16·6	4300
−1·7	−3360	8·3	3660
3·3	−2200	0·0	2550
7·5	500	−8·3	−1000
10·4	2000	−16·6	−3400
15·7	3400	−24·9	−4500
20·7	4100	−33·2	−5160
28·8	5000	—	—
37·3	5440	—	—
43·5	5700	—	—

For curve, see Fig. 129.

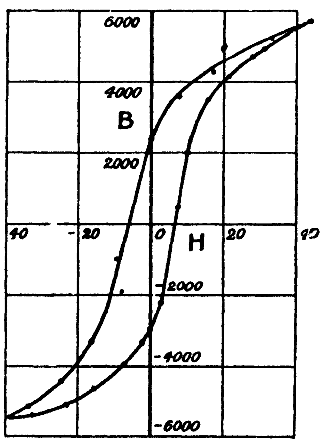


FIG. 129.

310. *Hopkinson's Method.*—One of the drawbacks to the ring method of measuring the magnetic properties of iron, is that a ring of each different sample must be carefully overwound with a magnetizing coil, this being somewhat laborious when many

samples are to be examined. This drawback is to some extent overcome in the ballistic method due to Dr. J. Hopkinson,[1] and usually known as the divided-bar method.

The apparatus consists of a block of soft annealed wrought iron, 18″ long by 6½″ wide and 2″ thick, out of the middle of which a rectangular groove is cut, into which are placed the magnetizing coils. The sample to be tested is in the form of a rod, 1·265 cm. diameter, carefully turned to slide through the holes at the ends of the block, and cut in two at the centre, the ends being faced up true where they meet in the space between the two magnetizing coils (see Fig. 130). To one end of

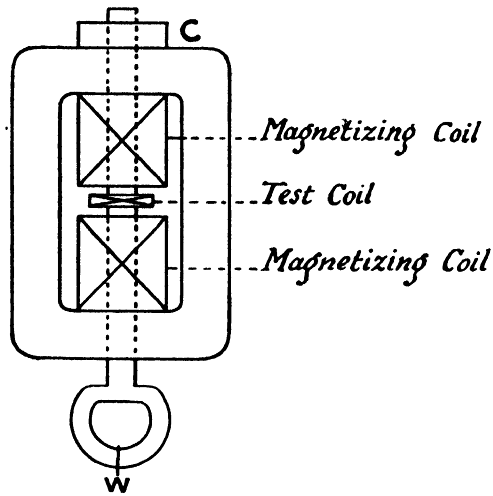


FIG. 130.

one of the rods a collar, C, is attached, in order to prevent it from slipping through the hole; to the other end, which passes through the hole at the opposite end of the wrought iron yoke, a ring to which weights can be fixed is attached. The test-coil encircles the bars at the point where they butt up against one another, and is held by means of stretched india-rubber bands. In order to make a measurement, a magnetizing current of known strength is sent through the coils; it is then broken and the rods pulled apart by the weight W; as

[1] *Phil. Trans.*, 1885.

the rod slips out of the test-coil, the indiarubber bands pull the latter right out of the rectangular space, thus causing it to cut all the lines of force in the iron bar. The reading on the ballistic galvanometer is then noted, and the process repeated for various magnetizing currents. In this way the entire cycle may be gone through, and various specimens examined rapidly. The object of having a heavy wrought-iron yoke or rectangle, is to provide a return path for the lines of force, so that the magnetic circuit is closed, and there are no free poles and consequent demagnetizing effect. The magnetic circuit is not so good, however, as in the ring form, on account of the various joints in it, and moreover it is extremely difficult to apply any correction, since so much depends on the nature of the joints. In one case Ewing found that it worked out to the same amount as that for the end effects on a rod 150 diameters long. The method cannot, therefore, be employed in the accurate testing of good soft iron, but for hard iron and steel the errors will, of course, not be so serious.

311. The two following tables are taken from Hopkinson's results.

ANNEALED WROUGHT IRON.

H	B	μ
1·66	5000	3000
4·0	9000	2250
5·0	10000	2000
6·5	11000	1692
8·5	12000	1412
12·0	13000	1083
17·0	14000	823
28·5	15000	526
50·0	16000	320
105·0	17000	161
200·0	18000	90
350·0	19000	54
666·0	20000	30

GREY CAST IRON.

H	B	μ
5	4000	800
10	5000	500
21·5	6000	279
42	7000	133
80	8000	100
127	9000	71
188	10000	53
292	11000	37

311*a*. *Ewing's Double Bar-and-Yoke Method.*—This method, devised by Professor Ewing, is by far the most satisfactory method of accurately testing the magnetic qualities of short iron bars. The specimen bars are fitted with adjustable yokes, so

that the length of the magnetic circuit may be altered at will. Two BH curves are then obtained for two different magnetic circuit lengths; from these two, each of which is affected by the air-gap effect at the joints, a third curve may be drawn in which this is entirely eliminated. The arrangement of the rods in the yokes is represented in Fig. 130*a*.

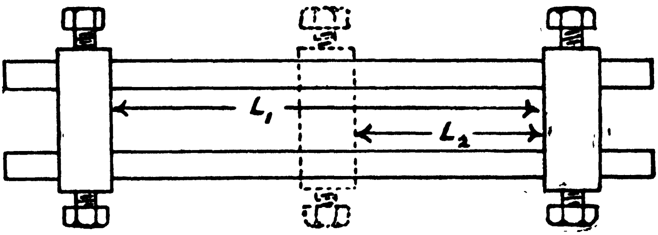


FIG. 130*a*.

Two sets of magnetizing coils and test-coils wound on formers are made, each with the same number of turns so as to fit the test-bars for two fixed lengths L_1 and L_2; these are not shown in the figure. The total magnetizing force, MMF, due to each coil may be regarded as performing two functions. (*a*) magnetizing the bar of length L_1, and (*b*) driving the lines of force through the yokes and air-gaps. Obviously it is the first of these we want to know; the second may be regarded as being constant whatever the length of the bars may be. If, therefore, we call H_1 the true magnetizing force per unit length in the rod L_1, and h the magnetizing force required for the yokes and air-gaps, we have—

$$\text{MMF} = H_1 L_1 + h$$

If now we shorten the bars to length L_2, then

$$\text{MMF}^1 = H_2 L_2 + h$$

$$\text{Hence } H_1 = \frac{\text{MMF}}{L_1} - \frac{h}{L_1} \quad \ldots \quad (1)$$

$$\text{and } H_2 = \frac{\text{MMF}^1}{L_2} - \frac{h}{L_2}$$

If now we take two values of MMF such that the value of B is the same for each, it follows that the value $H_1 = H_2$.

$$\text{Hence } \frac{\text{MMF}^1}{L_1} - \frac{\text{MMF}}{L_2} = \frac{h}{L_2} - \frac{h}{L_1}$$

Also generally $L_1 = 2L_2$,

$$\text{Hence } \frac{MMF^1}{L_1} - \frac{MMF}{L_2} = \frac{h}{L_1} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If now the values of $\frac{MMF}{L_1}$ and $\frac{MMF^1}{L_2}$, which are the apparent magnetizing forces, are plotted against the values of the flux density B, it is obvious from (1) and (2) that by taking the horizontal distance between the two curves and subtracting it from the corresponding abscissæ of the $\frac{MMF}{L_1}$ B curve, we shall get the true BH curve for the iron. The following example, taken from a paper by Professor Ewing,[1] will illustrate the method.

The bars were $\frac{3}{8}''$ diameter of Lowmoor iron. The yokes were rectangular blocks 2 cms. by 2·5 cms., and the distance between the centres of the bars was 2·2 cms. The two lengths L_1 and L_2 were 12·56 cms. and 6·28 cms. The following table gives the values of the apparent magnetizing forces for the various values of B :—

Length of bar L_1.		Length of bar L_2.	
MMF	B	MMF¹	B
2·05	1,650	2·05	1,360
3·07	4,600	3·07	3,160
4·10	7,440	4·10	5,660
5·12	9,460	5·12	7,920
6·15	10,750	6·15	9,560
8·20	12,280	8·20	11,590
10·25	13,200	10 25	12,700
15·40	14,410	15·40	14,160
20·0	14,950	20·0	14,750
30·0	15,700	30·0	15,550
50·0	16,550	50·0	16,470
70·0	17,150	70·0	17,050
120·0	18,100	120·0	17,960

In Fig. 130*b* curve (1) is drawn from the values MMF and B, curve (2) from MMF¹ and B, and curve (3), which is the true BH curve for the sample of iron, is obtained by taking,

[1] *Elect.*, vol. 38, p. 110.

as abscissæ for each value of B, the value MMF − (MMF^1 − MMF).

311*b*. *Ewing's Permeability Bridge.*—This instrument has been devised by Professor Ewing in order to enable a rapid and accurate measurement of permeability to be made by com-

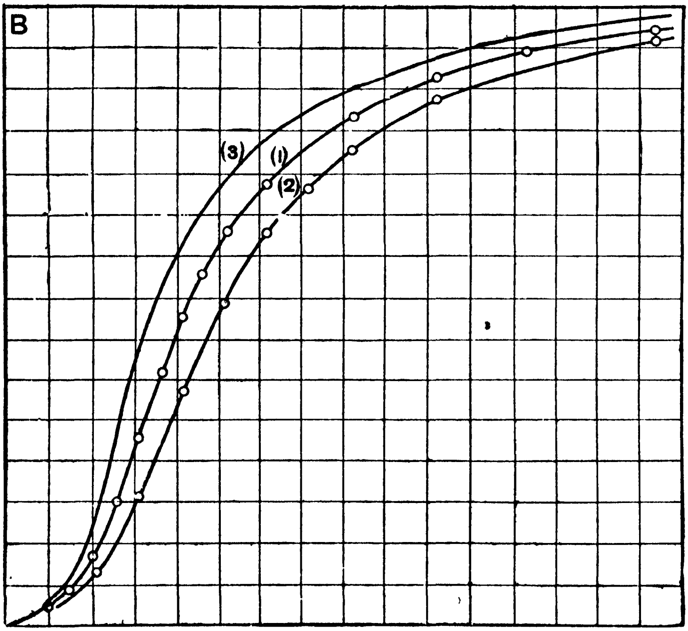


FIG. 130*b*.

paring the test-bar with one whose BH curve has been carefully determined by the bar-and-yoke method. The bars (test and standard) are placed side by side with their ends fixed in two yokes of soft iron, as in the last method. The yokes, however, are provided with long soft iron horns which almost meet over the top of the bars, and in the air-space between them is placed a small magnetic needle. The magnetizing coils on the bars are in series, so that the same current circulates through each; but the number of turns on the test-bar magnetizing coil can be altered by one turn at a time from 1 up to 210 turns by means of dial switches, which introduce or remove an equivalent resistance when a coil is cut out or

switched in the magnetizing circuit. The method of test consists in adjusting the magnetizing force on the test-bar until the flux density B has the same value in each of the bars, this being indicated by the needle in the air-space between the soft iron horns remaining at rest when the magnetizing current is reversed, since there will be no magnetic flux from yoke to yoke. The number of turns on the magnetizing coil of the standard bar is known, being either 100 or 50. From this H may be found for the standard, and thence, by the aid of the

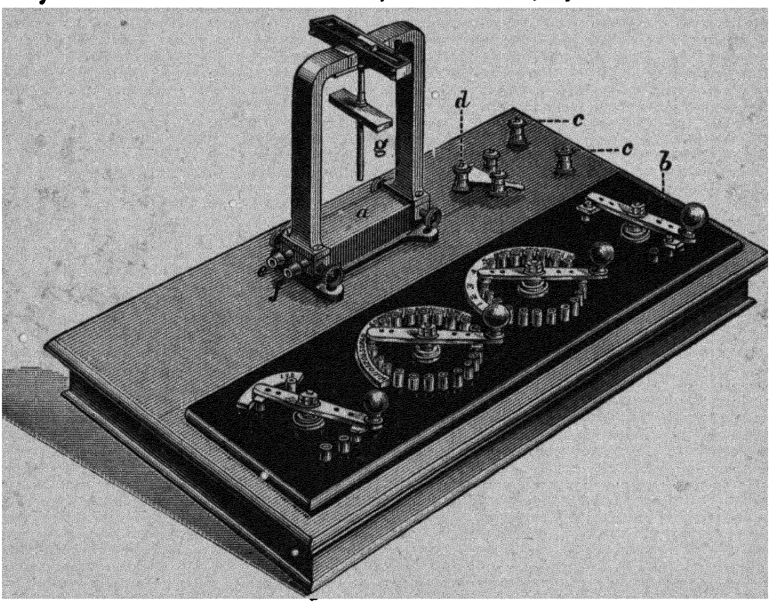


FIG. 130*c*.

magnetization curve of the standard bar supplied, also the value of B. The H required to produce this same value of B in the test-bar may now be found from the number of turns on the magnetizing coil of the test-bar, and the current strength. The length of each bar is 4π cms., so that for the test-coil $H = \frac{NC}{n}$, where N is the number of turns on that coil, and C the current in amperes, and n the number of turns on the standard bar magnetizing coil. Fig. 130*c* represents the form taken by the apparatus.

(3) *Traction Method of measuring Permeability.*

312. The last method of measuring the magnetic quality of a sample of iron to be described is the traction method, in which the induction is calculated from the force of attraction between two magnets. If A represents the area of contact of the magnets in square centimetres, and B the magnetic induction in lines per square centimetre, it can be shown[1] that the force in dynes required to separate them is—

$$W \text{ (dynes)} = \frac{B^2 A}{8\pi}$$

$$\text{or } W \text{ (grammes)} = \frac{B^2 A}{981 \times 8 \times \pi}$$

$$\text{Hence } B = \sqrt{\frac{W \times 981 \times 8\pi}{A}}$$

313. Measurements of induction by this method have been carried out by Bidwell[2] both on divided bars and rings. The ring was made of very soft charcoal iron rod, 0·64 cm. thick, the joint being carefully welded. It was then turned in a lathe to a uniform circular section, and when finished its external diameter was 8 cm. and its diameter of cross-section 0·482 cm. The ring was sawn into two equal portions, and the cut faces ground so as to be perfectly flat. Two brass tubes, 0·5 cm. long, were fitted over the ends so that they projected 1 mm. beyond the faces, in order to serve as guides in placing the two parts of the ring together. Each half-ring was then overwound with magnetizing coils, the total number of turns for the whole ring being 1929. The mean radius of the ring was 3·76 cm., and the mean circumference 23·6 cm. The upper half of the ring was rigidly fixed, while to the lower part a scale-pan was attached, the apparatus being connected as shown in Fig. 131. The battery B is connected in series with the magnetizing coils on the ring, the galvanometer or ammeter G, variable resistance R, and key K. The weights required in the scale-pan in order

[1] "Mathematical Theory of Electricity and Magnetism" (J. J. Thomson), p. 73.

[2] *Pro. Roy. Soc.*, vol. x'

to separate the two portions of the ring when various magnetizing currents are flowing in the coils, are noted, and the corresponding value of B calculated as above.

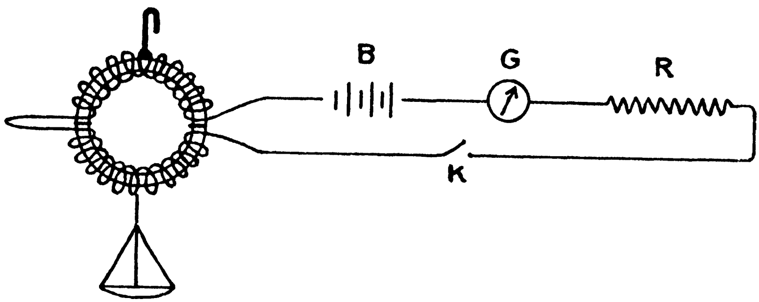


FIG. 131.

314. The following data were obtained for the above specimen :—

Grammes per sq. cm. area.	H	B	μ
2210	3·9	7390	1899·1
5400	10·3	11550	1121·4
9680	40·0	15460	386·4
12170	115·0	17330	150·7
13810	208·0	18470	88·8
15130	427·0	19330	45·3
15905	585·0	19820	33·9

314*a*. A simple workshop method of measuring the magnetic induction in a bar of iron by the traction method has been introduced by Professor Ewing. In this method the test-bar is placed across the poles of an electro-magnet, the magnetizing current in which has been adjusted to a known value. The force required to detach one end of the rod from one of the poles is measured by sliding a weight along a steelyard which is attached to one end of test-bar. The steelyard scale is graduated directly in values of B. Fig. 131*a* represents the apparatus.

The bars, which must all be the same size, are 4″ long

and $\frac{1}{4}''$ diameter, and since the pull measured is exerted between the cylindrical side of the test-bar and the slightly convex electro-magnetic pole, the contact surface has always a definite

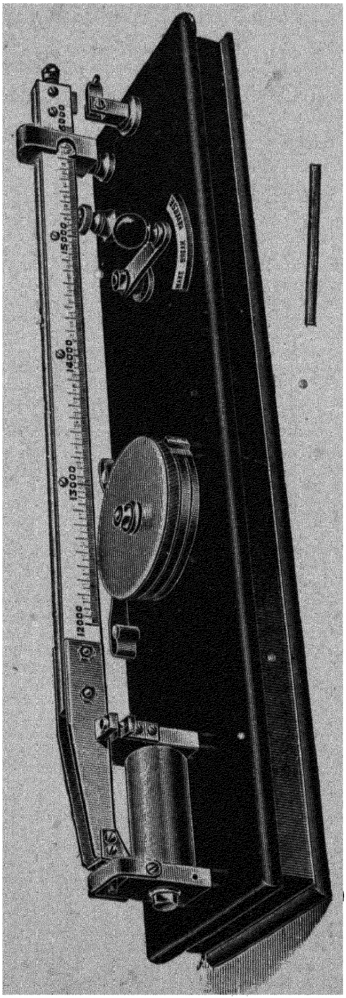


FIG. 131*a*

character, and no facing of ends is required. To adjust the magnetizing current to the proper strength, a standard bar is supplied, in which the value of B is known when H = 20

This bar is placed in the magnetic balance, and the current is adjusted until the bar is just pulled off the electro-magnetic pole when the sliding weight on the steelyard is opposite this value of B. The bars to be tested are now placed in, and keeping the current strength the same, the sliding weight is moved over the steelyard until the bars are just detached, and the corresponding value of B read off. A reversing key is included in the apparatus to enable the bars to be taken through a complete cycle of magnetization, and thus remove the residual effects of previous magnetizations.

The table below, given by Professor Ewing, enables the values of B at any other value of H to be approximately calculated when the value at H = 20 is known.

Value of H.	Value of B.				
20	12,000	13,000	14,000	15,000	16,000
25	12,700	13,700	14,600	15,500	16,350
30	13,300	14,200	15,100	15,900	16,600
40	14,200	15,000	15,700	16,400	17,000
50	14,900	15,600	16,300	16,900	17,400

In the above table a sample having an induction density of 16,000 for H = 20 may be regarded as exceptionally good, whilst that having only 12,000 is distinctly poor. A value of about 15,000 should be obtained with a good average specimen.

Magnetic Hysteresis.

315. In plotting out the rising and falling curves of magnetic induction in iron and steel, it will have been observed that the falling curve does not coincide with the rising one, and when a sample of iron is carried through a complete cycle of magnetization, the curves connecting the values of B and H enclose an S-shaped space. This phenomenon of the lagging of the induction behind the magnetizing force has received from Professor Ewing the name of hysteresis. One important consequence of magnetic hysteresis is that it involves a dissipation of energy, or, in other words, a certain amount of energy is

absorbed by the sample under test during a cyclic magnetization and goes to raise its temperature by a very small amount. The quantity of energy so absorbed can be calculated from the cyclic curves connecting the magnetizing force H with the intensity of magnetisation I and also from the BH cyclic curves.

Suppose that in any experiment I is increased by a very small amount, dI, the work done per unit volume of the material is that required to bring unit volume of the material with a magnetization dI from an infinite distance to that point. This work equals the product dI into the mean value of the force H, that is, $dw = HdI$. Now, if in Fig. 132 SP represents the value of dI on the magnetization curve OP, then the area $SPQQ^1$ represents the value HdI or the work done in increasing the magnetization by an amount dI. If the falling curve of magnetization coincided with the curve OP, then all this work would be recovered, and there would be no hysteresis loss. But the falling curve follows the path PR, hence the amount of work recovered is only the area P^1PQQ^1. Therefore the difference (area $SPQQ^1$ − area P^1PQQ^1) equals area P^1SP, and is the measure of the energy absorbed by the specimen.

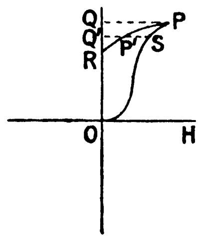


FIG. 132.

If instead of a small part of the curve SP we take the complete cycle, then the area enclosed by it represents the work done during the magnetization, and by plotting the curves of H against I, both being expressed in C.G.S. units, the area gives us the work done in ergs per cubic centimetre on the material.

In order to calculate the work done during the cycle from the BH curve, we employ the relation—

$$B = H + 4\pi I$$
$$\text{Hence } dB = dH + 4\pi dI$$
$$\text{and } HdB = HdH + 4\pi HdI$$
$$\text{therefore } HdI = \frac{HdB - HdH}{4\pi}$$

But for a cyclic process $\int HdH$ vanishes;

$$\text{therefore} \int H\,dI = \frac{1}{4\pi}\int H\,dB$$

and the work done per cubic centimetre on the material is $\frac{1}{4\pi}$ times the area of the BH curve. In general, in plotting the curve the scale of B or I will be many times that of H, thus in Fig. 128 the scale of B is 200 times the scale of H, so that the value of one small square is $\frac{2 \times 200}{4\pi} = \frac{400}{4\pi} = \frac{100}{\pi}$ ergs, and if there are n small squares enclosed by the curve $\frac{100}{\pi}n =$ ergs per cubic centimetre absorbed by the iron.

Thus in Fig. 128 the value of $\int H\,dI$ or $\frac{1}{4\pi}\int H\,dB$ is 16,300 ergs per cubic centimetre.

316. The following table, taken from Ewing's researches, give the approximate hysteresis loss for various materials per complete cycle of magnetization :—

Material.	Loss in ergs per c.c.
Iron, soft annealed	10000
Iron, annealed wrought	16000
Iron, cast	30000 to 40000
Mild steel	40000 to 60000
Pianoforte steel wire, annealed	94000
Pianoforte steel wire, commercial	116000
Pianoforte steel wire, glass-hard	117000
Chrome steel (1 % Cr.), annealed	65000
Chrome steel (1 % Cr.), oil-hardened	167000
Tungsten steel (3·4 % Tungsten, 0·5 % C, 0·6 % Mn.)	216800
Nickel, annealed	11000
Nickel, hardened	25000

316*a*. In the above the hysteresis loss is expressed in ergs per cubic centimetre per cycle of magnetization. It is, however, more usual for commercial purposes to express it in watts per pound of the iron at a certain frequency of magnetization cycle, usually 100 cycles per second, and with a certain flux density B, usually 2500.

The conversion from ergs per cubic centimetre per cycle to watts per pound is given below. Let e = ergs per cubic centimetre per cycle, $\sim$ = frequency, and Δ = specific gravity of iron.

$$\text{Then } e\sim = \text{ergs per cubic centimetre per second}$$

$$\frac{e\sim}{10^7} = \text{watts per cubic centimetre}$$

$$\frac{e\sim}{10^7 \times \cdot 0022 \times \Delta} = \text{watts per pound}$$

In the case of soft iron the loss is about 0·25 watt per pound at $\sim = 100$ and $B = 2500$, although with very good transformer strip it may be obtained as low as 0·15 watt per pound.

316*b*. *Steinmetz' Law.*—From a close examination of the results of many experiments on the amount of hysteresis loss in iron at various values of B, Steinmetz[1] arrived at the following empirical relationship between these quantities:—

$$e = KB^{1 \cdot 6}$$

K being a constant, the value of which depends on the nature of the iron, and is known as the "Steinmetz coefficient." The values given by Steinmetz are appended in the table below.

STEINMETZ COEFFICIENTS.

Material.	K
Very soft iron wire ...	0·002
Soft sheet iron	0·0024 to 0·003
Thick sheet iron	0·0033
Sheet iron for transformers	0·0045
Annealed cast steel ...	0·008
Soft steel	0·0090
Cast steel	0·0120
Cast iron	0·0162
Hard cast steel	0·025

[1] *Jour. Amer. Inst. Elect. Eng.*, Jan., 1892.

316*c*. The Steinmetz law enables us, knowing the hysteresis loss at any given value of B, to readily calculate what it would be at any other value. Thus, if the value of the hysteresis loss is W watts per pound at a flux density of value B_1, and it is desired

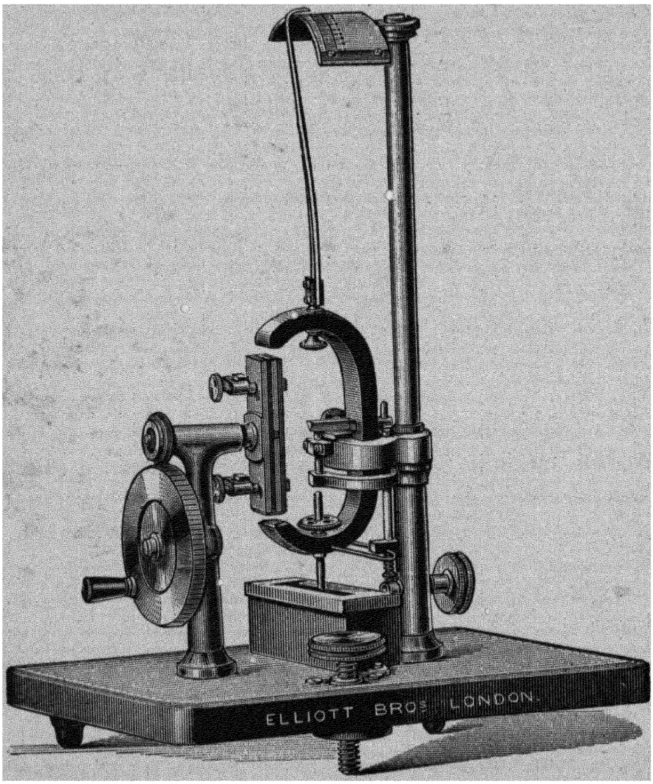


FIG. 132*a*.

to calculate it at a value B = 2500, then—

$$\text{Loss in watts per pound at B } 2500 = \frac{W \times B_1^{1 \cdot 6}}{2500^{1 \cdot 6}}$$

316*d*. An extremely simple apparatus for commercially

measuring the hysteresis loss in samples of sheet iron, due to Professor Ewing, is illustrated in Fig. 132*a*.

In this instrument the work done in causing the specimen to rotate between the poles of a permanent magnet is measured; this will be proportional to the hysteresis loss.

The test strips are made up into a small bundle 3″ long and $\frac{5}{8}$″ wide, and are placed in a carrier which is rotated between the poles of a horseshoe magnet, this latter being supported on knife edges and weighted. As the test-piece rotates, the magnet tends to follow it, and in consequence is permanently deflected through a certain angle. This angle is proportional to the hysteresis loss. The instrument is calibrated by means of two strips of known hysteresis loss being placed in the carrier, and the respective deflections they produce being plotted against their known hysteresis losses. From this curve the hysteresis loss for any deflection may be obtained.

316*e*. *Watt Meter Method of measuring Hysteresis Loss.*—The main objection to the hysteresis tester is that it is not an absolute method of measuring hysteresis loss, but one of comparison with the loss in a standard bar which must be assumed to remain constant. In the Watt meter method this objection is removed, and the bar to be tested may be subjected to exactly the same conditions of frequency of alternation of magnetic force as it will experience in actual practice. The method consists in measuring by means of a Watt meter the power expended in a magnetizing coil, first without, and afterwards with, the iron test-bar as a core, the difference of the two values being obviously the power lost due to hysteresis in the iron. The main difficulty in carrying out the measurement consists, not in measuring the power lost, but in estimating the value of the flux density in the bar to which the power lost corresponds, especially as the induction density varies in value from point to point along the bar. This difficulty has been solved by Dr. Fleming, who has shown [1] that there is always a definite point in such a bar, which he calls "the effective point,"

[1] *Elect.*, vol. 40, p. 587.

where the value of B is such that its 1·6 power is proportional to the true mean hysteresis loss of the whole bar. This value of B he terms the "effective value," and is found always to occur at a distance which is 0·22 of the whole length of the bar from either end.

To measure the value of B at this point a test-coil of known number of turns is slipped on the bar till it is 0·22 of the length from one end, and the E.M.F. induced in it is measured on an electrostatic voltmeter. If N = number of turns on the test-coil, S = cross-sectional area of the bar, n = frequency, and E = E.M.F. in volts,

$$\text{then } 10^8 \text{ E} = 4\text{NSB}n$$

$$\text{and B} = \frac{10^8\text{E}}{4\text{NS}n}$$

If the hysteresis loss at any other value of B is required, it may be calculated from the Steinmetz law, provided the value of B is below 9000.

The Watt meter employed must be wound to read at very low power factors. In Fleming's experiments the fixed coil consisted of 90 turns of No. 16 copper wire in five layers wound on a wooden block, whilst the movable coil had 20 turns of No. 24 copper wire, and was suspended by a bifilar suspension consisting of two silver wires 0·002″ diameter, 37 cms. long, and 3 mm. apart; this coil had also in series with it a non-inductive resistance of 1000 ohms. The deflection of the coil was read off by a mirror, lamp, and scale in the usual way.

Effects of Temperature on Magnets.

317. In permanent steel magnets the increase in temperature produces a very slight diminution in the magnetic moment of the magnet. This alteration may be determined experimentally, and expressed as a temperature coefficient. Thus

calling M_t the magnetic moment at a temperature t° C., M_0 the magnetic moment at 0°, and β the temperature coefficient of the magnet, we may write—

$$M_t = M_0 (1 - \beta t)$$

In order to determine the value of β for a particular magnet, it should be placed in a vessel of oil or water, the temperature of which may be altered by heating. A magnetometer needle is then arranged near it, so as to be in one of the Gauss positions, say the A tangent position, then $\frac{M}{H} = d^3 \tan \alpha$, where M is the magnetic moment, H the controlling force at the needle, d the distance of the magnet from the needle, and α the angle of deflection of the needle. The temperature of the vessel must be steadied for some time before the reading is taken, in order to allow the magnet to acquire the temperature of the liquid, which may be taken by a thermometer. The deflections for various temperatures are taken, and the coefficient calculated from them. If α_1 and α_2 represent two deflections for temperatures t_1° C. and t_2° C., then—

$$M_{t_1} \propto \tan \alpha_1 \text{ and } M_{t_2} \propto \tan \alpha_2$$

$$\text{therefore } \frac{\tan \alpha_1}{\tan \alpha_2} = \frac{1 - \beta t_1}{1 - \beta t_2}$$

$$\text{and } \beta = \frac{\tan \alpha_2 - \tan \alpha_1}{\tan \alpha_2 t_1^\circ - \tan \alpha_1 t_2^\circ}$$

In good steel magnets β varies from 0·0003 to 0·001.

318. The effect of temperature on the permeability of soft iron and steel is very interesting, and depends to a certain extent on the magnetizing force acting on the material. In order to investigate this, a ring of the material should be made and wound with magnetizing and secondary coils, which should be carefully insulated with asbestos paper, since the temperatures to which the specimen is to be carried would destroy ordinary insulation. The ring is then placed in an iron box, which can be heated by a gas furnace, the temperature of the ring being estimated from the resistance of the secondary coil, which is wound completely round the ring and should

be next the specimen, the magnetizing coil being wound over it. The temperature coefficient of the wire forming the secondary or test-coil should be previously determined by a separate experiment. The BH curve is then taken by the ballistic method, but before and after each reading on the ballistic galvanometer the resistance of the secondary coil is taken on a Wheatstone bridge, and the temperature calculated from the mean resistance, and the known temperature coefficient. Curves should be taken for both strong and weak magnetizing fields at various temperatures, since the variation of the permeability greatly depends on this.

319. The following tables, taken from curves given by Hopkinson, will give an idea of the variation of μ in the case of soft iron :—

MAGNETIZING FORCE, 0·3 C.G.S. UNIT.

Temperature.	μ
20° C.	500
480°	700
580°	900
730°	2500
750°	3800
770°	7700
775°	11000
785°	1

320. MAGNETIZING FORCE, 45 C.G.S. UNITS.

Temperature.	μ
20° C.	300
370°	300
570°	260
670°	250
720°	240
750	200
785°	1

321. In strong magnetic fields the value of μ steadily decreases as the temperature increases, and then drops suddenly

to unity at 785° C.; whilst in the weak fields it first increases slowly, then very suddenly, until it reaches the value 11,000, when it drops as suddenly again to unity at 785° C. At this temperature the iron becomes non-magnetic. In steel bars in weak and strong fields, the behaviour is similar, the temperature at which the steel loses its magnetic properties being about 700° C.

322. When the bar is allowed to cool down it is found to regain its magnetic properties at about the above temperatures. A most interesting physical change also occurs in the iron at this temperature, which was first observed by Professor Barrett,[1] and investigated by Dr. Hopkinson.[2] If a piece of iron is heated to a temperature above 785° C., and then allowed slowly to cool, it is observed to get perceptibly hotter and brighter red at a temperature about 785° C., this being obviously due to a disengagement of heat at this temperature. This phenomenon has received the name of recalescence.

In order to investigate it, a rod of steel is over-wound with an asbestos insulated coil of copper wire, and then jacketed with a number of layers of asbestos paper. The copper coil, the temperature variation of resistance of which should be accurately known, is connected up to a Wheatstone bridge and its resistance measured. The steel bar is then heated in a gas furnace to bright redness and allowed to cool slowly, time-measurements of the resistance of the wire being taken; on plotting a curve of temperature of the bar, as calculated from the resistance of the copper coil, against time as abscissæ, a curve of the shape shown in Fig. 133 is obtained, the temperature falling to 680° C. and then rising to 712° C., when it again falls. From this curve, also, the amount of heat disengaged may be estimated by finding the time corresponding to the distance between the two straight parts of the curve; during this time the steel bar has been giving out heat without fall or temperature, and, comparing this with the time-rate of fall of temperature of the curve after the bend, we can therefore estimate the relative quantities of heat given out in the two cases,

[1] *Phil Mag*, vol. xv., April, 1883.
[2] *Roy. Soc. Pro.*, March, 1889.

and if the latter quantity can be measured, knowing the mass of the metal and its specific heat at that temperature, the quantity disengaged during recalescence may be calculated. Thus Hopkinson showed that in a case where the rate of

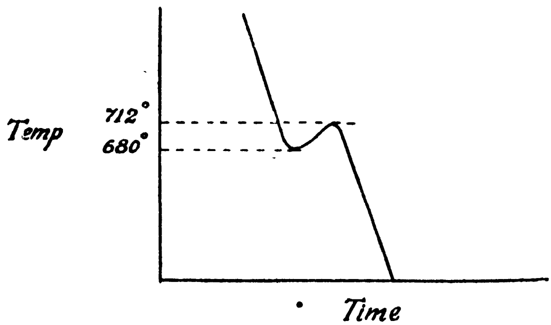


FIG. 133.

fall of temperature was 0·21° C. per second after the bend, the quantity of heat disengaged was 173 times the heat liberated during a fall of 1° C.

SELF AND MUTUAL INDUCTION.

323. When a current of electricity is started in a circuit, it does not at once reach its steady value, but gradually increases from zero to its maximum value, the time required for the growth of the current depending on the nature of the circuit. When a current is established in a circuit it sets up a magnetic field, but this magnetic field, according to Lenz's law, reacts inductively on the circuit during the time it is being set up, and induces a back E.M.F., which tends to stop the current. This is known as the back E.M.F. of self-induction, and its magnitude depends on the strength of the magnetic field set up by the given current; it therefore depends on the magnetic permeability of the field round the circuit, and will be greater the larger the value of μ.

In the case of a coil of wire consisting of n turns carrying a current C and producing a total flux of N lines of force, the total cutting of lines by the coil itself when the current is started in it, assuming all the lines to cut the coil, will be Nn,

and the ratio of this to the current C is defined as the coefficient of self-induction (L) of the coil, or the "inductance."

The coefficient of self-induction of a circuit may also be defined as the ratio of the back E.M.F. to the time rate of change of the current in the coil, or as the electro-kinetic energy possessed by the current. All these definitions lead to the same result when the permeability of the circuit is constant; but when μ varies with different values of B, as in the case of iron, they lead to different results. The following methods to be described of measuring the coefficient of self-induction are based on the first definition.

324. The practical unit of self-induction is called the "secohm," or sometimes the "henry," and corresponds to 10^9 C.G.S. units, *i.e.* to the cutting of 10^9 magnetic lines when unit current (10 ampères) is sent through the circuit.

If two coils are placed close together, and a current is started in one, the magnetic field set up will induce an E.M.F. in the other. The total number of lines cut by the second coil, when unit current is started in the first, is defined as the coefficient of mutual induction (M) between them. This quantity, like the other, also depends on the nature of the circuits and on the magnetic permeability of the medium between them.

325. *Lord Rayleigh's Method of measuring L.*—In the following method, due to Lord Rayleigh,[1] the coil whose coefficient of self-induction it is required to measure, is connected up to three other coils which are non-inductive, so as to form a Wheatstone bridge (see Fig. 134). A, B, and S represent the three non-inductive resistances, R the resistance whose self-induction is required, BG a ballistic galvanometer, B_1 a battery, and K and K_1 break-circuit keys.

The resistances are adjusted so as to give a balance for steady currents, *i.e.* so that on closing the battery key and *then* closing the galvanometer key no deflection is obtained on BG. Great care must be taken to get this balance exact; the final adjustment may be made by slipping a bare wire under the terminal at the junction of R and B until balance is obtained.

[1] *Trans. Roy. Soc.*, 1882.

If now the galvanometer circuit is kept closed and the battery circuit is opened, a ballistic swing, θ, will be obtained on BG, due to the E.M.F. of self-induction in the coil R.

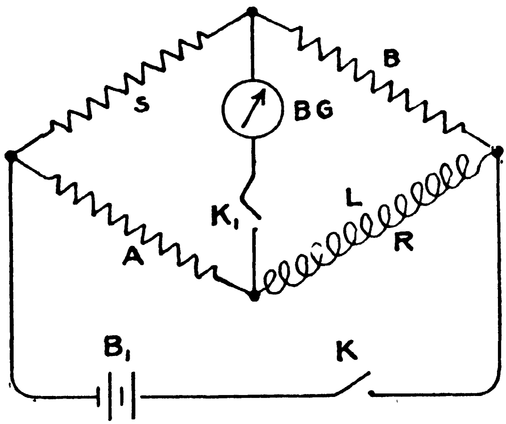


FIG. 134.

The quantity of electricity so discharged may be calculated as follows. An E.M.F. of value e in the coil will produce a current equal to, say, κe, but—

$$e = -\frac{d(\mathrm{L}c)}{dt}$$

where c = current in the coil. The current g in the galvanometer will therefore be—

$$g = -\kappa\frac{d(\mathrm{L}c)}{dt}$$

and the total quantity of electricity, Q, discharged through the galvanometer is—

$$\begin{aligned} \mathrm{Q} &= \int g dt \\ &= -\kappa\int\frac{d(\mathrm{L}c)dt}{dt} \\ &= -\kappa\mathrm{LC} \end{aligned}$$

where C = value of c when it has reached a steady value. But from the theory of the ballistic galvanometer (see par. 224)—

$$Q = \frac{HT}{2\pi G}\, 2 \sin \frac{6}{2}\left(1 + \frac{\lambda}{2}\right)$$

$$\text{Hence } \kappa LC = \frac{HT}{2\pi G}\, 2 \sin \frac{\theta}{2}\left(1 + \frac{\lambda}{2}\right)$$

$$\text{and } L = \frac{HT}{2\pi G\kappa C}\, 2 \sin \frac{\theta}{2}\left(1 + \frac{\lambda}{2}\right)$$

The steady current-balance of the bridge is now upset by inserting in series with the coil R a very small resistance of known value, r. This will produce a steady deflection, δ, on the galvanometer, due to a small current, g', flowing through it. Also, on account of the smallness of r, we may assume that the current flowing in the coil R is still C.

$$\text{Hence } g' = \kappa r C$$

$$\text{But } g' = \frac{H}{G} \tan \delta$$

where H and G have the same meanings as in the ballistic galvanometer formula—

$$\text{therefore } \kappa r C = \frac{H}{G} \tan \delta$$

$$\text{and } \kappa C = \frac{H \tan \delta}{Gr}$$

putting this value for κC in the equation for L, we get—

$$L = \frac{Tr}{2\pi} \frac{2 \sin \frac{\theta}{2}\left(1 + \frac{\lambda}{2}\right)}{\tan \delta}$$

If the periodic time of swing of the needle (T) is taken in seconds, and r in absolute units of resistance, the value of L will be obtained in C.G.S. units. If r is taken in ohms, L will be expressed in henrys. The angles θ and δ are the angular deflections of the needle, not the spot of light, and the logarithmic decrement is determined as described in par. 227.

The resistance r may either be a small coil of known resistance, or a piece of straight wire of known length, area, and specific resistance.

326. In order to measure the coefficient of self-induction of a coil of wire wound on a bobbin with non-magnetic core, it was connected up as a Wheatstone bridge to three other coils of nearly the same resistance, but wound non-inductively, and a steady current balance was obtained by slipping the bare end of one of the coils through the terminal until an exact balance was obtained. The galvanometer circuit being closed, the battery circuit was suddenly opened and a ballistic swing was obtained; the mean of six such swings was 52·4 scale-divisions. A standard 0·100-ohm resistance was now inserted in series with the coil whose self-induction was required, and a steady deflection of 48 scale-divisions was obtained. The periodic time of swing and logarithmic decrement of the galvanometer needle were then determined by the methods previously described, and were $T = 20{\cdot}15$ seconds and $\lambda = 0{\cdot}101$ respectively. The mirror was 1 metre distant from the scale, which was graduated in millimetres.

$$\text{Hence } \tan 2\theta = \frac{52{\cdot}4}{1000} = 0{\cdot}0524$$

$$\text{and } \therefore\ \theta = 3^\circ$$

$$\text{and } \sin\frac{\theta}{2} = 0{\cdot}0131$$

$$\text{also } \tan 2\delta = \frac{48}{1000} = 0{\cdot}048$$

$$\therefore\ \delta = 1{\cdot}38^\circ$$

$$\text{and } \tan\delta = 0{\cdot}024$$

$$\text{Therefore } L = \frac{20{\cdot}15 \times 0{\cdot}100 \times 10^9}{2 \times 3{\cdot}142} \times \frac{2 \times 0{\cdot}0131\,(1 + 0{\cdot}050)}{0{\cdot}024}$$

$$= 3{\cdot}68 \times 10^8 \text{ C.G.S. units}$$

$$= 0{\cdot}368 \text{ henrys}$$

327. *Maxwell's Method of measuring L.*—In Maxwell's method[1] the coefficient of self-induction of a coil is determined in terms of the capacity of a condenser. The coil R is, as in Lord Rayleigh's method, connected up to three non-inductive

[1] See Maxwell's "Electricity and Magnetism," vol. ii.

coils, so as to form a Wheatstone bridge (see Fig. 135). The arms A, B, and S represent the non-inductive resistances, across one of which is placed the condenser of capacity K., A

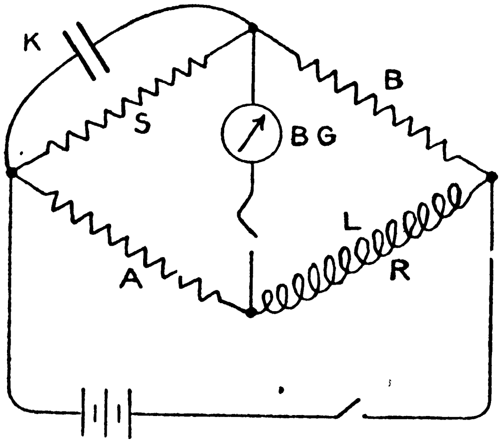


FIG. 135.

balance is first obtained for steady currents, when the following relation holds :—

$$RS = AB$$

A balance must now be obtained for both steady and transient currents, *i.e.* no deflection or swing will be obtained on the galvanometer BG, whether the battery key is pressed and *then* the galvanometer key, or *vice versâ.* There is only one possible arrangement of resistances which will fulfil this condition, and therefore the method is an exceedingly tedious one, since the experimenter must go on trying various combinations until he discovers the right one.

When a balance is obtained for both steady and transient currents, the rate of rise of the potential at each of the galvanometer terminals must be the same, since there is no current flowing through it. The rate of rise of potential of the terminal connected to the condenser is proportional to the time-constant of the condenser, *i.e.* it is proportional to KS, where K is the capacity of the condenser; also the rate of rise of

potential at the terminal connected to the coil R is proportional to its time-constant, which is $\frac{L}{R}$[1]

$$\text{Hence} \frac{L}{R} = KS$$

$$\text{and } L = KRS$$

If K, R, and S are all in C.G.S. units, L will be expressed in C.G.S. units.

This method of Maxwell's, although it leads to a very simple result, is so difficult to carry out in practice, on account of the adjustments required to get the balance, that it is almost useless in the laboratory, so that the following modifications are recommended.

328. *Rimington's Modification of Maxwell's Method of measuring L.*[2]—The arrangement of the apparatus in this modification is the same as in Maxwell's method, with the

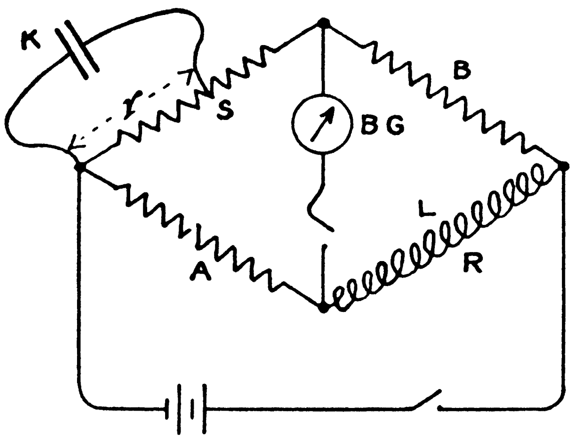


FIG. 136.

exception that the condenser, instead of being connected across the whole of the resistance S, is only connected across a part of it, *r* (see Fig. 136). The bridge is adjusted so as to balance for steady currents. It is then tested for balance with transient

[1] See Fleming's "Alternate Current Transformer," vol. i. p. 101.
[2] *Phil. Mag.*, vol. xxiv., July, 1887.

currents by closing the galvanometer circuit before the battery circuit; should a throw be obtained on the galvanometer, the resistance r must be altered, without altering the total resistance S of the arm, until a balance is obtained for transient, as well as for steady currents.

If we call the steady value of the current flowing in A and R x, and y that in B and S, also G the galvanometer resistance, then the quantity of electricity discharged through the galvanometer due to the self-induction of the coil R is—

$$Q = \frac{Lx}{S + R + \dfrac{G(A+S)}{G+A+S}} \times \frac{A+S}{G+A+S}$$

$$= \frac{LxS}{S(B+R) + G(S+B)} \text{ since } AB = SR$$

Also, the quantity which passes through the galvanometer due to the discharging of the condenser K is—

$$Q = Kyr\frac{r}{A + S + \dfrac{G(B+R)}{G+B+R}} \times \frac{B+R}{G+B+R}$$

$$= \frac{Kyr^2B}{S(B+R) + G(S+B)} \text{ since } AB = SR$$

But these quantities flow through the galvanometer in opposite directions, and, since there is no ballistic swing, they must be equal to one another.

$$\text{Hence } \frac{LxS}{S(B+R) + G(S+B)} = \frac{Kyr^2B}{S(B+R) + G(S+B)}$$

$$\text{and } LxS = Kyr^2B$$

$$\text{or } L = \frac{Kyr^2B}{xS}$$

$$\text{But } \frac{y}{x} = \frac{R}{B}$$

$$\text{therefore } L = \frac{Kr^2R}{S}$$

In order to simplify the adjustment, part of the resistance S

should include a straight calibrated wire on which a sliding contact from the condenser may make contact.

In carrying out the method, the values of R and G are usually fixed, and it can be shown mathematically[1] that for the most sensitive arrangement the following relation should obtain, viz.—

$$B^2 = \frac{RS(G + R)(r + S)}{(G + S)(r + S)}$$

329. *Sumpner's Method of measuring L.*—The connections in Dr. Sumpner's modification of Maxwell's method are shown in Fig. 137. S and B represent non-inductive resistances of

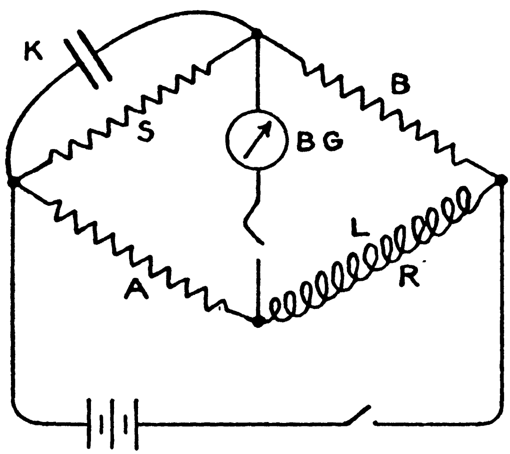


FIG. 137.

about 10,000 ohms each; A is a non-inductive resistance the value of which can be altered; BG is a ballistic galvanometer, also of about 10,000 ohms resistance; and K is a condenser of one-third microfarad capacity.

The resistances are first adjusted until the bridge is balanced for steady currents, the final adjustments being made, as in the other methods, by slipping a piece of bare wire, in series with R, under the terminal until the galvanometer gives no deflection when its circuit is completed *after* the battery circuit.

[1] Gray's "Absolute Measurements," vol. ii. p. 493.

Keeping the galvanometer circuit closed, the battery circuit is now opened and a ballistic throw may be obtained of value θ_1. This swing represents a quantity of electricity which is proportional to the difference between the time-constants of the coil and condenser, and is—

$$\frac{L}{R} - KS \propto \theta_1$$

One of the condenser terminals is now disconnected and the above operation repeated, the throw θ_2 this time being proportional to the time-constant of the coil alone, since the condenser is out of circuit;

$$\text{therefore } \theta_2 \propto \frac{L}{R}$$

Hence, from these we get—

$$\frac{\theta_1}{\theta_2} = \frac{\frac{L}{R} - KS}{\frac{L}{R}}$$

$$\text{and } L = KRS\frac{\theta_2}{\theta_2 - \theta_1}$$

This method is very much simpler to carry out than Maxwell's method, and is found to give the most satisfactory results, when—

$$B = S,\ B + R = 2G,\ KRS = 2L$$

(G = galvanometer resistance)

330. It must be borne in mind that in all these measurements of L the value obtained has no definite meaning, if the permeability of the medium is not constant, unless the permeability corresponding to the particular conditions under which the test was made is stated. This is well illustrated in the following measurements by Sumpner, of the coefficient of self-induction of an electro-magnet with an iron core for various magnetizing currents. The resistance of A in the following was 5 ohms, and $K = \frac{1}{3}$ microfarad, S, B, and G being 10,000 ohms each.

Current in R (ampères)	θ_2	θ_1	L (henrys)
0·220	688	508·0	0·0637
0·200	627	462·0	0·0634
0·147	444	321·0	0·0606
0·110	367	261·0	0·0577
0·073	230	160·0	0·0547
0·055	164	111·0	0·0517
0·020	49·5	29·75	0·0419
0·016	38	22·7	0·0414
0·0105	24	13·5	0·0381
0·0092	20·5	11·0	0·0360

The value of L when the iron core was removed was 0·0028 henry.

331. *Joubert's Method of measuring Self-Induction.*—The following modification of Joubert's method of measuring the coefficient of self-induction, due to Professor S. P. Thompson,[1] depends on the measurement of the apparent increase of resistance of a coil possessing self-induction when traversed by an alternating current. It can easily be shown[2] that, when a coil of resistance R ohms (as measured by a steady current on a bridge) is traversed by an alternating current which is a simple sine function of the time, its apparent resistance R′ is expressed by the following relation—

$$R^1 = \sqrt{R^2 + 4\pi^2 n^2 L^2}$$

where n is the frequency of alternation of the current, and L the coefficient of self-induction of the coil; the expression $\sqrt{R^2 + 4\pi^2 n^2 L^2}$ being known as the "impedance" of the coil. In order to produce an alternating current of known frequency, Professor Thompson employs a tuning-fork of known pitch as the interrupter of the current in the primary of an induction coil, the current in the secondary then alternating according to a simple sine function.

The following diagram (Fig. 138) shows the method of arranging the apparatus. The coil R is that of which the coefficient of self-induction is required, P, Q, and S are non-inductively wound bridge coils. The key K_1 is arranged so

[1] *Jour. Elect. Eng.*, vol. xvi. p. 385.
[2] "Alternate Current Transformer," Fleming, vol. i. p. 105.

that either the battery B_1 can be connected to the bridge or the secondary S_1 of the induction coil. Key K_2 connects either the bridge galvanometer G or the telephone T across the other diagonal. The battery B_2 is connected in series with the primary coil P and the tuning-fork interrupter F,

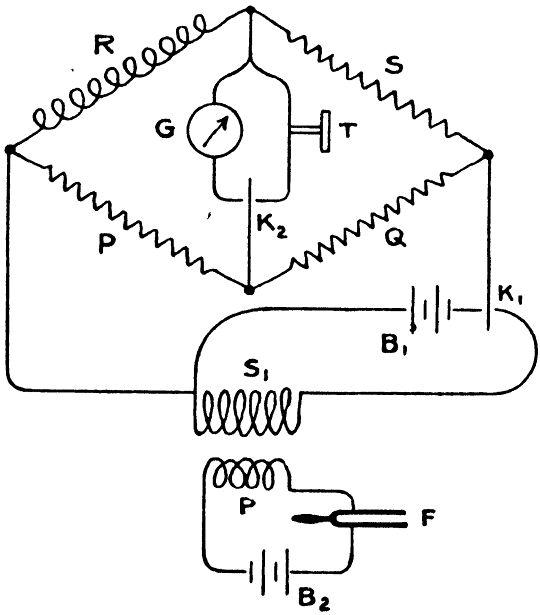


FIG. 138.

which may be arranged so as to be vibrated electro-magnetically by inserting an electro-magnet in series with P between the prongs of the fork.

In making a measurement, K_1 is connected to the battery B_1, and K_2 to G, this allows a steady current to flow through the bridge, and by adjusting the coils a balance is obtained in the ordinary way, and the resistance to steady currents calculated. Key K_1 is now connected to S_1, and K_2 to T, and the current in the induction coil started. A series of alternating currents now flow through the resistances, the frequency of which will be the same as that of the fork F. Balance must now be obtained by adjusting the non-inductive coils until there is a

minimum of sound in the telephone, and the apparent resistance, R_1, calculated. Then—

$$L = \frac{\sqrt{R_1^2 - R^2}}{2\pi n}$$

The value of n may be determined experimentally as described in vol. i. p. 103.

332. The following example will illustrate the method. A solenoid was placed in the bridge, and its resistance to steady currents was found to be 4·06 ohms. The tuning-fork in the primary circuit of the induction coil had a frequency of 100 vibrations per second; and the apparent resistance to alternating currents, as found by the arrangement of resistances required to give minimum sound in the telephone, was 13·8 ohms.

$$\text{Hence } L = \frac{\sqrt{R_1^2 - R^2}}{2\pi n}$$

$$= \frac{\sqrt{(13{\cdot}8)^2 - (4{\cdot}06)^2}}{2 \times 3{\cdot}142 \times 100}$$

$$= 0{\cdot}0209 \text{ henrys}$$

COMPARISON OF COEFFICIENTS OF SELF-INDUCTION.

333. If a coil of variable known self-induction is constructed, comparisons of the coefficients of self-induction of coils may be made on the Wheatstone bridge, in much the same way as comparisons of resistances, and the coefficients expressed in terms of that of the standard.

Thus if coils A and S, whose coefficients of self-induction are to be compared, are connected up to the non-inductive coils R_1 and R_2 so as to form a Wheatstone bridge (see Fig. 139), and the values of R_1 and R_2 adjusted so as to give a balance for steady currents, then, by adjusting the variable standard of self-induction S, we can also obtain a balance for transient currents, without altering the steady current balance. When the double balance is obtained, then—

$$\frac{L_A}{L_S} = \frac{R_1}{R_2}$$

where L_A and L_S are the coefficients of self-induction of the coils A and S respectively; and since L_S is known—

$$L_A = L_S \frac{R_1}{R_2}$$

In order to simplify the test for transient currents, and make it

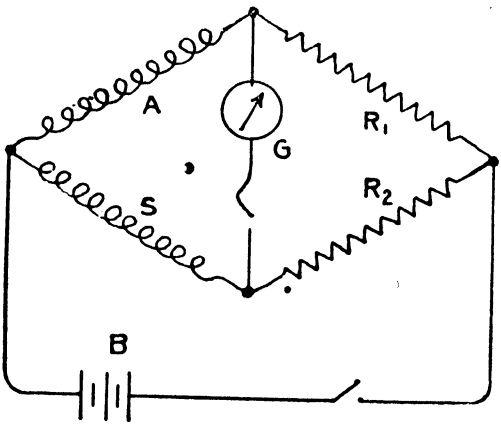


FIG. 139.

more sensitive, Professors Ayrton and Perry have devised an instrument known as a secohmmeter (see Fig. 140), in which

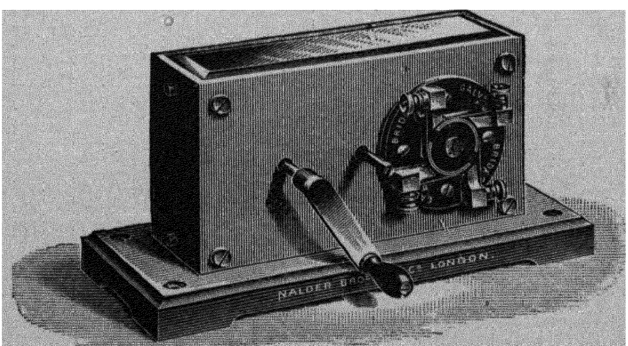


FIG. 140.

there are two commutators mounted on the same axis, one of which is placed in the galvanometer, and the other in the

battery circuit, so that by rotating a handle the galvanometer and battery connections are periodically reversed at rates varying from 200–6000 reversals per minute, the galvanometer terminals being reversed between each of the battery reversals, thus sending the transient current (if any) always in the same direction through the galvanometer, and so producing a steady deflection.

The resistances A and S are first balanced for steady currents with the secohmmeter at rest. The commutator is then rapidly rotated, and if the coefficients of self-induction of the coils are not also balanced, a steady deflection will be obtained in the galvanometer. The variable standard of self-induction S is now adjusted without altering the resistance of the arm S, until the galvanometer deflection has been reduced to zero, when the self-inductions balance, and—

$$L_A = L_S \frac{R_1}{R_2}$$

Fig. 141 represents the connections as applied to an ordinary

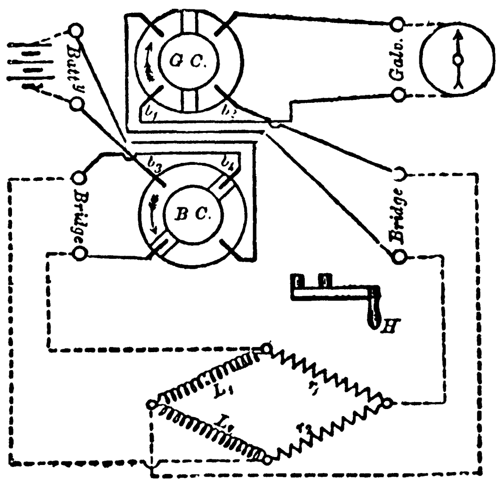


FIG. 141.

P.O. bridge. BC and GC represent the battery and galvanometer commutators respectively, r_1 and r_2 the ratio arms, L_1 and L_2 the coil whose coefficient is to be measured, and the

standard of self-induction. Exact balance for steady currents is in this case obtained by means of a slide wire adjustment.

STANDARDS OF SELF-INDUCTION.

334. The variable standard of self-induction designed by Ayrton and Perry is shown in Fig. 142, and consists of two

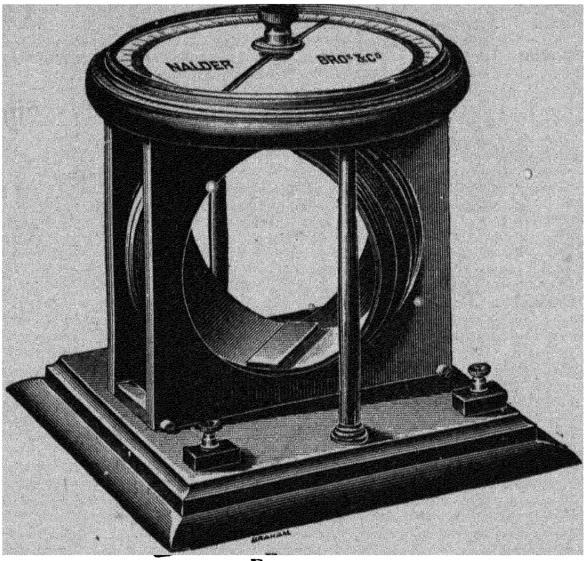


FIG. 142.

coils in series, one of which revolves inside the other about a vertical axis. The coils are wound on frames which are parts of spheres, so as to allow the coils to be very close together when they lie in the same plane. When the coils are in the same plane, and the current circulates in the same direction in both, the coefficient of self-induction has its maximum value. By rotating the inner coil through 180°, the coefficient passes through all values from the maximum to zero.

The coils are made of copper wire, so as to have a low resistance, and on the top of the instrument there is a dial over which moves a pointer connected to the movable coil, which indicates either the angular separation of the coils, or, if desired, the dial may be graduated directly in henrys, the usual range being from 0·004–0·04 henry.

For use in conjunction with the variable standard, there are supplied boxes of fixed standards (see Fig. 143) which are arranged in much the same way as resistances, and contain inductances of 10, 20, 30, and 40 millihenrys. These coils may be placed in series with the variable standard when the

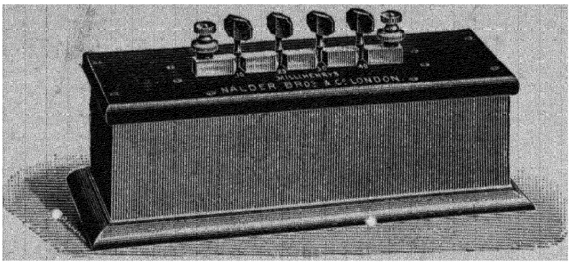


Fig. 143.

latter is not sufficiently large. In adding additional coils, care must be taken to rebalance the bridge for steady currents.

Great care must be exercised, when comparing the coefficients of self-induction of two coils, to place them sufficiently far apart so that they do not affect one another, and so produce a mutual induction effect in addition to that of self-induction.

Comparison of Coefficients of Self-Induction by Sumpner's Method.

335. A comparison of the coefficients of self-induction of two coils may also be made by a method similar to that employed by Dr. Sumpner, in comparing the self-induction with the capacity of a condenser.

The two coils, A and B, whose coefficients of self-induction are to be compared, are connected up to the non-inductive coils R and S so as to form a Wheatstone bridge (see Fig. 144). The non-inductive coils R and S are then adjusted so as to give a balance for steady currents. If, keeping the galvanometer circuit closed, the battery circuit is now opened, a swing, θ_1, will be obtained on the ballistic galvanometer BG which is proportional to the difference between the time-constants of the two coils, and—

$$\frac{L_A}{A} - \frac{L_B}{B} \propto \theta_1$$

where A and B are the resistances of the coils of self-induction L_A and L_B.

One of the coils, say B, is now removed, and replaced by a

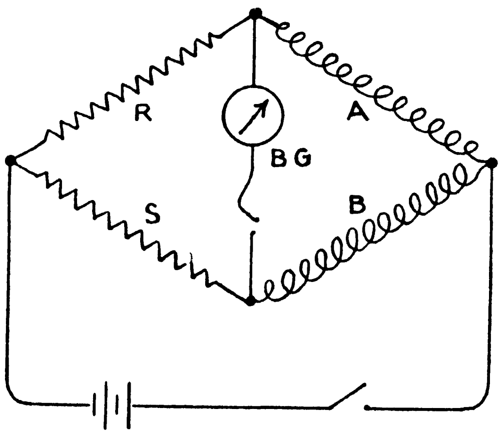


FIG. 144.

non-inductive resistance of equal value, so that the balance for steady currents remains undisturbed, and the above operation is repeated, this time the ballistic swing θ_2 is proportional to the time-constant of coil A, and—

$$\frac{L_A}{A} \propto \theta_2$$

$$\text{Hence } \frac{\dfrac{L_A}{A} - \dfrac{L_B}{B}}{\dfrac{L_A}{A}} = \frac{\theta_1}{\theta_2}$$

$$\text{and } \frac{BL_A - AL_B}{BL_A} = \frac{\theta_1}{\theta_2}$$

$$\text{or } \frac{L_A}{L_B} = \frac{A}{B} \times \frac{\theta_2}{\theta_2 - \theta_1}$$

$$= \frac{R}{S} \times \frac{\theta_2}{\theta_2 - \theta_1}$$

336. Two coils exactly similar were wound, one on a bar of iron and the other on a rod of wood; the resistances of both were exactly the same, 5 ohms. These coils were then connected up to two coils, R and S, each of 10,000 ohms, and a ballistic swing of 665 scale-divisions was obtained. When the coil wound on the wooden rod was replaced by an inductionless resistance of 5 ohms, a ballistic swing of 700 scale-divisions was obtained. Hence—

$$\frac{L_A}{L_B} = \frac{R}{S} \times \frac{\theta_2}{\theta_2 - \theta_1}$$

$$\frac{L_A}{L_B} = \frac{10000}{10000} \times \frac{700}{700 - 665}$$

$$= \frac{20}{1}$$

337. The following table of coefficients of self-induction of common pieces of apparatus has been compiled from data given in a paper on inductance by A. E. Kennelly.[1]

Instrument.	Approximate value of L (henrys).
Cardew voltmeter	0·000001
Doubly wound resistance coil	0·000001
Standard 10-ohm telegraphic relay ...	0·2 to 0·5
Mirror-speaking galvanometer, 2250 ohms	3·6
An 80-ohm telephone call-bell	1·4
Bell telephone receiver, 75 ohms ...	0·07 to 0·1
Dynamo field magnets	1 to 900
Dynamo armature	0·02 to 50
Primary of alternating current transformer	0·4 to 40
Secondary ,, ,, ,,	0·001 to 0·1
Primary small medical coil	0·005
Secondary ,, ,, ,, ...	0·100
Primary large induction coil	0·013
Secondary ,, ,, ,, ...	2000·0
Astatic mirror galvanometer, 5000 ohms	2·0
Electric bell, 2·5 ohms	0·012

[1] *Electrician*, vol. xxvi p. 267.

DETERMINATION OF THE COEFFICIENT OF MUTUAL INDUCTION OF TWO COILS.

338. The following method of determining the coefficient of mutual induction of two coils is due to Professor Carey Foster.[1]

The two coils A and B are connected up, one to a ballistic galvanometer BG (see Fig. 145), and the other to a non-

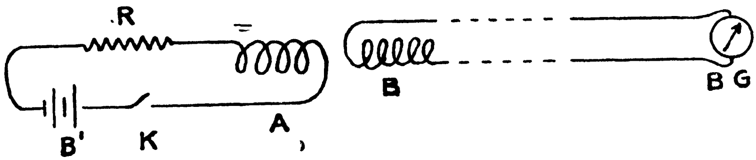


FIG. 145.

inductive resistance R, battery B′, and break-circuit key K. On opening or closing the key K, a quantity of electricity, Q_1, is induced in the coil B, which will produce a swing of amplitude, θ_1, on the ballistic galvanometer needle. If γ represents the strength of the current in the coil A, M the coefficient of mutual induction, and g the resistance of the coil B and the ballistic galvanometer, then—

$$Q_1 = \gamma \frac{M}{g}$$

If now a condenser of capacity K is connected in series with the ballistic galvanometer across the ends of resistance R, as

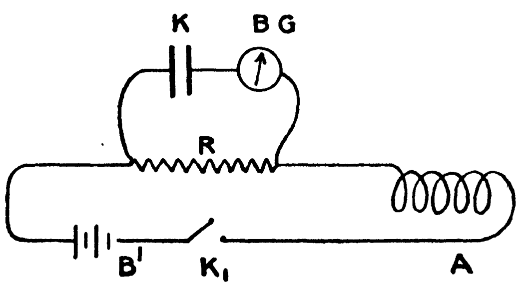


FIG. 146.

in Fig. 146, then, on closing or opening the key K_1, the

[1] *Phil. Mag.*, vol. xxiii., February, 1887.

condenser will be charged or discharged, a quantity of electricity, Q_2, passing through the ballistic galvanometer and producing a swing of amplitude θ_2, and—

$$Q_2 = \gamma R K$$

$$\text{hence } \frac{Q_1}{Q_2} = \frac{\frac{\gamma M}{g}}{\gamma R K}$$

$$= \frac{M}{RKg}$$

$$\text{But } \frac{Q_1}{Q_2} = \frac{\sin\frac{\theta_1}{2}}{\sin\frac{\theta_2}{2}}$$

$$\text{therefore } M = \frac{RKg \sin\frac{\theta_1}{2}}{\sin\frac{\theta_2}{2}}$$

The experimental conditions may be so chosen that $\theta_1 = \theta_2$ when the formula simplifies to—

$$M = RKg^1$$

In order to do this, and make the method a zero one, the connections are altered to those in Fig. 147.

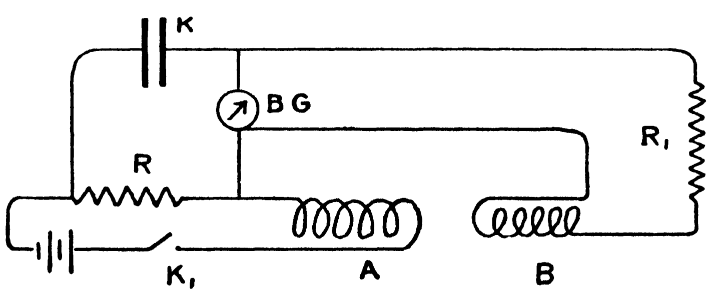


FIG. 147.

An additional resistance, R_1, is inserted in circuit with the

[1] g in this case represents the resistance of the coil B only.

coil B, this being adjusted till, on opening and closing the key K_1, no swing is observed on the galvanometer needle, when the induced quantity of electricity in B just balances the quantity passing in or out of the condenser K. Then—

$$M = RK(g + R_1)$$

g being the resistance of the coil B.

Should it not be found possible to get a balance for any value of R_1, then the resistance R must be altered and the test repeated. In order to save time when making the test, R_1 should first be made infinite, and the direction of the galvanometer swing noted on opening or closing the key K_1, this swing being entirely due to the condenser K; R_1 is then short circuited, and, if the throw is still to the same side, it shows that the quantity of electricity given to the condenser is greater than the maximum induced quantity in B, so that the charging potential of K must be diminished by diminishing the resistance of R between the condenser terminals. This should be done without altering the total resistance in the circuit of coil A, otherwise the current in the inducing circuit will be altered, and this will alter the quantity of electricity induced in B. The sensitiveness of this test depends on the sensitiveness of the ballistic galvanometer. If the values of R, K, and $(g + R_1)$ are expressed in absolute units, M will be calculated in absolute units.

339. The following example is taken from Professor Foster's paper.

A measurement was made of the coefficient of mutual induction between the primary and secondary coils of an Apps 6-inch spark induction coil of the following dimensions:—

Length of secondary coil = 21 cm.
Outside diameter of secondary coil = 11·3 cm.
Resistance of primary coil = 0·278 ohm (16·5° C.).
Resistance of secondary coil = 7394 ohms (16·5° C.).

The condenser used had a capacity of 4·926 microfarads. The following results were obtained:—

R (ohms).	$g + R_1$ (ohms).	$R(g + R_1)$ (absolute units).
27	7394 + 1550	$2{\cdot}415 \times 10^{23}$
28	7394 + 1250	2·420
29	7394 + 940	2·417
30	7394 + 650	2·413
31	7394 + 390	2·413
32	7394 + 150	2·414

$$\text{Mean } \frac{M}{K} = 2{\cdot}415 \times 10^{23}$$

$$M = 4{\cdot}926 \times 10^{-15} \times 2{\cdot}415 \times 10^{23}$$
$$= 1{\cdot}1896 \times 10^{9}$$

Comparison of a Capacity and Coefficient of Mutual Induction.

340. A comparison may be effected between the capacity of a condenser and the coefficient of mutual induction between two coils in a similar manner to the comparison of a capacity

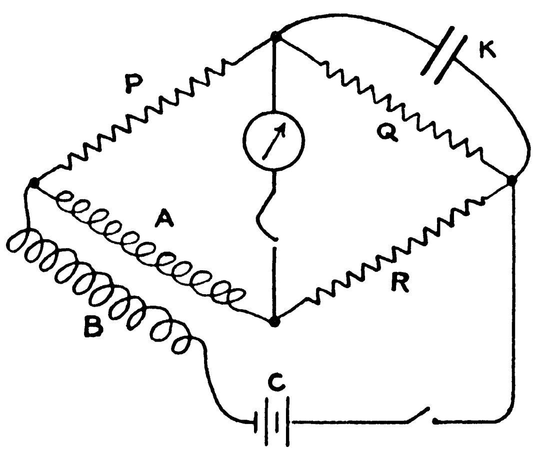


Fig. 148.

and coefficient of self-induction. One of the coils, A, is connected with three non-inductive coils, P, Q, and R, so as to form a Wheatstone bridge (see Fig. 148). The other coil,

B, is connected between the junction of A and P and the battery C. A condenser of known capacity, K, is placed across the end of coil Q.

The bridge is first balanced for steady currents in the usual way, and then, keeping the galvanometer circuit closed, the battery circuit is opened, when a ballistic swing, θ_1, will be obtained; and—

$$\theta_1 \propto \frac{Q+R}{Q} M - QAK$$

The condenser is now disconnected from the coil Q, and the galvanometer swing again taken; let this be θ_2, then—

$$\theta_2 \propto \frac{Q+R}{Q} M$$

$$\text{Hence } \frac{\theta_1}{\theta_2} = \frac{\frac{Q+R}{Q} M - QAK}{\frac{Q+R}{Q} M}$$

$$\text{and } \frac{M}{K} = \frac{Q^2 A}{Q+R} \times \frac{\theta_2}{\theta_2 - \theta_1}$$

If the capacity of K is known, the value of M may be found.

341. References to Scientific Papers.

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Complete Theory of Determination of H	Wiedemann	*Ibid.*, vol. 9, June, 1880.
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" " "	Smith	*Phil Mag.*, vol. 14, Sept., 1882.
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Specific Induction Constants of Magnets in Magnetic Fields of Different Strengths	Sack	*Phil. Mag.*, vol. 22 Oct., 1886.
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Permanent Magnetism	Bosanquet	*Ibid.*, vol. 15, Mar., 1883.
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Effects of Stress on Magnetization	Sir Wm. Thomson	*Ibid.*, 1876 and 1879.
Magnetic Induction	Tomlinson	*Ibid.*, 1891.
Experimental Determination of Magnetic Susceptibility and Maximum Magnetization in Absolute Measure	Shida	*Pro. Roy. Soc.*, vol. 34, p. 285; vol. 35, p. 404.
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On Magneto-Motive Force	,,	*Ibid.*, vol. 15, Mar., 1883.
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Magnetization of Iron in Strong Fields	Bidwell	*Ibid.*, vol. 29, May, 1890.
On Magnetism	Hopkinson	*Jour. Elect. Eng.*, vol. 19, p. 10; *Elect.*, vol. 24, p. 245.
Testing Iron	Swinburne	*Elect.*, vol. 25, p. 648.
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A Magnetic Balance	Ewing	*Ibid.*, vol. 41, p. 110.

Title of Paper.	Author.	Reference.
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342.—		
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Title of Paper.	Author.	Reference.
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Some Methods of comparing Coefficients of Self and Mutual Induction	Niven	*Ibid.*, vol. 24, Sept., 1887.
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VII.

ELECTRO-MAGNETIC WAVES.

343. WITHIN recent years the experimental part of this section of the subject has been developed to such an extraordinary extent, and its importance in connection with the relation of light to electricity is so great, that we do not consider it necessary to offer any apology for adding a chapter descriptive of experiments on electro-magnetic waves. It will also be found that in general the apparatus required is of such a simple character that the experiments can easily be repeated, provided a little care is exercised in making them.

344. Before describing the apparatus required for producing and detecting electro-magnetic waves, it will be as well to give a short account of the phenomenon itself, and by describing some of the phenomena peculiar to wave-motions in general indicate the nature of the experiments required to establish the wave-motion nature of electro-magnetic effects.

345. Suppose that an insulated metal sphere is set up at some distance from an uncharged electroscope, and the sphere suddenly charged; the gold leaves of the electroscope will diverge, since they are charged by induction, but in separating the leaves and causing them to diverge, work must be done against the force of gravity, which will be given out again when the leaves collapse. Now, the energy necessary to make the leaves diverge must have come from the sphere, and since in any case of the propagation of energy some medium is necessary to transmit it, we have to answer the questions, What is the medium in the case above mentioned? and how does the medium transmit the energy?

346. Again, suppose we set up a coil of wire, and connect it to a battery through a reversing key. At some distance from the coil let us place a pivoted magnetic needle, then when we send a current through the coil it becomes an electro-magnet, and will deflect the needle; by properly timing the direction of the current in the coil we can cause the needle to spin round on its pivot. Here, again, energy is transmitted across space, namely, from the coil to the magnet, and for the same reason as before we must assume the existence of some medium. In the first experiment mentioned above we were transmitting what was almost entirely an electrostatic effect, and in the second a magnetic effect. Strictly speaking, however, electrostatic and magnetic effects were propagated in both cases—a magnetic effect in charging the sphere, since that involved a flow of electricity for a very short interval of time; and an electrostatic effect in starting a current in the coil, since its potential was raised slightly above that of the earth. The same two effects are propagated in the charging and discharging of a Leyden jar or other condenser, but instead of considering each separately, the two are combined in the term electro-magnetic effect.

347. As regards the medium which transmits this electro-magnetic effect, we can prove that it is not atmospheric air, since the effect can be propagated through a vacuum; a determination of the velocity of propagation of the effect, however, gives us a valuable clue to the nature of the transmitting medium. The velocity of propagation of an electro-magnetic effect has been shown to be the same as the velocity of propagation of light, and therefore we might expect that the medium by which light is transmitted—the luminiferous ether—is also that which transmits electro-magnetic effects. The experimental proof of this supposition was the great work of the late Professor Hertz, to whose treatise on "Electric Waves," translated by Professor Jones, and also to the "Work of Hertz," by Professor Lodge, the student is recommended to refer.

348. Given a supply of energy and the means of transmitting it from one place to another we have next to consider how the transmission is effected. Here we find great assistance by

considering the method of the propagation of sound energy. Sound is propagated in its media—solids, liquids, or gases—by means of waves, and the experimental study of sound is on this account of great importance, apart from its musical interest, since we can study it as a typical case of energy propagation by waves. If, then, waves are *a* means of transmitting energy in a medium, we must examine our electro-magnetic phenomena for effects which we should expect from a wave-motion.

349. Firstly, with regard to the propagation of waves in homogeneous media, we find that the transmission is rectilinear, and that the velocity of propagation depends on the medium, and can be calculated, provided we know some of its physical properties. Newton was the first to enunciate the law respecting the velocity of propagation, which is—

$$v = \sqrt{\frac{e}{d}}$$

v = velocity of propagation; e = elasticity of the medium; d = density of the medium.

This law has been experimentally proved to hold in the propagation of sound-waves in ordinary matter; if, however, we wish to apply it to the case of electro-magnetic waves, we must remember that e and d will be the elasticity and density of the ether respectively. Now, elasticity is the reciprocal of pliability, and the pliability of a dielectric, we call its specific inductive capacity, K, hence $\frac{1}{K}$ will represent the elasticity of the ether; also, the density of the ether is what we call permeability, μ, and rewriting Newton's law with the above constants we get—

$$v = \frac{1}{\sqrt{K\mu}}$$

350. Secondly, when a wave which is being propagated through a homogenous medium arrives at the interface between that medium and another of different density, the following phenomena may occur:—

1. Reflection.
2. Transmission.
3. Absorption.

The laws which govern the reflection of waves from plane or curved surfaces are well known, and do not require any further mention.

351. As regards the transmission of waves in the new medium, if the latter is of greater density than the first medium, the velocity of propagation is decreased; if of smaller density it is increased. The consequence of this alteration of velocity is, that should the wave impinge normally on the surface of the new medium its velocity alone will be affected; should it, however, meet it at some other angle, then both velocity and direction will be altered. This phenomenon is usually, in the case of light or sound waves, termed refraction; the amount of bending for a given ray entering a medium at a given angle depending on the index of refraction of the medium, r, which we can define as follows—

$$r = \frac{\text{velocity of wave in space}}{\text{velocity of wave in medium}}$$

Now, we have just seen how to calculate the velocity of propagation of electro-magnetic waves, and the above definition of the refractive index gives us a means of proving that electro-magnetic waves and light waves are propagated by the same medium, and are very similar. Let K be the S.I.C. of a substance, μ its permeability, and r its index of refraction for light, then, since K and μ for air are both unity, we have—

$$r = \frac{\text{velocity of wave in space}}{\text{velocity of wave in medium}} = \frac{\frac{1}{\sqrt{1}}}{\frac{1}{\sqrt{K\mu}}} = \sqrt{K\mu}$$

or $r^2 = K\mu$

Taking as an example carbon bi-sulphide (CS_2), we find that $r^2 = 2{\cdot}678$, and $K\mu = 2{\cdot}67$, μ being $= 1$, which shows a remarkable agreement. There are, however, many cases in which experiment has failed to show that the above law holds; this, however, may be due to a difference in degree rather than in kind of the two wave-motions compared.

When the new medium which the wave meets is of smaller density than the other, and the angle of incidence is greater

than the critical angle, the wave is totally reflected with the loss of half a wave-length.

352. The third effect that may occur in the new medium is absorption, and we wish more particularly to call attention to one particular kind, namely, selective absorption, in which one particular frequency is absorbed more readily than any other; just as when a tuning-fork of definite pitch, placed in the vicinity of a second fork of the same pitch which is vibrating, absorbs these vibrations and itself commences to vibrate, exhibiting the phenomenon of resonance; if the forks were of different pitch this would not occur.

353. One other effect, peculiar to wave-motions, and one which is well illustrated in the case of sound-waves, is that of interference, which deals with the resultant effect of two or more waves at a point. If two waves of the same frequency, amplitude, and wave-length meet at a point so that one is half a wave-length in advance of the other, they completely annul one another and produce interference. Thus if waves are transmitted so as to fall normally on a reflecting surface, the outgoing waves meet the reflected waves, and at certain points half a wave-length apart produce interference. In the case of sound-waves this means silence, and by measuring the distance between these points the wave-length can be calculated.

We have now enumerated some of the more important effects produced by a typical wave-motion, and if we can reproduce these effects in our electro-magnetic phenomena, we will have strong evidence in favour of the view that it also is propagated by a wave-motion.

Apparatus for producing Electro-Magnetic Waves.

354. In making experiments with waves of any kind, it is necessary that we should suit our apparatus to the length of the waves with which we are working, or, in cases where we can employ waves of different lengths, that we should choose those lengths which can be most conveniently experimented upon; thus in sound-waves we should not think of trying interference experiments with waves twenty or thirty feet long but with

those of a few inches in length, so also with our electro-magnetic waves, they must be of a convenient length. We could, of course, produce electro-magnetic waves by means of our coil, battery, and reversing key, but the length would be so great—many miles in this case—that they would be perfectly useless; the more rapid the alternations of current the shorter the waves would be, but no commutator could be made to alter the direction of the current so rapidly as to produce waves of a few inches in length.

In order to get a discharge which is alternating with sufficient rapidity to produce waves of useful length, we have to fall back on the discharge of a Leyden jar. A charged Leyden jar corresponds very closely to a stretched spring, which, if released, oscillates backwards and forwards about its position of rest for some time before it finally stops. If, however, the motion of the spring was damped by placing it in some viscous liquid such as glue, there would be no oscillations, it would simply return slowly to its position of rest. A Leyden jar discharging behaves in the same way if the oscillations are not damped by self-induction, and the frequency of the oscillation produced can be calculated as follows[1]—

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{KL} - \frac{R^2}{4L^2}}$$

where n = frequency of oscillation in vibrations per sec.;
K = capacity of the jar in farads;
L = coefficient of self-induction in henrys;
R = resistance of discharge circuit in ohms;
the conditions for the production of an oscillating discharge being that—

$$R < \sqrt{\frac{4L}{K}}$$

In cases where R is small, the expression for frequency may be written—

$$n = \frac{1}{2\pi\sqrt{LK}}$$

[1] "Electricity and Magnetism" (Joubert, Foster, and Atkinson), pars. 329, 330.

355. If ordinary Leyden jars are employed for producing the oscillations, it is found that the wave-length is still far too great, being from 150–300 feet with ordinary sized jars. A better arrangement for producing oscillations is that first employed by Hertz, and known as a Hertz oscillator. This consists simply of two brass spheres attached to the ends of two rods which are attached to the secondary of an induction coil, the spark passing between the two brass knobs. In order to give capacity to this, metal spheres or plates are sometimes attached to the other ends of the rods.

356. The following formula is given by Lodge[1] for the calculation of L in the formula for the frequency of the oscillation:—

$$L = 2l \log \frac{4l}{d}$$

where $l =$ length of the entire rod portion of the oscillator in cases where the oscillator consists of metal spheres mounted on the ends of brass rods, to the other ends of which are attached small plates to give capacity, and $d =$ diameter of the rod. In measuring l, it is best to include the knobs and spark-gap, and about a quarter the diameter of each end plate.

The capacity K is practically half the static capacity of the plate or sphere at either end of the oscillator, since the two capacities are really in series. Thus for two oppositely charged spheres of radius r and distance s from centre to centre—

$$K = \frac{1}{2} r \left(1 + \frac{r}{s}\right)$$

357. For isolated bodies, the following values of K are given by Lodge:—

Condenser.	Value of K
Globe	radius
Thin circular disc ...	$\frac{2}{\pi}$ radius
Thin square disc ...	0·36 side of square
Thin oblong disc ...	slightly greater than square of same area

[1] Lodge, *Phil. Mag.*, vol. xxviii., July, 1889.

358. The following example will illustrate the calculation, and applies to an oscillator consisting of two rods each 6 cm. long and 1 cm. diameter, to which knobs 2 cm. diameter and plates 8 cm. diameter have been attached (see Fig. 149.) The spark-gap is 0·8 cm.

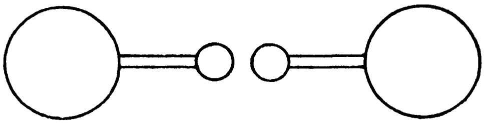

FIG. 149.

$$\text{Static capacity } K = 1{\cdot}4 \text{ cm.}$$
$$\text{Self-induction } L = 190 \text{ cm.}$$
$$\text{Log } \frac{4l}{d} = 4{\cdot}5$$
$$\text{Rate of vibration } n = 300 \text{ million per sec.}$$
$$\text{Wave-length } \lambda = 100 \text{ cm.}$$

359. A still more convenient form of oscillator, originally due to Professor Lodge, consists of two small brass spheres connected to the secondary of an induction coil, and arranged so as to spark across the diameter of a larger brass sphere placed between them (see Fig. 150). This form has also been adopted by Bose.[1] The small spheres in his case were 0·3 cm. diameter, and the large one 0·78 cm. diameter. The distance from centre to centre of the small spheres was 1·2 cm. The waves produced by this oscillator were 1·84 cm. long. It is advisable to hinge the supports of the small spheres so that their distance apart may be varied.

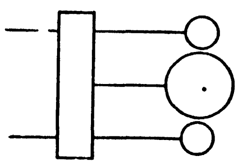

FIG. 150.

360. Another form of oscillator due to Lodge consists of two metal spheres sparking to the interior of a copper cylinder; with the dimensions in Fig. 151 waves of 3″ long are produced.

361. A form of oscillator used by Marconi, and described by Preece,[2] consists of two solid brass spheres fixed in an oil-tight

[1] See *Pro. Roy. Soc.*, vol. lx. p. 167.
[2] *Electrician*, vol. xxxix. p. 17.

insulating case filled with vaseline oil, two smaller spheres placed close to the outside of the others are connected to the induction coil, the spark passing across the three gaps. With the large spheres, 4″ diameter, waves 1·2 metres long were obtained.

362. The great difficulty with all these oscillators is that when the sparking surfaces get roughened, as they do with use,

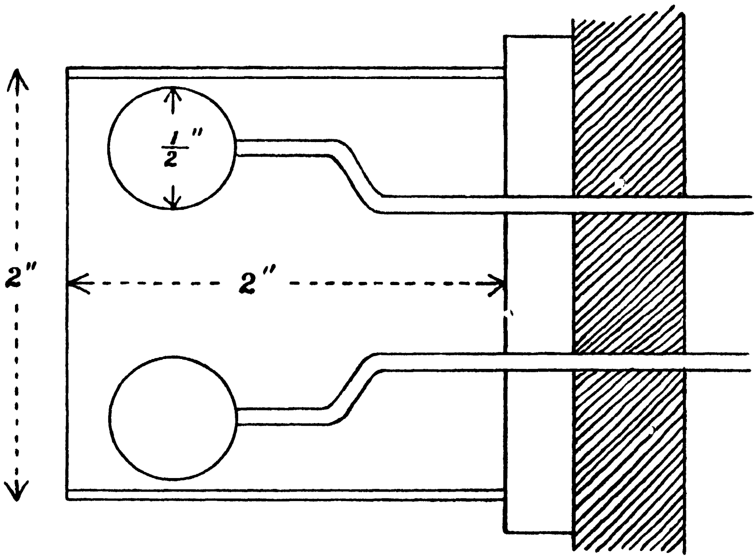

FIG. 151.

the discharge ceases to be oscillatory; the brass knobs should therefore be made easily removable, to admit of their being burnished up. It is also necessary in all quantitative experiments to completely enclose the oscillating apparatus—battery, coil, and wires—inside a copper-lined box, leaving only the spark-gap outside, since oscillations may escape from other parts of the apparatus. For this reason the form of apparatus designed by Bose [1] is recommended.

363. The oscillator is fixed to the outside of a box containing the coil and battery; a copper tube open at both ends is slipped over the oscillator so as to direct the path of the waves.

[1] See *Phil. Mag.*, vol. xliii., Jan., 1897; also see *Pro. Roy. Soc.*, vol. lix. p. 160.

The box containing the battery and coil is encased inside two metal cases—an inner one of iron, which forms a magnetic shield round it, and an outer one of copper, which prevents stray radiation from escaping. Also, instead of having the usual vibrating make and break on the coil, an ordinary break-circuit key is employed, the condenser being connected to it as in the vibrating break; the object of this is to enable single flashes of radiation to be obtained instead of a continuous stream, thus saving the sparking surfaces. The key can be manipulated from the outside of the box by a string.

Apparatus for detecting Electro-Magnetic Waves.

364. If the electro-magnetic waves are produced by a Leyden jar discharging through a wire circuit, the presence of the waves may be detected by placing a second similar jar, provided with a discharging circuit of the same dimensions, in the vicinity of the first. On the first jar being discharged, the oscillations produced set up electro-magnetic waves which, on arriving at the second jar, induce discharges in it, which can be detected by the presence of a spark at the discharging knobs. This effect, however, is only possible when the jars and their discharging circuits are identical with one another, the second jar having to be "tuned into resonance" with the first before it will act as a detector.

365. A much more sensitive form of detector, and one which is easier to work with, is that known as a coherer.

The action of the coherer was first demonstrated by Branly,[1] who showed that a tube containing metallic filings, and which had a very high resistance, became a tolerably good conductor when placed near a sparking coil. This phenomenon has been utilized by Lodge, Bose, Marconi, and others, as a means of detecting electro-magnetic waves.

A simply made form of coherer consists of a glass tube about eight inches long and half-inch diameter, filled with small clippings of copper. Passing into the clippings through corks at either end of the tube are copper wires, which connect the

[1] Branly, *Comptes Rendus*, vol. cxi. p. 785; vol. cxii. p. 90.

coherer in series with a galvanometer and battery. The tube in its normal state is a very bad conductor, due to the number of bad contacts at the copper clippings, and no deflection will be obtained on the galvanometer. If a sparking coil is now set going in the vicinity of the tube, the galvanometer will be observed to deflect, this being due to the sudden acquisition of conducting power by the tube of clippings. On slightly tapping the tube, it returns at once to its badly conducting condition. This action, according to Lodge, is due to the breaking down of the insulating layer of oxide between the clippings by the electrical surgings set up in them by the electro-magnetic waves.

366. A much more satisfactory form of coherer is that devised by Professor Bose,[1] in which the copper clippings are replaced by steel springs. The construction of the instrument is as follows. A rectangular block of ebonite, about 3 cm. long, of square cross-section, with 1·0 cm. side, has a small rectangular groove cut in one side, 2 cm. long by 0·4 cm. broad. Into this groove are placed eight or nine short steel wire springs, 0·4 cm. in length, of No. 34 wire. A glass plate is fixed over them to prevent them from dropping out of the groove. Fig. 152 shows the spirals in position.

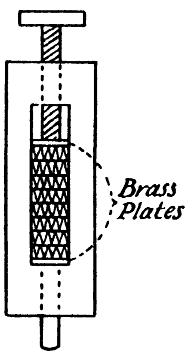

FIG. 152.

Two brass plates make contact with the upper and lower spirals respectively, the upper brass plate being attached to a screw, by means of which it can be made to compress the spirals. The upper and lower brass plates are connected in series with a battery and sensitive galvanometer. If the spirals are loose, no deflection will be obtained on the galvanometer. The screw is now turned so as to compress the spirals until a small deflection is obtained; the coherer is now in a sensitive condition, and if exposed to electro-magnetic radiation the galvanometer will deflect violently, indicating a great diminution of resistance at the spirals. In order to restore them

[1] *Phil. Mag.*, vol. xliii., January, 1897.

to their normal state, the coherer may be tapped, or the compressing screw may be slightly unscrewed. It will be found, as has been pointed out by Bose, that there is a certain value for the E.M.F. acting in the circuit, for which the coherer is most sensitive when detecting waves of given length. This must be found experimentally, and the battery, instead of being connected directly to the coherer, should be joined up to the ends of a high resistance, from which the coherer circuit is taken as a shunt; thus, by altering the resistance included between the coherer wires, various E.M.F.'s may be obtained, as has already been explained in par. 20.

367. In experimenting with the above, or, in fact, with any form of coherer, the greatest difficulty will be experienced in protecting it from stray radiation. Thus, covering the face of the spirals with a copper plate, which is opaque to electro-magnetic waves, will not be found sufficient to prevent the coherer from working, since the waves impinge on the connecting wires, or on the battery or galvanometer, and transmit their effects through the circuit to the coherer. We must therefore completely shield all the instruments in the coherer circuit.

The following Fig. (153) illustrates the method adopted by the author.

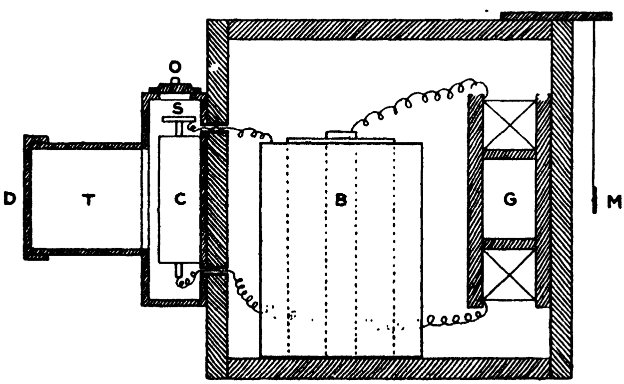

FIG. 153.

The battery B, consisting of a small iron-zinc cell (the iron in a solution of ferric chloride and the zinc in sulphuric acid,

this giving an E.M.F. of about 0·5 volt, which is best for all-round work), and the galvanometer coil G, are enclosed inside a square brass box, the needle attached to the mirror M being suspended outside. The coherer C is enclosed inside a circular brass box attached to one side of the square box, an opening, protected by the brass tube T, being made in front of the sensitive spirals. On the open end of T various diaphragms, D, can be attached, so as to limit the amount of radiation received by C. The wires from the coherer pass through two ebonite bushed holes into the interior of the brass box, where they are attached to the battery and galvanometer respectively. An opening, O, provided with a sliding brass lid in the circular box containing the coherer, admits of entrance to the adjusting screw S. The whole apparatus can be attached to a spectrometer circle, taking the place of the observing telescope. Before using, the apparatus should be tested for electro-magnetic wave tightness, by placing a brass cap with no hole in it over the end of the tube, T, and adjusting the coherer to its maximum sensitiveness. Then, on producing electro-magnetic waves in its vicinity, no effect should be obtained on the coherer. This receiver is used in conjunction with the oscillator described in par. 363.

368. In performing the various optical experiments with the electro-magnetic waves, it is necessary that we should employ a parallel beam, and not a divergent one. In order to obtain this, a lens of some refracting material must be placed in front of the oscillator, so that the spark-gap is at its principal focus. The lens should be in the form of a cylinder or semi-cylinder, and should be enclosed in a square metal tube, which fits on to the tube surrounding the oscillator. Such a lens may be made of sulphur, the index of refraction of which is 1·73. Employing the usual lens formula—[1]

$$\frac{1}{F} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{S}\right)$$

where F = focal length;

R = radius of curvature of side nearest source;

S = radius of curvature of side farthest from source;

[1] Vol. i, Exp. 72.

a surface concave to the incident rays being reckoned positive, we can calculate the necessary curvature for our sulphur lens, and, making a metal mould, proceed to cast it to the required size.

TRANSMISSION, REFLECTION, AND ABSORPTION.

369. (1) *Rectilinear Propagation.*—In order to demonstrate the rectilinear propagation of the rays, the oscillator and coherer should be set up opposite each other, and at some distance apart, as A B in Fig. 154. The coherer B is then adjusted so

FIG. 154.

as to respond to the waves from A, which are supposed to consist of a parallel ray. If a thick plate of some good conductor, such as copper, is now interposed between A and B no effect will be obtained at B, because a conductor is opaque to the electro-magnetic waves. If, however, a non-conductor, as ebonite sheet, or a wooden board, etc., be interposed the effect is obtained as if there was nothing between A and B. Non-conductors are transparent to the waves. Various substances should be tried, and the relative thicknesses of the plates required to just stop the passage of the waves determined. If now a number of copper plates are taken, and holes of the same diameter as the radiator tube drilled in them, and the plates placed parallel to each other between A and B, then, if the holes coincide, the effect will still be obtained at B; but if they are displaced, so that the holes are not opposite one another, then no effect will be obtained at the coherer, thus demonstrating the rectilinear propagation of the waves.

370. (2) *Reflection and Absorption.*—In order to prove experimentally the laws of reflection of the waves, a spectrometer should be employed; but they may be demonstrated without, in the following manner. On a large flat board a graduated circle of about 12″ diameter is drawn (see Fig. 155). The

oscillator and coherer A and B are placed opposite each other, and B adjusted to respond to the waves from A. The lens must be placed in front of A, in order to get a parallel beam, and the smallest diaphragm which can be used should be put in front of A, so as to get a small ray. B is then rotated through 90° to B′, and on working the oscillator, no effect should be obtained. A small copper plate, C, is now placed at the centre of the circle, and rotated about a vertical axis until an effect is

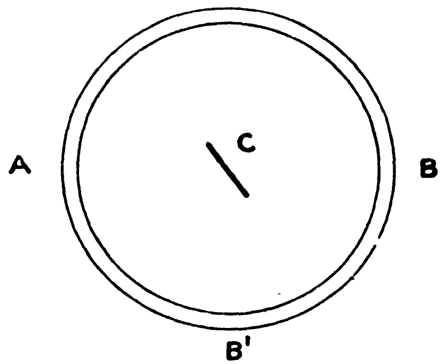

FIG. 155.

obtained at B′. On measuring the angles it will be found that the angles of incidence and reflection are equal. The effect should be tried with B in various other positions, also using different substances for the reflector C. It will be found that while almost every substance reflects more or less of the incident waves, the effect is much more marked in the case of the good conductors than of the insulators. Tests should be made for substances that are electrically black to the waves; *i.e.* substances which do not either reflect or transmit to any marked extent, but absorb the waves. Blotting-paper soaked in water will be found to do this to some extent.

REFRACTION.

371. Of all the experiments with the electro-magnetic waves the measurement of the index of refraction is one of the most important. The methods employed are similar to those when

using light waves. We may, for instance, make a prism of the refracting substance, and determine the angle of minimum deviation of the electro-magnetic ray and the angle of the prism, when—

$$r = \frac{\sin \frac{A + D}{2}}{\sin \frac{A}{2}}$$

where A = angle of prism, and D = angle of minimum deviation. Such measurements have been made by Hertz, Lodge, and others, but, as Bose[1] has shown, it is much more satisfactory to measure the index of refraction by determining the critical angle a of the medium, from which we get—

$$r = \frac{1}{\sin a}$$

In the case of solids, the refracting medium should take the form of two semi-cylinders with an air-space between them. This shape is chosen because, in order that all the ray should be totally reflected at the same instant, it should impinge normally on the air film, this being obtained by using semi-cylindrical lenses as the medium, and placing the spark-gap at the principal focal line.

In the case of substances whose index of refraction is not approximately known, a rough measurement is first made, with the spark-gap somewhere near the semi-cylinders; from the approximate value of r thus obtained, the focal length of the lens may be calculated, and, placing the spark-gap at that point, a new and more accurate value of r may be obtained. In order only to utilize the central rays, a copper diaphragm, with a slit cut in it, is placed between the semi-cylinders, thus stopping all rays but those that get through the slit.

The arrangement of the apparatus is shown in Fig. 156. The oscillator A and coherer B are placed diametrically opposite one another, and tested for proper adjustment. The semi-cylinders, with the diaphragm D between them, are then placed in position, so that the diaphragm is at right angles to

[1] *Pro. Roy. Soc.*, vol. lix. p. 160.

the line joining AB. The tubes surrounding the oscillator and coherer should end quite close up to the semi-cylinders. On starting the oscillator the coherer will at once respond, due to the rays which pass through the hole in the copper diaphragm. The diaphragm D and semi-cylinders are now rotated in a clockwise direction, and when they reach some position, D″D″, it

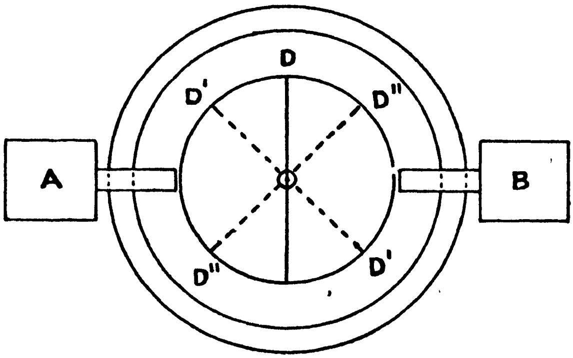

FIG. 156.

will be found that the coherer no longer responds, the ray being just totally reflected in that position. This position is noted on the graduated circle. The semi-cylinders are then rotated in a counter-clockwise direction, till, on reaching the position D′D′, it is found that total reflection just occurs again, this position being noted; the angle D′OD″ is twice the critical angle, and from this *r* may be calculated. In practice it is better to start with the ray totally reflected, and find the position where the coherer just begins to act; this gives, of course, the same result. The dimensions of the semi-cylinders depend considerably on the size of the rest of the apparatus, but they may be about 25 cm. diameter, and 10 cm. high. Solids like wood, ebonite, etc., should be turned up in a lathe, and then cut in two, whilst substances like sulphur should be cast in a mould.

Liquids are placed in a cylindrical glass vessel divided by two glass partitions, which enclose an air-space between them.

372. *Measurement of Wave-Length.*—In order to measure the length of the waves emitted by a Hertz oscillator, there are

several methods available. We might, for instance, employ the relationship $v = n\lambda$ where $v =$ velocity of propagation, $n =$ frequency, and $\lambda =$ wave-length; this would involve a measurement of velocity and frequency, both of which measurements present great difficulties, rendering this method unsuitable.

373. Another method that has been adopted, and successfully carried out, by Hertz, was to produce a stationary wave by reflecting a wave back on itself from a metal sheet; then, by finding the positions in front of the reflector where interference occurs, and measuring the distance between them, a value may be obtained for the wave-length. Some doubts have, however, been thrown on this method of measuring wave-length, as some experiments tend to show that the wave-length measured depends to some extent on the free period of vibration of the resonator employed.

374. A much more satisfactory method, and one that has given good results, is that employed by Bose,[1] in which a curved diffraction grating is used. The theory of the curved diffraction grating has been fully worked out,[2] since it is employed for the measurement of the length of light waves, so that only the dimensions of the grating have to be altered to meet the changed conditions of using it with much longer waves.

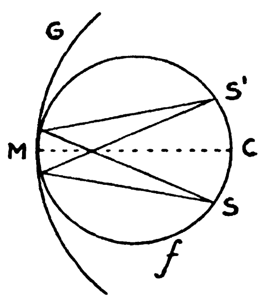

FIG. 157.

Let G in Fig. 157 represent the curved grating, and c the centre of curvature. The circle f, drawn with Mc as diameter, is the focal curve. Any source of radiation placed on this curve will give a diffracted spectrum situated somewhere on the same curve, defined by the equation—

$$n\lambda = (a + b) (\sin i \pm \sin \theta)$$

where $n\lambda = n$th spectrum;

$(a + b) =$ sum of the breadths of a strip and space in grating;

$i =$ angle of incidence;

$\theta =$ angle of diffraction.

[1] *Pro. Roy. Soc.*, vol. lx. p. 167.

[2] See Preston's "Light," par. 136.

The sign of θ is + if the diffracted image s' lies on the same side of Mc as s the source, and − if it lies on the opposite side.

375. The grating employed consists of a sheet of ebonite on which a number of strips of tinfoil have been pasted, these being about 2·5 to 3 cm. broad, and the space between two adjacent tinfoil strips is equal in breadth to the strips. The ebonite sheet before pasting on the strips must be bent into a curve of about 100–150 cm. radius.

This grating is then placed vertically on a flat surface, as G (Fig. 157), and the circle f drawn to give the positions for s and s'. The coherer should be placed at c, thus making θ in the calculation equal to zero. The radiator must now be moved about on the curve f, until the diffracted image falls on the coherer, when it will respond. The angle of incidence, i, must now be measured, and the wave-length calculated.

376. The following numbers obtained by Bose for a 3 cm. grating will serve to illustrate the measurement:—

i	θ	λ	
21·5	0	1·832	mean 1·845
29·5	−7	1·852	
33·0	−10	1·854	
34·0	−11	1·841	

The radiator employed in the above measurement was the same as that described in par. 359.

Polarization of Electro-Magnetic Waves.

377. All the phenomena previously described are common to sound, light, and radiant heat waves; the phenomenon of polarization, however, is common only to light and radiant heat waves, and indicates that the nature of the wave-motion in these two cases differs essentially from that in sound propagation. Sound is propagated in solids, liquids, or gases, by means of longitudinal waves whose direction of vibration is the same as the direction of propagation. The phenomenon of polarization indicates that light and radiant heat waves are

propagated by transverse vibrations, or those whose direction of motion is at right angles to the direction of propagation, and moreover that the direction of vibration is not confined solely to one plane.

378. If a ray of light falls on a plate of tourmaline cut parallel to the axis of the crystal, it is transmitted with a slight diminution of intensity. The light that is transmitted, however, differs essentially from the incident light on the tourmaline, since it will only pass through a second plate of tourmaline if the latter is parallel to the first; if placed at right angles, the ray is totally stopped by the second plate. The ray of light after passing through the first tourmaline plate has therefore acquired a two-sided property that it did not possess before, and is said to be plane polarized. The first tourmaline is called the "polarizer," and the second one which is used to discover the polarization is called the analyzer.

379. In order to polarize the electro-magnetic waves, they are sent through a screen of parallel copper wires, which should be constructed as described by Bose,[1] by winding copper wire of 2 mils. diameter round a thin piece of mica, which is then immersed in melted paraffin, the object being to fix the wires. Four circular pieces are cut out of this, and two are fixed parallel to each other at the ends of a tube which will fit into the tube surrounding the spark-gap, this constituting the polarizer. There are about twenty-five wires to the centimetre. This polarizer only transmits vibrations at right angles to the wires, and by means of it, it can be shown that the waves from the sparking knobs are partly plane polarized parallel to the spark-gap.

The analyzer is made in the same way as the polarizer, and can be attached to the tube surrounding the coherer.

When the polarizer is placed with the wires at right angles to the direction of the spark, the ray proceeding from the oscillator will be completely plane-polarized, as can be tested by turning the analyzer so that its wires are (1) parallel, (2) perpendicular to the direction of the wires in the polarizer, when in the second case, no effect will be obtained at the coherer.

[1] Bose, *Electrician*, vol. xxxvi. p. 291.

We have now seen that all the ordinary optical experiments can be performed with electro-magnetic waves as produced above, and that light and these electro-magnetic waves are identical, except in so far as regards frequency and wave-length, the latter vibrating more slowly and being considerably longer than the visible ether waves; they will therefore occupy a place far to the left of the red in the spectrum.

380. References to Original Papers.

Title of Paper.	Author.	Reference.
Discharge of Leyden Jars	Lodge	*Pro. Roy. Soc.*, vol. 50, p. 2.
Indices of Refraction for Electric Rays	Bose	*Ibid.*, vol. 59, p. 160.
Determination of Wave-Length of Electric Radiation	,,	*Ibid.*, vol. 60, p. 167.
Selective Conductivity	,,	*Ibid.*, vol. 60, p. 433.
Kerr's Experiments on Relation between Light and Electricity	Gordon	*Phil. Mag.*, vol. 2, Sept., 1876.
Influence of Light on Electrical Resistance of Metals	Börnstein	*Ibid.*, Supp., vol. 3, 1877.
Influence of Light on the Electric Tension of Metals	,,	*Ibid.*, vol. 4, Nov., 1877.
Electrical Absorption of Crystals	Rowland and Nichols	*Ibid.*, vol. 11, June, 1881.
Transmission of Radiation of Low Refrangibility through Ebonite	Abney	*Ibid.*, vol. 11, June, 1881.
Refractive Index of Ebonite	Ayrton and Perry	*Ibid.*, vol. 12, Sept., 1881.
Reflection of Electric Rays	Goldstein	*Ibid.*, vol. 14, Dec., 1882.
Indices of Refraction of Metals	Kund	*Ibid.*, vol. 26, July, 1888.
Calorimetric Investigations on Electric Discharge	Staub	*Ibid.*, vol. 30, Sept., 1890.
Investigation of Electric Vibrations with Thermo-Elements	Klemencic	*Ibid.*, vol. 30, Sept., 1890.
Refraction and Dispersion in Metals	Du Bois	*Ibid.*, vol. 30, Nov., 1890.
Measurement of Electro-Magnetic Phenomena	Boys, Briscoe, and Watson	*Ibid.*, vol. 31, Jan., 1891.
Experiments on Photo-Electricity	Minchin	*Ibid.*, vol. 31, Mar., 1891.
Method of Determining Electro-magnetic Radiation	Klemencic	*Ibid.*, vol. 33, April, 1892.
Electro-Magnetic Radiation on Films	Minchin	*Ibid.*, vol. 37, Jan., 1894.
Sudden Acquisition of Conducting Power by Metallic Particles	Lodge	*Ibid.*, vol. 37, Jan., 1894.
On a Complete Apparatus for the Study of Electric Waves	Bose	*Ibid.*, vol. 43, Jan., 1897.
Concentration of Electric Radiation by Lenses	Lodge	*Elect.*, vol. 23, p. 32.
Variations of Electrical Conductivity under Electric Influence	Branly	*Ibid.*, vol. 27, pp. 221, 448.
Sensitive Cells	Minchin	*Ibid.*, vol. 28, p. 85.
Photo-Electric Researches	Righi	*Ibid.*, vol. 33, p. 297.

Title of Paper.	Author.	Reference.
Electric Waves	Trowbridge	*Elect.*, vol. 35, pp. 712, 812.
Electric Oscillations	Klemencic	*Ibid.*, vol. 35, p. 812.
Resistance of Filings of Metals	Lhuillier	*Ibid.*, vol. 35, p. 570.
Contact Resistance of Metals	Branly	*Ibid.*, vol. 35, p. 4.
Heat developed by Electric Oscillations in Wires	Cardani	*Ibid.*, vol. 36, p. 292.
Tinfoil Grating for Electric Oscillations	Mezuno	*Ibid.*, vol. 36, p. 187.
Apparatus for the Study of Electric Waves	Bose	*Ibid.*, vol. 37, p. 788.
Multiple Resonance in connection with Hertz Experiments	Sarasin and De la Rive	*Ibid.*, vol. 24, p. 3 7.
An Electric Radiation Meter	Gregory	*Ibid.*, vol. 24, p. 16.
Experiments to determine Wave Velocity in certain Dielectrics	Trouton	*Ibid.*, vol. 25, p. 557.
On the Speed of Propagation of Electric Waves in Insulating Dielectrics	Arons and Rubens	*Ibid.*, vol. 26, p. 157.
Experimental Determination of the Rate of Propagation of Electro-Magnetic Waves	Blondlot	*Ibid.*, vol. 28, p. 85.
Refractive Index of Electric Waves in Alcohol	Ellinger	*Ibid.*, vol. 30, p. 387.
The Equality of the Velocities of Propagation of very Short Electric Waves in Free Space and along Conductors	Dufour	*Ibid.*, vol. 33, p. 485.
On the Refraction and Dispersion of Rays of Electric Force	Garbasso	*Ibid.*, vol. 34, p. 39.
A Carbon Detector for Hertz Waves	Jervis-Smith	*Ibid.*, vol. 40, p. 84.
The History of the Coherer Principle	Lodge	*Ibid.*, vol. 40, p. 87.
The Practical Applications of the Coherer	A. C. Brown	*Ibid.*, vol. 40, p. 91.
Study of Cohering Action in Different Metals	Bose	*Ibid.*, vol. 43, p. 441.
Coherence and Re-Coherence	Shaw and Garrett	*Phil. Mag.*, No. 94, Aug., 1904.

LOGARITHMS — ANTILOGARITHMS — TABLES OF SQUARES—RECIPROCALS—TANGENTS—SINES —CONVERSION TABLES—CONSTANTS.

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9816	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9078	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	12	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

SQUARES OF NUMBERS FROM 1 TO 10000, CORRECT TO FOUR SIGNIFICANT FIGURES.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	1000	1020	1040	1061	1082	1102	1124	1145	1166	1188	3	5	7	9	11	13	15	17	19
11	1210	1232	1254	1277	1300	1322	1346	1369	1392	1416	3	5	7	10	12	14	17	19	21
12	1440	1464	1488	1513	1538	1562	1588	1613	1638	1664	3	5	8	10	13	15	18	20	23
13	1690	1716	1742	1769	1796	1822	1850	1877	1904	1932	3	6	9	11	14	17	19	22	25
14	1960	1988	2016	2045	2074	2102	2132	2161	2190	2220	3	6	9	12	15	18	21	24	27
15	2250	2280	2310	2341	2372	2402	2434	2465	2496	2528	4	7	10	13	16	19	22	25	28
16	2560	2592	2624	2657	2690	2722	2756	2789	2822	2856	4	7	10	14	17	20	24	27	30
17	2890	2924	2958	2993	3028	3062	3098	3133	3168	3204	4	7	11	14	18	21	25	28	32
18	3240	3276	3312	3349	3386	3422	3460	3497	3534	3572	4	8	12	15	19	23	26	30	34
19	3610	3648	3686	3725	3764	3802	3842	3881	3920	3960	4	8	12	16	20	24	28	32	36
20	4000	4040	4080	4121	4162	4202	4244	4285	4325	4368	5	9	13	17	21	25	29	33	37
21	4410	4452	4494	4537	4580	4622	4666	4709	4752	4796	5	9	13	18	22	26	31	35	39
22	4840	4884	4928	4973	5018	5062	5108	5153	5198	5244	5	9	14	18	23	27	32	36	41
23	5290	5336	5382	5429	5476	5522	5570	5617	5664	5712	5	10	15	19	24	27	33	38	43
24	5760	5808	5856	5905	5954	6002	6052	6101	6150	6200	5	10	15	20	25	30	35	40	45
25	6250	6300	6350	6401	6452	6502	6554	6605	6656	6708	6	11	16	21	26	31	36	41	46
26	6760	6812	6864	6917	6970	7022	7076	7129	7182	7236	6	11	16	22	27	32	38	43	48
27	7290	7344	7398	7453	7508	7562	7618	7673	7728	7784	6	11	17	22	28	33	39	44	50
28	7840	7896	7952	8009	8066	8122	8180	8237	8294	8352	6	12	18	23	29	35	40	46	52
29	8410	8468	8526	8585	8644	8702	8761	8821	8880	8940	6	12	18	24	30	36	42	48	54
30	9000	9060	9120	9181	9242	9302	9364	9425	9486	9548	7	13	19	25	31	37	43	49	55
31	9610	9672	9734	9797	9860	9922	9986	1005*	1011*	1018*	7	13	19	26	32	38	45	51	57
32	1024	1030	1037	1043	1050	1056	1063	1069	1076	1082	1	1	2	3	3	4	5	5	6
33	1089	1096	1102	1109	1115	1122	1129	1136	1142	1149	1	1	2	3	4	4	5	6	6
34	1156	1163	1170	1176	1183	1190	1197	1204	1211	1218	1	2	2	3	4	4	5	6	6
35	1225	1232	1239	1246	1253	1260	1267	1274	1282	1289	1	2	2	3	4	4	5	6	7
36	1296	1303	1310	1318	1325	1332	1339	1347	1354	1362	1	2	2	3	4	5	5	6	7
37	1369	1376	1384	1391	1399	1406	1414	1421	1429	1436	1	2	2	3	4	5	5	6	7
38	1444	1452	1459	1467	1474	1482	1490	1498	1505	1513	1	2	2	3	4	5	6	6	7
39	1521	1529	1537	1544	1552	1560	1568	1576	1584	1592	1	2	3	3	4	5	6	6	7
40	1600	1608	1616	1624	1632	1640	1648	1656	1665	1673	1	2	3	3	4	5	6	7	7
41	1681	1689	1697	1706	1714	1722	1730	1739	1747	1756	1	2	3	3	4	5	6	7	8
42	1764	1772	1781	1789	1798	1806	1815	1823	1832	1840	1	2	3	4	4	5	6	7	8
43	1849	1858	1866	1875	1883	1892	1901	1910	1918	1927	1	2	3	4	5	5	6	7	8
44	1931	1945	1954	1962	1971	1980	1989	1998	2007	2016	1	2	3	4	5	5	6	7	8
45	2025	2034	2043	2052	2061	2070	2079	2088	2098	2107	1	2	3	4	5	6	7	7	8
46	2116	2125	2134	2144	2153	2162	2171	2181	2190	2200	1	2	3	4	5	6	7	8	9
47	2209	2218	2228	2237	2247	2256	2266	2275	2285	2294	1	2	3	4	5	6	7	8	9
48	2304	2314	2323	2333	2342	2352	2362	2372	2381	2391	1	2	3	4	5	6	7	8	9
49	2401	2411	2421	2430	2440	2450	2460	2470	2480	2490	1	2	3	4	5	6	7	8	9
50	2500	2510	2520	2530	2540	2550	2560	2570	2581	2591	1	2	3	4	5	6	7	8	9
51	2601	2611	2621	2632	2642	2652	2662	2673	2683	2694	1	2	3	4	5	6	7	8	9
52	2704	2714	2725	2735	2746	2756	2767	2777	2788	2798	1	2	3	4	5	6	8	9	10
53	2809	2820	2830	2841	2852	2862	2873	2884	2894	2905	1	2	3	4	6	7	8	9	10
54	2916	2927	2930	2948	2959	2970	2981	2992	3003	3014	1	2	3	5	6	7	8	9	10

Squares from 1 to 3 contain 1 figure.			Squares from 100 to 316 contain 5 figures.		
,, ,, 4 to 9 ,, 2 figures.			,, ,, 317 to 999 ,, 6 ,,		
,, ,, 10 to 31 ,, 3 ,,			,, ,, 1000 to 3163 ,, 7 ,,		
,, ,, 32 to 99 ,, 4 ,,			,, ,, 3163 to 10000 ,, 8 ,,		

* The differences for squares from 3171 to 3199 are 1, 1, 2, 3, 3, 4, 5, 5, 6.

SQUARES OF NUMBERS FROM 1 TO 10000, CORRECT TO FOUR SIGNIFICANT FIGURES.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	3025	3036	3047	3058	3069	3080	3091	3102	3114	3125	1	2	3	5	6	7	8	9	10
56	3136	3147	3158	3170	3181	3192	3204	3215	3226	3238	1	2	4	5	6	7	8	9	10
57	3249	3260	3272	3283	3295	3306	3318	3329	3341	3352	1	2	4	5	6	7	8	9	11
58	3364	3376	3387	3399	3411	3422	3434	3446	3457	3469	1	2	4	5	6	7	8	10	11
59	3481	3493	3505	3516	3528	3540	3552	3564	3576	3588	1	3	4	5	6	7	8	10	11
60	3600	3612	3624	3636	3648	3660	3672	3684	3697	3709	1	3	4	5	6	7	9	10	11
61	3721	3733	3745	3758	3770	3782	3795	3807	3819	3832	1	3	4	5	6	8	9	10	11
62	3844	3856	3869	3881	3894	3906	3919	3931	3944	3956	1	3	4	5	6	8	9	10	11
63	3969	3982	3994	4007	4020	4032	4045	4058	4070	4083	1	3	4	5	7	8	9	10	12
64	4096	4109	4122	4134	4147	4160	4173	4186	4199	4212	1	3	4	5	7	8	9	10	12
65	4225	4238	4251	4264	4277	4290	4303	4316	4330	4343	1	3	4	5	7	8	9	11	12
66	4356	4369	4382	4396	4409	4422	4436	4449	4462	4476	1	3	4	5	7	8	9	11	12
67	4489	4502	4516	4529	4543	4556	4570	4583	4597	4610	2	3	4	6	7	8	10	11	12
68	4624	4638	4651	4665	4679	4692	4706	4720	4733	4747	2	3	4	6	7	8	10	11	12
69	4761	4775	4789	4802	4816	4830	4844	4858	4872	4886	2	3	4	6	7	8	10	11	13
70	4900	4914	4928	4942	4956	4970	4984	4998	5013	5027	2	3	4	6	7	9	10	11	13
71	5041	5055	5069	5084	5098	5112	5127	5141	5155	5170	[illegible]	3	4	6	7	9	10	12	13
72	5184	5198	5213	5227	5242	5256	5271	5285	5300	5314	2	3	5	6	7	9	10	12	13
73	5329	5344	5358	5373	5388	5402	5417	5432	5446	5461	2	3	5	6	8	9	10	12	13
74	5476	5491	5506	5520	5535	5550	5565	5580	5595	5610	2	3	5	6	8	9	11	12	14
75	5625	5640	5655	5670	5685	5700	5715	5730	5746	5761	2	3	5	6	8	9	11	12	14
76	5776	5791	5806	5822	5837	5852	5868	5883	5898	5914	2	3	5	6	8	9	11	12	14
77	5929	5944	5960	5975	5991	6006	6022	6037	6053	6068	2	3	5	6	8	9	11	13	14
78	6084	6100	6115	6131	6147	6162	6178	6194	6209	6225	2	3	5	6	8	10	11	13	14
79	6241	6257	6273	6288	6304	6320	6336	6352	6368	6384	2	3	5	7	8	10	11	13	14
80	6400	6416	6432	6448	6464	6480	6496	6512	6529	5545	2	3	5	7	8	10	11	13	15
81	6561	6577	6593	6610	6626	6642	6659	6675	6691	6708	2	3	5	7	8	10	12	13	15
82	6724	6740	6757	6773	6790	6806	6823	6839	6856	6872	2	3	5	7	8	10	12	13	15
83	6889	6906	6922	6939	6956	6972	6989	7006	7022	7039	2	3	5	7	9	10	12	14	15
84	7056	7073	7090	7106	7123	7140	7157	7174	7191	7208	2	4	5	7	9	10	12	14	15
85	7225	7242	7259	7276	7293	7310	7327	7344	7362	7379	2	4	5	7	9	10	12	14	16
86	7396	7413	7430	7448	7465	7482	7500	7517	7534	7552	2	4	5	7	9	11	12	14	16
87	7569	7586	7604	7621	7639	7656	7674	7691	7709	7726	2	4	5	7	9	11	12	14	16
88	7744	7762	7779	7797	7815	7832	7850	7868	7885	7903	2	4	5	7	9	11	13	14	16
89	7921	7939	7957	7974	7992	8010	8028	8046	8064	6082	2	4	6	7	9	11	13	14	16
90	8100	8118	8136	8154	8172	8190	8208	8226	8245	8263	2	4	6	7	9	11	13	15	16
91	8281	8299	8317	8336	8354	8372	8391	8409	8427	8446	2	4	6	7	9	11	13	15	17
92	8464	8482	8501	8519	8538	8556	8575	8593	8612	8630	2	4	6	8	9	11	13	15	17
93	8649	8668	8686	8705	8724	8742	8761	8780	8798	8817	2	4	6	8	10	11	13	15	17
94	8836	8855	8874	8892	8911	8930	8949	8968	8987	9006	2	4	6	8	10	11	13	15	17
95	9025	9044	9063	9082	9101	9120	9139	9158	9178	9197	2	4	6	8	10	12	14	15	17
96	9216	9235	9254	9274	9293	9312	9331	9351	9370	9390	2	4	6	8	10	12	14	16	18
97	9409	9428	9448	9467	9487	9506	9526	9545	9565	9584	2	4	6	8	10	12	14	16	18
98	9604	9624	9643	9663	9683	9702	9722	9742	9761	9781	2	4	6	8	10	12	14	16	18
99	9801	9821	9841	9860	9880	9900	9920	9940	9960	9980	2	4	6	8	10	12	14	16	18

Squares from 1 to 3 contain 1 figure.	Squares from 100 to 316 contain 5 figures.
,, ,, 4 to 9 ,, 2 figures.	,, ,, 317 to 999 ,, 6 ,,
,, ,, 10 to 31 ,, 3 ,,	,, ,, 1000 to 3162 ,, 7 ,,
,, ,, 32 to 99 ,, 4 ,,	,, ,, 3163 to 10000 ,, 8 ,,

The differences for squares from 3171 to 3199 are 1, 1, 2, 3, 3, 4, 5, 5, 6.

RECIPROCALS OF NUMBERS FROM 1 TO 10000.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0100	9901	9804	9709	9615	9524	9434	9346	9259	9174	9	18	27	36	45	54	63	72	81
11	9091	9009	8928	8849	8772	8696	8621	8547	8474	8403	8	16	23	31	38	46	53	61	68
12	8333	8264	8197	8130	8064	8000	7936	7874	7812	7752	6	13	19	26	32	38	45	51	57
13	7692	7633	7576	7519	7463	7407	7353	7299	7246	7194	5	11	16	22	27	32	38	43	49
14	7143	7092	7042	6993	6944	6896	6849	6803	6757	6711	5	9	14	19	23	28	33	37	42
15	6667	6622	6579	6536	6493	6452	6410	6369	6329	6289	5	9	13	17	21	25	29	34	38
16	6250	6211	6173	6135	6097	6061	6024	5988	5952	5917	4	8	11	15	19	22	26	30	33
17	5882	5848	5814	5780	5747	5714	5682	5650	5618	5586	3	6	10	13	16	19	23	26	29
18	5555	5529	5494	5464	5435	5405	5376	5347	5319	5291	3	6	8	11	14	17	20	23	26
19	5263	5236	5208	5181	5155	5128	5102	5076	5050	5025	3	5	8	10	13	16	18	21	23
20	5000	4975	4950	4926	4902	4878	4854	4831	4808	4785	2	5	7	10	12	14	17	19	21
21	4762	4739	4717	4695	4673	4651	4630	4608	4587	4566	2	4	6	9	11	13	15	17	19
22	4545	4525	4504	4484	4464	4444	4425	4405	4386	4367	2	4	6	8	10	11	13	15	17
23	4348	4329	4310	4292	4273	4255	4237	4219	4202	4184	2	3	5	7	9	11	12	14	16
24	4167	4149	4132	4115	4098	4082	4065	4048	4032	4016	2	4	5	7	9	10	12	14	15
25	4000	3984	3968	3952	3937	3921	3906	3891	3876	3861	1	3	4	6	7	9	10	12	13
26	3846	3831	3817	3802	3788	3773	3759	3745	3731	3717	1	2	4	5	7	8	9	11	12
27	3704	3690	3676	3663	3650	3636	3623	3610	3597	3584	1	2	4	5	6	8	9	10	12
28	3571	3559	3546	3533	3521	3509	3496	3484	3472	3460	2	3	4	5	6	8	9	10	11
29	3448	3436	3378	3413	3401	3390	3378	3367	3356	3344	1	3	4	5	6	7	8	9	11
30	3333	3322	3311	3300	3289	3279	3268	3257	3247	3237	1	3	4	5	6	7	8	9	10
31	3226	3215	3205	3195	3185	3175	3164	3154	3145	3135	2	3	4	5	6	7	8	9	10
32	3125	3115	3105	3096	3086	3077	3067	3058	3049	3039	1	2	3	4	5	6	7	8	9
33	3030	3021	3012	3003	2994	2985	2976	2967	2958	2950	1	2	3	4	4	5	6	7	8
34	2941	2932	2924	2915	2907	2898	2890	2882	2873	2865	0	1	2	3	4	5	5	6	7
35	2857	2849	2841	2839	2825	2817	2809	2801	2793	2785	1	2	3	3	4	5	6	7	7
36	2778	2770	2762	2755	2747	2740	2732	2725	2717	2710	1	2	3	3	4	5	6	6	7
37	2703	2695	2688	2681	2674	2667	2659	2652	2645	2638	1	2	3	3	4	5	5	6	7
38	2631	2625	2618	2611	2604	2597	2591	2584	2577	2571	0	1	2	2	3	4	4	5	6
39	2564	2557	2551	2544	2538	2532	2525	2519	2512	2506	1	2	2	3	4	4	5	6	6
40	2500	2494	2487	2481	2475	2469	2463	2457	2451	2445	1	1	2	2	3	4	4	5	5
41	2439	2433	2427	2421	2415	2410	2404	2398	2392	2387	1	2	2	3	3	4	5	5	6
42	2381	2375	2370	2364	2358	2353	2347	2342	2336	2331	1	1	2	2	3	3	4	5	5
43	2325	2320	2315	2309	2304	2299	2293	2288	2283	2278	1	1	2	2	3	3	4	4	5
44	2273	2267	2262	2257	2252	2247	2242	2237	2232	2227	0	1	1	2	2	3	3	4	4
45	2222	2217	2212	2207	2203	2198	2193	2188	2183	2179	1	1	2	2	3	3	4	4	5
46	2174	2169	2164	2160	2155	2150	2146	2141	2137	2132	0	0	1	1	2	2	3	3	4
47	2128	2123	2119	2114	2110	2105	2101	2096	2092	2088	0	1	1	2	2	2	3	3	4
48	2083	2079	2075	2070	2066	2062	2058	2053	2049	2045	1	1	2	2	2	3	3	4	4
49	2041	2037	2032	2028	2024	2020	2016	2012	2008	2004	0	1	1	2	2	2	3	3	4
50	2000	1996	1992	1988	1984	1980	1976	1972	1968	1965	0	1	1	1	2	2	3	3	3
51	1961	1957	1953	1949	1945	1942	1938	1934	1930	1927	1	1	1	2	2	3	3	3	4
52	1923	1919	1916	1912	1908	1905	1901	1897	1894	1890	1	1	1	2	2	3	3	3	4
53	1887	1883	1880	1876	1873	1869	1866	1862	1859	1855	0	1	1	1	2	2	2	3	3
54	1852	1848	1845	1842	1838	1835	1831	1828	1825	1821	1	1	1	2	2	2	3	3	3

Reciprocals from 2 to 10 = 0·
" " 11 to 100 = 0·0
Reciprocals from 101 to 1000 = 0·00
" " 1001 to 10000 = 0·000

Numbers in difference columns to be subtracted, not added.

RECIPROCALS OF NUMBERS FROM 1 TO 10000.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	1818	1815	1811	1808	1805	1802	1798	1795	1792	1789	1	1	1	2	2	2	3	3	3
56	1786	1782	1779	1776	1773	1770	1767	1764	1760	1757	1	1	1	1	2	2	2	3	3
57	1754	1751	1748	1745	1742	1739	1736	1733	1730	1727	0	1	1	1	1	2	2	2	3
58	1724	1721	1718	1715	1712	1709	1706	1703	1701	1698	0	0	1	1	1	1	2	2	2
59	1695	1692	1689	1686	1683	1681	1678	1675	1672	1669	1	1	1	2	2	2	2	3	3
60	1667	1664	1661	1658	1656	1653	1650	1647	1645	1642	0	1	1	1	2	2	2	2	3
61	1639	1637	1634	1631	1629	1626	1623	1621	1618	1615	0	1	1	1	1	2	2	2	2
62	1613	1610	1608	1605	1602	1600	1597	1595	1592	1590	0	1	1	1	1	2	2	2	2
63	1587	1585	1582	1580	1577	1575	1572	1570	1567	1565	1	1	1	1	2	2	2	2	3
64	1562	1560	1558	1555	1553	1550	1548	1545	1543	1541	0	0	0	1	1	1	1	2	2
65	1538	1536	1534	1531	1529	1527	1524	1522	1520	1517	1	1	1	1	2	2	2	2	2
66	1515	1513	1510	1508	1506	1504	1501	1500	1497	1495	1	1	1	1	1	2	2	2	2
67	1492	1490	1488	1486	1484	1481	1479	1477	1475	1473	0	0	0	0	1	1	1	1	2
68	1470	1468	1466	1464	1462	1460	1458	1456	1453	1451	0	1	1	1	1	2	2	2	2
69	1449	1447	1445	1443	1441	1439	1437	1435	1433	1431	0	1	1	1	1	1	2	2	2
70	1428	1426	1424	1422	1420	1418	1416	1414	1412	1410	0	0	0	0	1	1	1	1	1
71	1408	1406	1404	1402	1400	1399	1397	1395	1393	1391	1	1	1	1	1	2	2	2	2
72	1389	1387	1385	1383	1381	1379	1377	1375	1374	1372	0	0	0	1	1	1	1	1	1
73	1370	1368	1366	1364	1362	1360	1359	1357	1355	1353	0	0	0	0	0	1	1	1	1
74	1351	1349	1348	1346	1344	1342	1340	1339	1337	1335	0	0	0	1	1	1	1	1	1
75	1333	1331	1330	1328	1326	1324	1323	1321	1319	1317	0	0	0	0	0	1	1	1	1
76	1316	1314	1312	1311	1309	1307	1305	1304	1302	1300	0	0	0	1	1	1	1	1	1
77	1299	1297	1295	1294	1292	1290	1289	1287	1285	1284	0	0	0	0	1	1	1	1	1
78	1282	1280	1279	1277	1275	1274	1272	1271	1269	1267	0	1	1	1	1	1	1	2	2
79	1266	1264	1263	1261	1259	1258	1256	1255	1253	1251	0	1	1	1	1	1	1	2	2
80	1250	1248	1247	1245	1244	1242	1241	1239	1238	1236	0	0	0	0	1	1	1	1	1
81	1234	1233	1231	1230	1228	1227	1225	1224	1222	1221	0	0	1	1	1	1	1	1	1
82	1219	1218	1216	1215	1213	1212	1211	1209	1208	1206	0	0	0	1	1	1	1	1	1
83	1205	1203	1202	1200	1199	1198	1196	1195	1193	1192	1	1	1	1	1	1	1	2	2
84	1190	1189	1188	1186	1185	1183	1182	1181	1179	1178	0	0	0	0	0	1	1	1	1
85	1176	1175	1174	1172	1171	1169	1168	1167	1165	1164	0	0	0	0	0	0	0	1	1
86	1163	1161	1160	1159	1157	1156	1155	1153	1152	1151	0	0	0	1	1	1	1	1	1
87	1149	1148	1147	1145	1144	1143	1141	1140	1139	1138	0	1	1	1	1	1	1	1	1
88	1136	1135	1134	1132	1131	1130	1129	1127	1126	1125	0	0	1	1	1	1	1	1	1
89	1123	1122	1121	1120	1118	1117	1116	1115	1113	1112	0	0	0	0	0	1	1	1	1
90	1111	1110	1109	1107	1106	1105	1104	1102	1101	1100	0	0	0	1	1	1	1	1	1
91	1099	1098	1096	1095	1094	1093	1092	1090	1089	1088	0	0	1	1	1	1	1	1	1
92	1087	1086	1084	1083	1082	1081	1080	1079	1077	1076	0	0	0	0	1	1	1	1	1
93	1075	1074	1073	1072	1071	1069	1068	1067	1066	1065	0	0	0	0	0	0	0	0	1
94	1064	1063	1061	1060	1059	1058	1057	1056	1055	1054	0	0	0	0	0	1	1	1	1
95	1053	1051	1050	1049	1048	1047	1046	1045	1044	1043	0	0	0	0	1	1	1	1	1
96	1042	1040	1039	1038	1037	1036	1035	1034	1033	1032	0	0	0	0	0	0	1	1	1
97	1031	1030	1029	1028	1027	1026	1024	1023	1022	1021	1	1	1	1	1	1	1	1	1
98	1020	1019	1018	1017	1016	1015	1014	1013	1012	1011	0	0	0	0	0	0	1	1	1
99	1010	1009	1008	1007	1006	1005	1004	1003	1002	1001	0	0	0	0	1	1	1	1	1

Reciprocals from 2 to 10 = 0·
,, ,, 11 to 100 = 0·0

Reciprocals from 101 to 1000 = 0·00
,, ,, 1001 to 10000 = 0·000

Numbers in difference columns to be subtracted, not added.

NATURAL TANGENTS.

	·0°	·1°	·2°	·3°	·4°	·5°	·6°	·7°	·8°	·9°
0°	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662
41	8693	8724	8754	8785	8816	8847	8878	8910	8941	8972
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965

Natural Tangents.

	·0°	·1°	·2°	·3°	·4°	·5°	·6°	·7°	·8°	·9°
45°	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542
63	1·9626	9711	9797	9883	9970	0057	0145	0233	0323	0413
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878
71	2·9042	9208	9375	9544	9714	9887	0061	0237	0415	0595
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646
78	4·7046	7453	7867	8288	8716	9152	9594	0045	0504	0970
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140
80	5·6713	7297	7894	8502	9124	9758	0405	1066	1742	2432
81	6·3138	3859	4596	5350	6122	6912	7920	8548	9395	0264
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	0285
83	8·1443	2636	3863	5126	6427	7769	9152	0579	2052	3572
84	9·5144	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0

NATURAL SINES.

	·0°	·1°	·2°	·3°	·4°	·5°	·6°	·7°	·8°	·9°
0°	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332
2	0349	0366	0384	0401	0419	0436	0454	9471	0488	0506
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854
5	0872	0889	0906	0924	0941	0958	0976	0993	1011	1028
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719
10	1736	1754	1771	1788	1805	1822	1840	1857	1874	1891
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2232
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571
15	2588	2605	2622	2639	2656	2672	2689	2706	2723	2740
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239
19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404
20	3420	3437	3453	3469	3486	3502	3518	3535	3551	3567
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730
22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891
23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051
24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210
25	4226	4242	4258	4274	4289	4305	4321	4337	4352	4368
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833
29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985
30	5000	5015	5030	5045	5060	5075	5090	5105	5120	5135
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284
32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432
33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577
34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721
35	5736	5750	5764	5779	5793	5807	5821	5835	5850	5864
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004
37	6018	6032	6046	6060	6074	6088	6101	6115	6129	6143
38	6157	6170	6184	6198	6211	6225	6239	6252	6266	6280
39	6293	6307	6320	6334	6347	6361	6374	6388	6401	6414
40	6428	6441	6455	6468	6481	6494	6508	6521	6534	6547
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934
44	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059

NATURAL SINES.

	·0°	·1°	·2°	·3°	·4°	·5°	·6°	·7°	·8°	·9°
45	7071	7083	7096	7108	7120	7133	7145	7157	7169	7181
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302
47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420
48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536
49	7547	7558	7570	7581	7593	7604	7615	7627	7638	7649
50	7660	7672	7683	7694	7705	7716	7727	7738	7749	7760
51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869
52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976
53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080
54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181
55	8192	8202	8211	8221	8231	8241	8251	8261	8271	8281
56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377
57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471
58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652
60	8660	8669	8678	8686	8695	8704	8712	8721	8729	8738
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056
65	9063	9070	9078	9085	9092	9100	9107	9114	9121	9128
66	9135	9143	9150	9157	9164	8171	9178	9184	9191	9198
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391
70	9397	9403	9409	9415	9421	9426	9432	9438	9444	9449
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505
72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558
73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608
74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655
75	9659	9664	9668	9673	9677	9681	9686	9690	9694	9699
76	9703	9707	9711	9715	9720	9724	9728	9732	9736	9740
77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813
79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845
80	9848	9851	9854	9857	9860	9863	9866	9869	9871	9874
81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900
82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923
83	9925	9928	9930	9932	9934	9936	9938	9940	9942	9943
84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960
85	9962	9963	9965	9966	9968	9969	9971	9972	9973	9974
86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985
87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993
88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9998
89	9998	9999	9999	9999	9999	1·000 nearly	1·000 nearly.	1·000 nearly.	1·000 nearly.	1·000 nearly.

CONVERSION TABLES, CONSTANTS, ETC.

To convert inches to centimetres	×	2·54
,, centimetres to inches	×	0·3937
,, grains to grammes	×	0·0648
,, grammes to grains	×	15·432
,, oz. (avoir.) to grammes	×	28·35
,, grammes to oz. (avoir.)	×	0·0353
,, lbs. to grammes	×	453·59
,, grammes to lbs.	×	0·0022
,, gallons to c.c.	×	4541

The weight of 1 grain	=	63·57	dynes
,, 1 oz.	=	$2{\cdot}78 \times 10^4$	,,
,, 1 lb.	=	$4{\cdot}45 \times 10^5$	,,
,, 1 gramme	=	981	,,

1 foot-pound	=	$1{\cdot}356 \times 10^7$	ergs
1 kilogrammetre	=	$9{\cdot}81 \times 10^7$	,,
1 joule	=	10^7	,,
1 Hp.	=	$7{\cdot}46 \times 10^9$	ergs per sec.
1 watt (volt ampère)	=	10^7	,,

To convert common into Naperian logs × 2·3026
,, Naperian into common ,, × 0·4343

"g" (Manchester)	=	981·34
Latitude (Manchester)	=	53° 29′
Length of seconds pendulum (Manchester)	=	99·430 cm.
π	=	3·1416
Area of a circle	=	0·7854 (diam.)^2

TABLE OF SPECIFIC GRAVITIES.

Substance.	Specific gravity.	Substance.	Specific gravity.
Platinum	21·5	Sulphur	2·0
Mercury at 0° C. ...	13·596	Ivory	1·9
Lead	11·3	Sand	1·42
Silver	10·5	Hooper's I.R. ..	1·18
Nickel	8·9	Ebony	1·1–1·2
Copper	8·5–8·9	Ebonite	1·15
German silver ...	8·5	Boxwood	0·91–1·03
Brass	8·1–8·6	Oak	0·7–1·0
Steel (cast)	7·8	Guttapercha ...	0·97–0·98
Iron (wrought) ...	7·8	Wax	0·96
,, (wire)	7·7	Indiarubber (pure)	0·93
,, (cast)	7·1–7·6	Cork	0·24
Tin	7·3	H_2SO_4 at 0° C.	1·85
Zinc	7·1	HNO_3 ,,	1·56
Glass (flint)	3·0–3·5	HCl ,,	1·27
,, (crown) ...	2·5–2·7	CS_2 ,,	1·293
Marble	2·5–2·8	Glycerine ,,	1·27
Aluminium	2·6	Linseed oil ,,	0·94
Porcelain	2·4	Oil of turpentine ,,	0·87
Chalk	1·8–2·8	Alcohol ,,	0·806
Carbon (graphite) ...	2·3	Mineral oil ,,	0·76–0·85
Gas carbon	1·9	Ether ,,	0·736
Brick	2·1		

DENSITY OF WATER AT DIFFERENT TEMPERATURES.

4° C. = 1·00000
6° C. = 0·99997
10° C. = 0·99974
12° C. = 0·99955
14° C. = 0·99930

15° C. = 0·99915
17° C. = 0·99884
20° C. = 0·99827
25° C. = 0·99713
30° C. = 0·99577

MANCHESTER, 1900 A.D.

Dip = 69°
Declination = 18° West of North
H = 0·1712

TABLE OF SPECIFIC GRAVITIES OF AQUEOUS SOLUTIONS AT 15° C.

Weight of su's. in 100 parts by weight of solution.	KOH	KCl	KNO_3	$K_2Cr_2O_7$	NH_3	NH_4Cl	$NaCl$	Na_2CO_3	$CaCl_2$	$ZnSO_4$	$CuSO_4$	HCl	HNO_3	H_2SO_4	Weight of subs. in 100 parts by weight of solution.
0	0·999	0·999	0·999	0·999	0·999	0·999	0·999	0·999	0·999	0·999	0·999	0·999	0·999	0·999	0
5	1·045	1·0316	1·031	1·036	0·978	1·0149	1·035	1·052	1·042	1·052	1·050	1·0242	1·029	1·0334	5
10	1·092	1·0649	1·064	1·072	0·958	1·0299	1·072	1·105	1·086	1·108	1·103	1·0490	1·058	1·0687	10
15	·441	1·0994	1·099	1·1[illegible]9	0·941	1·0443	1·110	—	1·133	1·168	1·161	1·0744	1·089	1·1048	15
20	1·191	1·1351	1·135	—	0·924	1·0584	1·150	—	1·181	1·236	—	1·1001	1·121	1·1430	20
25	1·242	—	—	—	0·910	1·0721	1·191	—	1·23[illegible]	1·307	—	1·1262	1·154	1·1816	25
30	1·295	—	—	—	0·897	—	—	—	1·286	1·382	—	1·1524	1·187	1·223	30
35	1·349	—	—	—	0·885	—	—	—	1·343	—	—	1·1775	1·220	1·264	35
40	1·406	—	—	—	—	—	—	—	1·382	—	—	1·2007	1·253	1·307	40
45	1·466	—	—	—	—	—	—	—	—	—	—	—	1·287	1·352	45
50	1·528	—	—	—	—	—	—	—	—	—	—	—	1·320	1·399	50
55	—	—	—	—	—	—	—	—	—	—	—	—	1·350	1·449	55
60	—	—	—	—	—	—	—	—	—	—	—	—	1·377	1·503	60
65	—	—	—	—	—	—	—	—	—	—	—	—	1·402	1·559	65
70	—	—	—	—	—	—	—	—	—	—	—	—	1·424	1·616	70
75	—	—	—	—	—	—	—	—	—	—	—	—	1·443	1·675	75
80	—	—	—	—	—	—	—	—	—	—	—	—	1·461	1·733	80
85	—	—	—	—	—	—	—	—	—	—	—	—	1·479	1·785	85
90	—	—	—	—	—	—	—	—	—	—	—	—	1·497	1·819	90
95	—	—	—	—	—	—	—	—	—	—	—	—	1·514	1·839	95
100	—	—	—	—	—	—	—	—	—	—	—	—	1·530	1·838	100

Wire Table No. 1.

Gauge.	Diameter.		Copper.			Manganin.		
B.W.G.	Inches.	mm.	Yds per lb. (bare).	Ohms per lb. (bare).	Ohms per yd	Yds per lb. (bare).	Ohms per lb. (bare).	Ohms per yd.
10	0·134	3·403	6·1	0·01057	0·0017	—	—	—
12	0·109	2·770	9·3	0·02416	0·0025	—	—	—
14	0·083	2·108	16·0	0·07186	0·0045	15·98	1·9545	0·0971
16	0·065	1·651	26·0	0·19104	0·0073	26·06	5·1964	0·1994
18	0·049	1·244	46·0	0·59157	0·0129	45·86	16·0907	0·3508
20	0·035	0·889	89·9	2·2725	0·0253	89·89	61·8131	0·4109
22	0·028	0·711	140·4	5·5484	0·0397	140·44	150·918	1·075
24	0·022	0·560	227·5	14·557	0·0643	227·53	395·97	1·740
26	0·018	0·457	339·9	32·486	0·0961	339·83	88[illegible]·63	2·606
28	0·014	0·355	561·8	88·772	0·1580	561·83	2414·61	4·296
30	0·012	0·304	764·7	164·46	0·2150	761·71	4473·36	5·849
32	0·011	0·279	908·0	234·24	0·2574	910·06	6372·67	7·001
34	0·0095	0·241	1226·0	421·07	0·3451	1220·14	11455·1	9·389
36	0·0079	0·200	1760·0	880·62	0·4991	1764·42	23954·3	13·58
38	0·0068	0·172	2375·0	1604·1	0·6736	2381·22	43631·5	18·32
40	0·0058	0·147	3190·0	3031·1	0·9260	3273·41	82445·9	25·19

Wire Table No. 2.

Gauge.	Diameter.		German silver.			Platinoid.		
L.S.G.	Inches.	mm.	Yds. per lb. (bare).	Ohms per lb. (bare).	Ohms per yd.	Yds per lb. (bare).	Ohms per lb. (bare).	Ohms per yd.
10	0·128	3·25[illegible]	6·7	0·167	0·0249	6·7	0·3031	0·0450
12	0·104	2·642	10·2	0·384	0·0377	10·2	0·6957	0·0682
14	0·080	2·032	17·2	1·09	0·0637	17·2	1·9869	0·1154
16	0·064	1·626	26·9	2·67	0·0995	26·9	4·8520	0·1803
18	0·048	1·219	47·9	8·46	0·1769	47·9	15·3312	0·3206
20	0·036	0·914	72·5	26·75	0·3146	72·5	48·46	0·5699
22	0·028	0·711	85·0	73·11	0·5201	85·0	132·43	0·9422
24	0·022	0·559	140·5	191·8	0·8435	140·5	347·47	1·5261
26	0·018	0·457	227·7	428·1	1·2586	227·7	775·37	2·2798
28	0·0148	0·376	340·2	936·6	1·8517	340·2	1696·49	3·3722
30	0·0124	0·315	503·2	1900·0	2·6521	503·2	3442·80	4·8039
32	0·0108	0·274	946·1	3302·0	3·4962	946·1	5982·72	6·3329
34	0·0092	0·233	1302·0	6272·0	4·8181	1302·0	11362·0	8·7271
36	0·0076	0·193	1908·0	13469·0	7·0603	1908·0	24398·0	12·7896
38	0·0060	0·152	3058·0	34673·0	11·327	3058·0	62805·0	20·5185
40	0·0048	0·122	4785·0	84652·0	17·699	4785·0	153333·0	32·0601

A single cotton covering adds 0·006 to 0·008 inch to the diameter
A double „ „ „ 0·014 „ „ „
A single silk „ „ 0·002 to 0·003 „ „ „
A double „ „ „ 0·004 to 0·006 „ „ „

TABLE OF SPECIFIC RESISTANCES.

Material.	Remarks.	Density.	Specific resistance in C.G.S. units.	Temperature variation. α	Temperature variation. β	Authority.
Copper	Hard-drawn	8·95	1642	0·00388	+0·00000126	Mattheissen
„	—	9·0	1504	0·00382	+0·00000126	„
„	—	—	—	0·00367	+0·00000058	Benoit
„	Electrolytic	9·05	1527	0·00445	—	Lagarde
„	Expts. between −23° C. to −123° C.	—	—	0·00423	—	Cailletet and Bouty
„	Annealed (Same material)	8·86	1743	—	—	Fitz Patrick
„	Annealed (Same material)	8·87	1745	—	—	„
„	Hard-drawn (Same material)	8·89	1776	—	—	„
„	Hard-drawn (Same material)	8·89	1770	—	—	„
„	Annealed	8·89	1713	—	—	„
„	Hard-drawn	8·94	1731	—	—	„
„	Hard-drawn electrolytic	8·91	1766	—	—	„
„	Electrolytic +200° to −200° C.	—		0·00410	—	Dewar and Fleming
„	Electrolytic hard-drawn	8·958	1603	0·00408	—	Swan and Rhodin
„	„ soft	8·958	1566	0·00418	—	„ „
„	„ soft	8·958	1566	0·00415	—	„ „
Platinum ...	—	21·5	8957	0·003824	+0·00000126	Mattheissen
„	—	—	—	0·00354	—	Dewar and Fleming
„	—	—	—	0·00340	—	Cailletet and Bouty
„	—	—	—	0·003448	−0·000000533	Callendar
Silver	Annealed	10·5	1488	0·00377	—	Mattheissen
„	Hard-drawn	—	1616	0·00377	—	„
„	Chemically pure	—	—	0·00384	—	Dewar and Fleming
„	Between +30° to −102° C.	—	—	0·00385	—	Cailletet and Bouty
„	—	—	—	0·003972	+0·000000687	Benoit
Mercury	Liquid	13·595	94070	0·0007485	−0·000000398	Mattheissen
„	Between 0° to 12° C.	—	—	0·000861	—	Rayleigh
„	„ −40° to −92° C.	—	—	0·00407	—	Cailletet and Bouty
„	„ 0° to 28° C.	—	—	0·0008827	+0·00000126	Kreichgauer and Jaegar

,, ...	—	—	—	0·0008767	+0·00000106	Guillaume
Iron	Annealed	7·7	9611	0·005425	+0·0000083	Mattheissen
,, ...	—	—	—	0·005131	+0·00000815	Tomlinson
,, ...	Between 0° to −92° C.	—	—	0·00490	—	Cailletet and Bouty
,, ...	—	—	—	0·004516	+0·0000058	Benoit
Platinum silver ...	2 parts Pt, 1 part Ag, by weight	—	24120	0·00031	—	Mattheissen
,, ,, ...		—	31582	0·00024	—	Dewar and Fleming
,, ,, ...	—	—	—	0·0002327	+0·000000033	Callendar
German silver ...	—	—	20710	0·000443	+0·000000152	Mattheissen
,, ,, ...	Cu 60 %, Zn 25·3 %, Ni 14 %, Fe 0·3 %	—	30000	0·00036	—	St. Lindeck
,, ,, ...	—	—	29982	0·000273	—	Dewar and Fleming
Platinoid ...	—	—	29000	0·00022	—	Bottomley
,, ...	—	—	37000	0·00022	—	,,
,, ...	—	—	41731	0·00031	—	Dewar and Fleming
,, ...	—	—	—	0·00026	—	Elliotts
Manganese Copper	Cu 70 %, Mn 30 %	—	100600	0·00004	—	St. Lindeck
Manganin ...	Cu 84 %, Mn 12 %, Ni 4 %	—	42000	—	—	,,
,, ...	10° to 20°	—	—	0·000025	—	,,
,, ...	20° to 30°	—	—	0·000014	—	,,
,, ...	30° to 35°	—	—	0·000004	—	,,
,, ...	35° to 40°	—	—	0·000003	—	,,
,, ...	40° to 45°	—	—	0·000001	—	,,
,, ...	45° to 50°	—	—	−0·000001	—	,,
,, ...	50° to 55°	—	—	−0·000002	—	,,
,, ...	55° to 60°	—	—	−0·000004	—	,,
,, ...	Cu 73 %, Mn 24 %, Ni 3 %	—	47700	0·000003	—	,,
,, ...	—	—	46678	—	—	Dewar and Fleming

Mattheissen, *Phil. Mag.*, May, 1865; Benoit, *Comptes Rendus*, vol. lxvi. p. 343; La Garde, *Annales Telegraphiques*, vol. xx. p. 235; Guillaume, *Elect.*, vol. xxix. p. 553; Cailletet and Bouty, *Jour. de Phys.*, July, 1885; Fitz Patrick, *B.A. Report*, 1890; Dewar and Fleming, *Phil. Mag.*, October, 1892; Tomlinson, *Elect.*, vol. xxiv. p. 97; Bottomley, *Pro. Roy. Soc.*, May, 1885; Swan and Rhodin, *Pro. Roy. Soc.*, July, 1894; Callendar, *Phil. Trans.*, 1887, p. 261; Kreichgauer and Jaegar, *Elect.*, vol. xxx. p. 567; St. Lindeck, *Elect.*, vol. xxvi. p. 493, and *B.A. Report*, 1892.

TABLE OF SPECIFIC RESISTANCES OF LIQUIDS.

Liquid.	Density.	Temperature.	Specific resistance (ohms per c.c.)	Authority.
H_2SO_4	0·9985	22° C.	70·41	Kohlrausch *
,, ...	1·0000	,,	41·05	,,
,, ...	1·0504	,,	3·25	,,
,, ...	1·0989	,,	1·787	,,
,, ...	1·1430	,,	1·414	,,
,, ...	1·2045	,,	1·239	,,
,, ...	1·3163	,,	1·347	,,
,, ...	1·3994	,,	1·672	,,
,, ...	1·4482	,,	1·962	,,
,, ...	1·5026	...	2·412	,,
$CuSO_4$	1·0167	10° C.	164·4	Ewing and Macgregor †
,, ...	1·0216	,,	134·8	,, ,,
,, ...	1·0318	,,	98·7	,, ,,
,, ...	1·0622	,,	59·0	,, ,,
,, ...	1·1174	,,	38·0	,, ,,
,, (saturated)	1·2054	,,	29·0	,, ,,
$ZnSO_4$,,	1·4220	,,	33·7	,, ,,
HCl	1·109	,,	1·31	,, ,,
HNO_3	1·185	,,	1·28	,, ,,

TABLE OF SPECIFIC DIELECTRIC RESISTANCES.

Material.	Specific resistance.	Density.	Temp.	Authority.
Glass (Bohemian)	$4·25 \times 10^{22}$	2·427	60° C.	Gray
,, ,,	$7·15 \times 10^{22}$	2·587	60°	,,
,, (test-tube)	$1·44 \times 10^{20}$	2·435	60°	,,
,, ,, ...	$3·50 \times 10^{20}$	2·44	60°	,,
,, (flint) ...	$3·89 \times 10^{22}$	2·753	60°	,,
,, ,, ...	$1·02 \times 10^{24}$	3·172	60°	,,
Mica	$8·40 \times 10^{22}$	—	20°	Ayrton and Perry
Guttapercha ...	$4·50 \times 10^{23}$	0·97 to 0·98	24°	Latimer Clark
Shellac	$9·00 \times 10^{24}$	—	28°	Ayrton and Perry
Ebonite	$2·80 \times 10^{25}$	1·15	46°	,, ,,
Paraffin	$3·40 \times 10^{25}$	—	46°	,, ,,
Indiarubber ...	$1·09 \times 10^{25}$	—	24°	Jenkin
Hooper's core ...	$1·50 \times 10^{25}$	—	24°	,,
Parchment ...	$3·00 \times 10^{20}$	—	20°	Uppenborn
Ordinary paper	$4·85 \times 10^{18}$	—	20°	,,

* Kohlrausch and Nippoldt, "Leilfaden der Praktschen Physick, p. 298.

† Ewing and Macgregor, *Trans. Roy. Soc.*, Edinburgh, vol. xxvii., 1873.

TABLE OF DIELECTRIC RESISTANCE AND CAPACITY OF G.P. COVERED WIRE AFTER ONE MINUTE ELECTRIFICATION.

External diameter / Internal diameter.	Resistance per mile in megohms.	Capacity per mile in microfarads.
2·5	366·1	0·394
2·6	384·8	0·375
2·7	396·9	0·363
2·8	411·4	0·351
2·9	425·4	0·339
3·0	439·0	0·329
3·5	500·6	0·288
4·0	553·9	0·260

TABLE OF ELECTRO-MOTIVE FORCES OF CELLS.

Cell.	E.M.F. (volts).	Cell.	E.M.F. (volts).
Grove	1·9 to 1·95	E.C.C. ...	1·45
Bunsen ...	1·9 to 1·95	Obach ...	1·5
Daniell ...	1·07 to 1·14	Lessing ...	1·47
Bichromate ..	2	Secondary ...	1·85 to 2·1
Leclanche ...	1·5	Clark... ...	1·434
Gassner ...	1·3	Cadmium ...	1·019
Hellesen ...	1·45		

TABLE OF CONTACT DIFFERENCES OF POTENTIAL.

Metals.	Difference of potential volts at 18° C.
Copper to iron... ...	0·146
Copper to platinum ...	−0·238
Copper to zinc... ...	0·750
Copper to brass ...	0·087
Platinum to brass ...	0·287

Table of Potential Difference required to Spark between two Parallel Plates.

Distance (in cms.).	E.M.F. (volts).	Distance. (in cms.).	E.M.F. (in volts).
0·0205	1000	0·1800	7000
0·0430	2000	0·2146	8000
0·0660	3000	0.2495	9000
0 0914	4000	0·2863	10000
0·1176	5000	0·3245	11000
0·1473	6000	0·3378	11330

Table of Electro-Chemical Equivalents.

Element.	Grammes per coulomb.	Element.	Grammes per coulomb.
Silver	0·001118	Lead	0·001071
Copper (cupric)	0·0003279	Tin (stannic) ...	0·000304
,, (cuprous)	0·000655	,, (stannous)	0·000609
Iron (ferric) ...	0·0001934	Hydrogen ...	0·00001035
,, (ferrous) ...	0·0002900	Oxygen ...	0·00008286
Nickel	0·0003042	Water	0·00009321
Zinc	0·0003367	Iodine	0·0013134

Element.	Cub. cm. per coulomb at 0° C. and 760 mm.
Hydrogen	0·1156
Oxygen	0·0578
Water	0·1734

TABLE OF SPECIFIC INDUCTIVE CAPACITIES.

Material.	Density.	S.I.C.	Authority.
Glass (flint)	4·5	9·896	Hopkinson, *Phil. Trans.*, 1881
" "	3·66	7·376	" " "
" "	3·20	6·72	" " "
" "	2·87	6·61	" " "
" (crown)	2·48	6·96	" " "
" (plate)	—	8·45	" " "
Mica	—	6·64	Klemencic, *Beiblätter*, vol. xii., 1888
Ebonite	1·15	{4·2 2·284	Faraday Gordon, *Phil. Trans.*, 1879
Guttapercha (best) ...	0·91 to 0·98	2·462	" " "
Indiarubber (black) ...	0·93	2·220	" " "
" (grey vulcanized)	—	2·497	" " "
Paraffin wax (M.P. 68° C.)	0·9109	1·9936	" " "
" "	0·9080	1·977	Gibson and Barclay, *Phil. Trans.*, 1871
Shellac	—	2·740	Gordon, *Phil. Trans.*, 1879
Petroleum spirit	—	1·92	Hopkinson, *Phil. Trans.*, 1881
" oil	0·88	2·10	" " "
Turpentine	0·87	2·23	" " "
Castor oil	—	4·78	" " "
Sperm oil	—	3·02	" " "
Olive oil	0·91	3·16	" " "
Benzene	0·85	2·198	Silow, *Pogg. Ann.*, 1875
Water (distilled)	1·00	76·0	Quincke, *Wied. Ann.*, 1888
Alcohol (ethyl)	0·80	26·5	" " "

TABLE OF COEFFICIENTS OF CUBICAL EXPANSION.

Material.	Coefficient.
Glass	0·0000258
Alcohol	0·0011
Water at 5° C. ...	0·000022
" at 100° C ...	0·00755
Mercury	0·00018

TABLE OF SPECIFIC HEATS.

Substance.	Specific heat.	Substance.	Specific heat.
Brass	0·094	Glass	0·19
Copper	0·094	Alcohol	0·58
Zinc	0·094	Mercury	0·034
Iron	0·113	Turpentine ..	0·43
Silver	0·057	Paraffin oil ...	0·434
Platinum 0° to 100°	0·0335	Aniline	0·49
" 0° to 300°	0·0343	Water 0° to 40° ...	1·0013
" 0° to 500°	0·0352	" 0° to 80° ...	1·0035
" 0° to 1000°	0·0373		

TABLE OF MELTING-POINTS.

Substance.	Melting-point.	Substance.	Melting-point.
Platinum	1775° C.	Zinc	412° C.
Iron	1600°	Lead	326°
Nickel	1450°	Cadmium	315°
Steel	1370°	Bismuth	260°
Copper	1054°	Tin	230°
Glass	1100°	Selenium	217°
Silver	954°	Sulphur	115°
Aluminium	600°	Paraffin	54°
Antimony	432°		

TABLE OF HEAT CONSTANTS.

Latent heat of water (gramme degree Centigrade) = 80
,, ,, ,, (pound ,, Fahrenheit) = 143
,, ,, steam (gramme ,, Centigrade) = 536
,, ,, ,, (pound ,, Fahrenheit) = 966

Mechanical equivalent of heat = 772 ft.-lbs. per ° F.
,, ,, ,, = 1390 ,, ,, ° C.
,, ,, ,, = 42400 gramme cms. per ° C.

TABLE OF INDICES OF REFRACTION FOR SODIUM LIGHT.

Material.	Density.	Index.
Glass (crown)... ...	2·5	1·51
,, (flint)	2·8 to 4·2	1·54 to 1·71
Water	1·0	1·33
Carbon bisulphide ...	1·27	1·6303
Benzol	0·877	1·4972
Alcohol	0·800	1·3633
Ether	0·713	1·3594
Chloroform	1·501	1·4492

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THE END

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